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A note on the problem of semiglobal practical stabilization of uncertain nonlinear systems via dynamic output feedback

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Abstract

It is known that if a system can be (robustly) globally asymptotically stabilized by means of a feedback that is driven by functions that are *uniformly completely observable* (UCO), then this system can be practically semiglobally stabilized by means of (possibly dynamic) output feedback. This papers discusses a significant structural hypothesis under which the existence of a dynamic feedback driven by UCO functions is guaranteed. The class of systems which satisfy this hypothesis includes any stabilizable and detectable linear system and any relative degree one nonlinear system which is stabilizable by dynamic output feedback. In particular, the hypothesis does not require the system to be minimum phase. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

One of the most active research issues in nonlinear feedback theory is the synthesis of feedback laws which robustly stabilize an uncertain system with limited measurement information. In the case of output feedback without uncertainty, one of the major achievements in this area of research has been the "nonlinear separation principle" proved in [6], where it is shown that (semi)global stabilizability via state feedback and a property of uniform observability imply the possibility of semiglobal stabilization via output feedback. To cope with the restricted information structure, the stabilization scheme of [6] includes an approximate state observer (whose role is actually that of producing approximate estimates of a number of "higher-order" derivatives of the output) earlier developed in [3] to cope with a similar (though more restricted) stabilization problem. A "robust" version of this stabilization result was given in [5], where it was shown that, in the presence of parameter uncertainties, semiglobal stabilization via output feedback is still possible if a state feedback law is known which robustly globally stabilizes the system and its value, at any time, can be expressed as a (fixed) function of the values, at this time, of a fixed number of derivatives of input and output (a *uniformly completely observable* (UCO) state feedback, in the terminology of [5]).

The design tools introduced and developed in [3,5] have been recently used in [2], where a new

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(iterative) procedure has been proposed for the robust stabilization of certain classes of nonlinear systems. This procedure is not based on the idea of solving separately a problem of state feedback stabilization and a problem of asymptotic state reconstruction. Rather, it is based on the recursive update of a sequence of "dynamic" output feedback stabilizers: specifically, the basic result of [2] is that if a suitable subsystem of lower dimension is robustly stabilizable by dynamic output feedback, so is the entire system.

From the point of view of the approach of [5], the contribution of [2] can be interpreted as the identification of a natural (and, in fact, necessary in the case of linear systems) assumption that guarantees the *existence* of a dynamic feedback that is expressible in terms of the output and its derivatives, i.e. a dynamic feedback driven by UCO functions. The purpose of this note is to highlight this interpretation.

2. Dynamic UCO feedback

2.1. Preliminaries

This subsection summarizes a number of standing hypotheses and basic properties used throughout the paper.

- For simplicity all nonlinear functions in this paper will be assumed to be sufficiently smooth so that all needed derivatives exist and are continuous, all differential equations have solutions, etc.
- We will use $\bar{\mathscr{B}}_n(r)$, with $r > 0$, to denote a closed ball of radius r in $\mathbb{R}^n$.
- Unless otherwise noted, $\mu(t)$ is a measurable function taking values in a compact set $\mathscr{P} \subset \mathbb{R}^p$. The set of such functions is denoted $\mathscr{M}_{\mathscr{P}}$.
- The origin of a nonlinear dynamical system

$$\dot{x} = f(x, \mu(t), k) \tag{1}$$

with $x \in \mathbb{R}^n$ and $k \in \mathbb{R}^c$, is said to be *uniformly semiglobally practically asymptotically stable in the parameter k* if for each pair of strictly positive real numbers $0 < r < R < \infty$ there exist $\bar{k} \in \mathbb{R}^c$, an open set $\mathscr{O} \supset \bar{\mathscr{B}}_n(R)$, a function $V : \mathscr{O} \to \mathbb{R}_{\geqslant 0}$ that is proper on $\mathscr{O}$ and strictly positive real numbers $0 < q < Q < \infty$ such that

(i) $\bar{\mathscr{B}}_n(R) \subset \{\xi \in \mathscr{O}\colon V(\xi) \leqslant Q\}$,

(ii) $\bar{\mathscr{B}}_n(r) \supset \{\xi \in \mathscr{O}\colon V(\xi) \leqslant q\}$,

(iii) $(\partial V/\partial x) f(x, \mu, \bar{k}) < 0 \quad \forall \mu \in \mathscr{P}, \forall x \in \{\xi \in \mathscr{O}\colon q \leqslant V(\xi) \leqslant Q\}$.

Uniform semiglobal practical asymptotic stability implies:

> for each pair of strictly positive real numbers $0 < r < R < \infty$, there exist $\bar{k} \in \mathbb{R}^c$ and $T > 0$ such that, for all initial conditions in $\bar{\mathscr{B}}_n(R)$, all resulting trajectories $x(t)$ of (1) with $k = \bar{k}$ are such that $x(t) \in \bar{\mathscr{B}}_n(r)$ for all $t \geqslant T$.

It also can be shown to imply:

> for each pair of strictly positive real numbers $0 < r < R < \infty$, there exist $\bar{k} \in \mathbb{R}^c$, a compact set $\mathscr{A} \subseteq \bar{\mathscr{B}}_n(r)$ and an open set $\mathscr{G} \supset \bar{\mathscr{B}}_n(R)$ such that, for system (1) with $k = \bar{k}$, the set $\mathscr{A}$ is uniformly asymptotically stable with basin of attraction [3] $\mathscr{G}$.

In fact, due to recent converse Lyapunov function results (see [4,1,7]), these latter properties are equivalent characterizations of uniform semiglobal practical asymptotic stability. However, we are using the Lyapunov formulation here so that we can more directly appeal to the results on semiglobal practical asymptotic stabilization like [5, Proposition 3.1] where a Lyapunov formulation was used.

2.2. Stabilization via UCO feedback

Consider multi-input–multi-output nonlinear control systems

$$\dot{x} = f(x, u, \mu(t)),$$
$$y = h(x, u, \mu(t)) \tag{2}$$

with $\mu(\cdot) \in \mathscr{M}_{\mathscr{P}}$. The definition of uniformly completely observable (UCO) dynamic feedback, given next, at times implicitly constrains $\mu(t)$ to be sufficiently smooth, where sufficiently smooth has to do with the number of times the output needs to be differentiated to reconstruct the UCO function.

Definition. A function $\varphi(x, u, \mu)$ is said to be *uniformly completely observable* (UCO) with respect to system (1) if it can be expressed as a function of a

[3] By this we mean:

- for each $\varepsilon > 0$ there exists $\delta > 0$ such that all trajectories starting in a δ-neighborhood of $\mathscr{A}$ remain in an ε-neighborhood of $\mathscr{A}$ for all time, and
- for each $\varepsilon > 0$ and each compact subset of $\mathscr{G}$ there exists $T > 0$ such that all trajectories starting in the compact subset enter within T seconds and remain thereafter in an ε-neighborhood of $\mathscr{A}$.

finite number of derivatives of the output y and the input u, i.e., if there exist two integers n_y and n_u and a function

$$\Psi(y,\dots,y^{(n_y)},u,\dots,u^{(n_u)})$$

such that, for each solution of

$$\dot{x}=f(x,u,\mu(t)),$$
$$u^{(n_u+1)}=v,$$
$$y=h(x,u,\mu(t)), \tag{3}$$

we have, for all t where the solution makes sense,

$$\varphi(x(t),u(t),\mu(t)) = \Psi(y(t),\dots,y^{(n_y)}(t),u(t),\dots,u^{(n_u)}(t))), \tag{4}$$

where $y^{(i)}$ denotes the ith time derivative of y at time t (and similarly for $u^{(i)}$).

Remark. As in [5, Footnote 6], note the strong requirement that Ψ is independent of $\mu(t)$.

Our next definitions, on uniform semiglobal practical asymptotic *stabilizability* by dynamic UCO or output feedback, are closely related to our definition of uniform semiglobal practical asymptotic stability. However, as was the case in [5], we do not insist that the states of the dynamic compensator eventually become small in the closed loop. We formulate the definition in Lyapunov function terms but, again, the definition could be formulated in terms of trajectories.

Definition. The origin of (2) is said to be *uniformly semiglobally practically asymptotically stabilizable by dynamic UCO feedback* if for each pair of strictly positive real numbers $0<r<R<\infty$ there exist:

- a UCO function $\alpha(x,u,\mu)$,
- functions θ and κ,
- compact sets $\mathscr{C}_{\eta s}$ and $\mathscr{C}_{\eta l}$, with $\mathscr{C}_{\eta s}$ a subset of the interior of $\mathscr{C}_{\eta l}$,
- an open set $\mathcal{O}\supset\bar{\mathscr{B}}_n(R)\times\mathscr{C}_{\eta l}$,
- a function $V:\mathcal{O}\to\mathbb{R}_{\geqslant 0}$ that is proper on $\mathcal{O}$, and
- strictly positive real numbers $0<q<Q<\infty$ such that
  (i) $(\bar{\mathscr{B}}_n(R)\times\mathscr{C}_{\eta l})\subset\{\xi\in\mathcal{O}\colon V(\xi)\leqslant Q\}$,
  (ii) $(\bar{\mathscr{B}}_n(r)\times\mathscr{C}_{\eta s})\supset\{\xi\in\mathcal{O}\colon V(\xi)\leqslant q\}$,
  (iii) $$\frac{\partial V}{\partial X}F(X,\mu)<0 \qquad \forall\mu\in\mathscr{P},\quad \forall X\in\{\xi\in\mathcal{O}\colon q\leqslant V(\xi)\leqslant Q\}, \tag{5}$$

where X and $F(X,\mu)$ are defined by

$$\dot{X}=\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}x\\ \eta\end{pmatrix}=\begin{pmatrix}f(x,u,\mu(t))\\ \theta(\eta,\alpha(x,u,\mu(t)))\end{pmatrix} =: F(X,\mu(t)) \tag{6}$$

with

$$u=\kappa(\eta,\alpha(x,u,\mu(t))) \tag{7}$$

(and where, for simplicity, we assume the right-hand side of (7) is independent of u).

Definition. The origin of (2) is said to be *uniformly semiglobally practically asymptotically stabilizable by dynamic output feedback* if, in the previous definition, we can always take $\alpha(x,u,\mu)=h(x,u,\mu)=y$.

Remark. In these definitions, we could allow the right-hand side of (7) to depend on u if we impose an extra condition that guarantees a solution to (7).

It will follow from the proof of [5, Proposition 3.1] (much like what is suggested by Teel and Praly [5, Footnote 5]) that we have:

Theorem 1. *Let $\mu(\cdot)\in\mathscr{M}_{\mathscr{P}}$ be sufficiently smooth with a uniform bound on an appropriate number of derivatives. If the origin of system* (2) *is uniformly semiglobally practically asymptotically stabilizable by dynamic UCO feedback then it is uniformly semiglobally practically asymptotically stabilizable by dynamic output feedback.*

Sketch of proof. Fix $0<r<R<\infty$. From the assumption of uniform semiglobal practical asymptotic stabilizability by dynamic UCO feedback, this fixes a UCO function $\alpha(x,u,\mu)$, a corresponding function Ψ that is used to reconstruct α from derivatives of y and u, functions θ and κ, compact sets $\mathscr{C}_{ns}$ and $\mathscr{C}_{nl}$, an open set $\mathcal{O}$, a function V and strictly positive real numbers $0<q<Q<\infty$. Now we apply the proof of [5, Proposition 3.1] to the control system

$$\begin{pmatrix}\dot{x}\\ \dot{\eta}\end{pmatrix}=\begin{pmatrix}f(x,u_1,\mu(t))\\ \theta(\eta,u_2)\end{pmatrix},$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}h(x,u_1,\mu(t))\\ \eta\end{pmatrix},$$

where the UCO feedback

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \kappa(\eta, \alpha(x, u_1, \mu(t))) \\ \alpha(x, u_1, \mu(t)) \end{pmatrix}$$

induces the properties for the function V that are assumed in the proof of [5, Proposition 3.1] if we define the objects $\mathscr{K}_{zs}$, $\mathscr{K}_{zl}$, v_l, c_s and c_l used in the proof of [5, Proposition 3.1] as

$$\mathscr{K}_{zs} := \bar{\mathscr{B}}_n(r) \times \mathscr{C}_{ns}, \quad \mathscr{K}_{zl} := \bar{\mathscr{B}}_n(R) \times \mathscr{C}_{nl}$$

and

$$v_l := q, \quad c_s := Q, \quad c_l := Q + 1.$$

From here we follow the proof of [5, Proposition 3.1], but noting that dynamic extension is only needed on the input u_1 and no estimates of the derivatives of $y_2 = \eta$ are needed.

3. A class of nonlinear systems that are uniformly semiglobally practically stabilizable by dynamic UCO feedback

3.1. Some motivations

It is well known that a nonlinear system having relative degree one can be robustly semiglobally stabilized via output feedback if its zero dynamics are globally asymptotically stable. The reason why this hypothesis is invoked is that, in order to offset the effect of matched uncertainties, "high-gain" output feedback is often recommended, and this – in turn – enforces a closed-loop behavior whose asymptotic properties are essentially determined by the asymptotic properties of the zero dynamics of the system. In particular, asymptotic stabilization occurs only if the latter is asymptotically stable, i.e., if the system is minimum phase. Consider robust (with respect to disturbances $\mu(t)$) stabilization of the origin for the system

$$\begin{aligned} \dot{z} &= f_0(z, y, \mu(t)), \\ \dot{y} &= q(z, y, \mu(t)) + b(y)u, \end{aligned} \tag{8}$$

where $z \in \mathbb{R}^{n-1}$, $y \in \mathbb{R}$, $u \in \mathbb{R}$, $\mu(\cdot) \in \mathscr{M}_{\mathscr{P}}$ and $b(y) \neq 0$ for all y. In the case of uniformly globally asymptotically stable zero dynamics, i.e. (see [4]) when there exists a smooth, positive definite and proper function $V(z)$ such that

$$\frac{\partial V}{\partial z} f_0(z, 0, \mu) < 0 \quad \forall z \neq 0, \quad \forall \mu \in \mathscr{P},$$

the control law

$$u = -\frac{1}{b(y)} k y,$$

where k is a sufficiently large number, solves the problem of semiglobal practical asymptotic stabilization of the origin. This follows from the fact that, given a compact set in (z, y) not containing the origin, for large enough k the negative-definite term $(\partial V/\partial z) f_0(z, 0, \mu) - k y^2$ in the derivative of the composite Lyapunov function

$$U(z, y) = V(z) + y^2,$$

i.e., in

$$\frac{\partial V}{\partial z} f_0(z, y, \mu) + 2y[q(z, y, \mu) - k y],$$

is able to dominate all nonnegative terms on the given compact set.

Note, however, that dominating all such terms implies dominating in particular $q(z, y, \mu)$, which is the only term through which the information about the z subsystem is made available to the measurement y. This is why a control law of the form indicated above requires the upper subsystem of (8) to be already asymptotically stable.

In case the original output does not yield an asymptotically stable zero dynamics, an (output feedback) stabilizing law should not offset the term $q(z, y, \mu)$, but rather should – if possible – take explicit advantage of it. As a simple explanation of why this is the case, suppose the system in question – which is a system having relative degree one – has been (for instance, uniformly globally asymptotically) stabilized by some dynamic output feedback law

$$\begin{aligned} \dot{\eta} &= \theta(\eta, y), \\ u &= \kappa(\eta, y). \end{aligned} \tag{9}$$

Then, looking at the interconnection of (8) and(9), we see that the "subsystem"

$$\begin{aligned} \dot{z} &= f_0(z, v, \mu(t)), \\ w &= q(z, v, \mu(t)) \end{aligned} \tag{10}$$

viewed as a system with input v and output w, has been necessarily (uniformly globally asymptotically) stabilized by a dynamic output feedback, with input w and output v, which has the form

$$\begin{aligned} \dot{y} &= w + b(y)\kappa(\eta, y), \\ \dot{\eta} &= \theta(\eta, y), \\ v &= y. \end{aligned} \tag{11}$$

In other words, we see that if system (8) is stabilizable at all, via dynamic output feedback, then subsystem (10) must *necessarily* be stabilizable by dynamic output feedback. This obviously implies that, if the dynamics of (10) with $v=0$, which coincide with the zero dynamics of (8), are unstable, the latter are necessarily "observable" through the map $q(z,v,\mu(t))$ and any successful stabilization scheme should aim at taking explicit advantage of this property.

3.2. A sufficient condition for semiglobal practical stabilizability via dynamic UCO feedback

Consider a nonlinear system modeled by equations of the form

$$\begin{aligned}
\dot z &= f(z,\zeta_1,\dots,\zeta_r,\mu(t)),\\
\dot\zeta_1 &= \zeta_2,\\
\dot\zeta_2 &= \zeta_3,\\
&\vdots\\
\dot\zeta_r &= q(z,\zeta_1,\dots,\zeta_r,\mu(t)) + b(\zeta)u,\\
y &= \zeta_1
\end{aligned} \tag{12}$$

in which $\zeta=(\zeta_1,\dots,\zeta_r)$, $z\in\mathbb{R}^{n-r}$, $\mu(\cdot)\in\mathscr{M}_{\mathscr{P}}$ and $b(\zeta)\neq 0$ for all ζ. This normal form may result from applying a globally defined, perhaps μ dependent, coordinate transformation to a nonlinear system given in some other form.

With system (12), we associate an *auxiliary system*

$$\begin{aligned}
\dot x_a &= f_a(x_a,u_a,\mu(t)),\\
y_a &= h_a(x_a,u_a,\mu(t)),
\end{aligned} \tag{13}$$

in which

$$x_a = \begin{pmatrix} x_{a,1}\\ \hline x_{a,2}\end{pmatrix} := \begin{pmatrix} z\\ \hline \zeta_1\\ \vdots\\ \zeta_{r-2}\\ \zeta_{r-1}\end{pmatrix}$$

and

$$f_a(x_a,u_a,\mu(t)) = \begin{pmatrix} f_{a,1}(x_a,u_a)\\ \hline f_{a,2}(x_{a,2},u_a)\end{pmatrix} := \begin{pmatrix} f(z,\zeta_1,\dots,\zeta_{r-1},u_a,\mu(t))\\ \hline \zeta_2\\ \vdots\\ \zeta_{r-1}\\ u_a\end{pmatrix},$$

and

$$h_a(x_a,u_a,\mu) := q(z,\zeta_1,\dots,\zeta_{r-1},u_a,\mu(t)).$$

About this system, we assume the following:

Assumption 1. The controller

$$\begin{aligned}
\dot\varphi &= L(\varphi,x_{a,2}) + My_a,\\
u_a &= N(\varphi,x_{a,2}),
\end{aligned} \tag{14}$$

in which $L(0,0)$ and $N(0,0)=0$, is such that the origin of system (13),(14) is uniformly globally asymptotically stable.

Then, we have:

Theorem 2. *If Assumption* 1 *holds, system* (12) *is uniformly semiglobally practically stabilizable via dynamic UCO feedback. More precisely, the origin of the state space of system* (12) *with control*

$$\begin{aligned}
\dot\varphi &= L(\varphi,x_{a,2}) + Mk[\zeta_r - N(\varphi,x_{a,2})],\\
u &= \frac{1}{b(\zeta)}\left[\frac{\partial N}{\partial\varphi}(L(\varphi,x_{a,2}) + Mk[\zeta_r - N(\varphi,x_{a,2})])\right.\\
&\qquad \left. + \frac{\partial N}{\partial x_{a,2}} f_{a,2}(x_{a,2},\zeta_r) - k[\zeta_r - N(\varphi,x_{a,2})]\right]
\end{aligned} \tag{15}$$

is uniformly semiglobally practically asymptotically stable in the control parameter k.

Proof. Consider the closed-loop system (12),(15)

$$\begin{aligned}
\dot x_a &= f_a(x_a,\zeta_r,\mu(t)),\\
\dot\varphi &= L(\varphi,x_{a,2}) + Mk[\zeta_r - N(\varphi,x_{a,2})],\\
\dot\zeta_r &= h_a(x_a,\zeta_r,\mu(t)) + \left[\frac{\partial N}{\partial\varphi}(L(\varphi,x_{a,2})\right.\\
&\qquad + Mk[\zeta_r - N(\varphi,x_{a,2})]) + \frac{\partial N}{\partial x_{a,2}} f_{a,2}(x_{a,2},\zeta_r)\\
&\qquad \left. - k[\zeta_r - N(\varphi,x_{a,2})]\right].
\end{aligned} \tag{16}$$

Define a new state variable $\theta=\zeta_r - N(\varphi,x_{a,2})$, and note that the resulting system can be interpreted as a system with input v and output θ

$$\begin{aligned}
\dot x_a &= f_a(x_a,\theta + N(\varphi,x_{a,2}),\mu(t)),\\
\dot\varphi &= L(\varphi,x_{a,2}) - Mv,\\
\dot\theta &= h_a(x_a,\theta + N(\varphi,x_{a,2}),\mu(t)) + v
\end{aligned} \tag{17}$$

controlled by $v=-k\theta$.

System (17) has relative degree one with high-frequency gain identically equal to one. Hence, this system has a globally defined zero dynamics manifold, the set

$$Z^* = \{(x_a, \varphi, \theta)\colon\ \theta = 0\},$$

which is rendered invariant by

$$v = v^*(x_a, \varphi, \mu(t)) = -h_a(x_a, N(\varphi, x_{a,2}), \mu(t)).$$

Its zero dynamics, those of

$$\begin{aligned} \dot{x}_a &= f_a(x_a, N(\varphi, x_{a,2}), \mu(t)),\\ \dot{\varphi} &= L(\varphi, x_{a,2}) + M h_a(x_a, N(\varphi, x_{a,2}), \mu(t)) \end{aligned} \tag{18}$$

are uniformly globally asymptotically stable by assumption.

The global diffeomorphism

$$\phi = \varphi + M\theta$$

changes system (17) into

$$\begin{aligned} \dot{x}_a &= f_a(x_a, \theta + N(\varphi - M\theta, x_{a,2}), \mu(t)),\\ \dot{\phi} &= L(\phi - M\theta, x_{a,2})\\ &\quad + M h_a(x_a, \theta + N(\phi - M\theta, x_{a,2}), \mu(t)),\\ \dot{\theta} &= h_a(x_a, \theta + N(\phi - M\theta, x_{a,2}), \mu(t)) + v \end{aligned}$$

which is a system having the form considered in [5, Lemma 2.2 (semiglobal backstepping I)] and the hypotheses of this Lemma hold. Thus, the result follows. □

Remark. Indeed, the functions $\zeta_1, \ldots, \zeta_r$ are trivially UCO functions and, hence, system (15) is an UCO dynamic feedback.

Remark. Controller (14) is affine in the input y_a. However, this hypothesis is not restrictive. In fact, observe, using again [5, Lemma 2.2 (semiglobal backstepping I)], that if a system

$$\begin{aligned} \dot{x} &= f(x, u),\\ y_1 &= h_1(x, u),\\ y_2 &= h_2(x), \end{aligned} \tag{19}$$

in which $f(0,0) = 0$, $h_1(0,0) = 0$ and $h_2(0) = 0$, is globally asymptotically stabilized by a controller

$$\begin{aligned} \dot{\varphi} &= \theta(\varphi, y_1, y_2),\\ u &= \kappa(\varphi, y_2), \end{aligned}$$

in which $\varphi(0,0,0) = 0$ and $\kappa(0,0) = 0$, the origin of the state space of the system

$$\begin{aligned} \dot{x} &= f(x, \kappa(\varphi, h_2(x))),\\ \dot{\xi} &= -k\xi + k h_1(x, \kappa(\varphi, h_2(x))),\\ \dot{\varphi} &= \theta(\varphi, \xi, h_2(x)) \end{aligned} \tag{20}$$

is semiglobally practically asymptotically stable in the parameter k. This system can be viewed as system (19) with control

$$\begin{aligned} \dot{\xi} &= -k\xi + k y_1,\\ \dot{\varphi} &= \theta(\varphi, \xi, y_2),\\ u &= \kappa(\varphi, y_2), \end{aligned}$$

which is affine in the input y_1.

On the basis of this observation it is not difficult to see that the result of Theorem 2 holds under the assumption that the auxiliary system (13) is uniformly globally asymptotically stabilized by a controller of the form

$$\begin{aligned} \dot{\varphi} &= L(\varphi, x_{a,2}, y_a),\\ u_a &= N(\varphi, x_{a,2}). \end{aligned}$$

Remark. We have shown above that any nonlinear relative degree one system which is stabilizable by dynamic output feedback necessarily satisfies Assumption 1. It can also be shown, by means of simple calculations, that the hypothesis in question is always fulfilled by any stabilizable and detectable linear system.[4] In the light of these properties, Assumption 1 can be regarded as a very natural hypothesis for the existence of dynamic output feedback stabilizers.

4. Conclusions

An important contribution of the recent paper [5] to the problem of robust stabilization via output feedback is the proof that the existence of a (possibly dynamic) stabilizing a feedback driven by UCO functions guarantees practical semiglobal stabilizability by means of output feedback. This paper discusses a significant structural hypothesis, introduced in [2], under which the existence of a dynamic feedback driven by UCO functions is guaranteed. The class of systems which satisfy this hypothesis includes any stabilizable and detectable linear system and any

[4] For further details, see [2].

relative degree one nonlinear system which is stabilizable by dynamic output feedback. In particular, the hypothesis does not require the system to be minimum phase. In the light of this fact, the hypothesis in question can be regarded as a very natural point of departure for the design of dynamic output feedback stabilizers.

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