PLAYING WITH THE REGULATION ZEROS IN THE STABILIZATION OF A DOUBLE INVERTED PENDULUM

BRIGITTE D'ANDRÉA-NOVEL, L. PRALY

State-space methods are well adapted for solving stabilization problems. In the linear multivariable case, when the poles are placed, some degrees of freedom are left which can be used to place some zeros in the transfer functions from the perturbations to the outputs, called *regulation zeros*, to reject some perturbations. It is therefore interesting to use a suitable representation to compute the closed-loop transfer functions from the perturbations to the outputs. The Youla-Jabr-Bongiorno parametrization of the controller turns out to be very appropriate for this task.

We have applied this method to the stabilization of a double inverted pendulum, fixed on a carriage moving on an horizontal bench. We have obtained a minimal controller which stabilizes the system and rejects asymptotically on the position of the carriage some perturbations, for example: measurements noises on the angle, the slope of the bench. All the results we have obtained on a full-size realization, (which can be seen at the permanent exhibition of the Cité des Sciences et de l'Industrie de La Villette), show that the behavior of the system closely depends on the choice of the regulation zeros.

1. CLOSED-LOOP TRANSFER FUNCTIONS OF A MINIMAL SYSTEM

The linear dynamical system that we consider is supposed to be controllable and observable. Using a polynomial formulation, it can be described by:

$$A(s)\,y = B_c(s)\,u + B_d(s)\,w \tag{1}$$

where y is the output p-vector, u is the input m-vector and w is a disturbance d-vector. $A_{p\times p}$, $(B_c)_{p\times m}$ and $(B_d)_{p\times d}$ are polynomial matrices.

System (1) being observable, A and B_c are left-prime matrices, the following Bezout identities are then satisfied:

$$\begin{aligned} A(s)\,E_0(s) + B_c(s)\,F_0(s) &= I \\ A(s)\,B_1(s) &= B_c(s)\,A_1(s) \end{aligned} \tag{2}$$

where A_1 and B_1 are right-prime matrices associated to the polynomial controller form of the system:

$$
\begin{aligned}
A_1(s)\,\xi &= u + T_1(s)\,w \\
y &= B_1(s)\,\xi + T_2(s)\,w
\end{aligned}
\tag{3}
$$

with ξ the partial state m-vector and

$$
\begin{aligned}
T_1(s) &= F_0(s)\,B_d(s) \\
T_2(s) &= E_0(s)\,B_d(s)
\end{aligned}
\tag{4}
$$

This representation is well adapted to the computation of the closed-loop transfer functions, the feedback law being expressed directly from the partial state ξ. More precisely, the most general linear, rational and causal control law is of the form:

$$P(s)\,u + Q(s)\,y = r \tag{5}$$

where r is obtained from a set-point vector $\bar{r}$, and can contain n derivatives, n being the degree of the determinant of $P(s)$:

$$r = Q_r(s)\cdot\bar{r} \tag{6}$$

$P(s)$, $Q(s)$ and $Q_r(s)$ are polynomial matrices with suitable dimensions, such that $P^{-1}Q$ and $P^{-1}Q_r$ are strictly proper to ensure the strict causality of the controller, for piecewise continuous input functions.

Let us now compute the closed-loop transfer functions. To be more exhaustive, we consider also an error vector v on the outputs measurements, i.e.:

$$
\begin{aligned}
A_1(s)\,\xi &= u + T_1(s)\,w \\
y &= B_1(s)\,\xi + T_2(s)\,w + v
\end{aligned}
\tag{7}
$$

Replacing y and u by (7) in (5) and denoting $T_{\nu\mu}$ the transfer matrix from ν to μ, we obtain:

$$y = T_{ry}r + T_{wy}w + T_{vy}v \tag{8}$$

with

$$
\begin{aligned}
T_{ry}(s) &= B_1 T^{-1} \\
T_{wy}(s) &= (B_1 T^{-1}(PF_0 - QE_0) + E_0)\,B_d \\
T_{vy}(s) &= (I - B_1 T^{-1} Q)
\end{aligned}
\tag{9}
$$

where

$$T(s) = PA_1 + QB_1 \tag{10}$$

is an invertible $m \times m$ polynomial matrix: the roots of its determinant $\delta(s)$, are the closed-loop poles, which are of course chosen stable.

We find again a well known result (see [5, 7, 8]), namely:
the open-loop zeros, (those of the open-loop transfer from u to y), are also present after feedback in the T_{ry} transfer; they are called the *tracking zeros*. Let us now parametrize the controller to point out the degrees of freedom which will be used to place some *regulation zeros*, i.e. the zeros of the T_{wy} and T_{vy} transfers.

2. AFFINE PARAMETRIZATION OF THE CONTROLLER TO PLACE REGULATION ZEROS WITH FIXED POLES

In the case of a multivariable system, we have obviously:

$$m + p > 2 \tag{11}$$

Let us denote by n the dimension of the system which is supposed to be minimal. Using the separation principle, a pair $C_{m \times n}$, $L_{n \times p}$ of gain matrices can be chosen to ensure complete and arbitrary pole placement, and consequently the stability of the $2n$-dimensional closed-loop system made of the given system compensated by a Luenberger observer of the entire state (see for example [4]). Some degrees of freedom are then left, said n_d:

$$n_d = n(m + p - 2) \tag{12}$$

The idea is now to apply the Youla-Jabr-Bongiorno parametrization of the controller (see [6]) to display these degrees of freedom.

A_1 and B_1 introduced in (2) are right-prime matrices, consequently there exist polynomial matrices P_0 and Q_0 such that the following Bezout identity is satisfied:

$$P_0 A_1 + Q_0 B_1 = I \tag{13}$$

On the other hand, using (2), the general form of the solution of (10) is:

$$\begin{aligned} P &= TP_0 + KB_c \\ Q &= TQ_0 + KA \end{aligned} \tag{14}$$

K being a free polynomial matrix. Using this parametrization, the closed-loop transfer functions given by (9) can be rewritten:

$$\begin{aligned} T_{ry}(s) &= B_1 T^{-1} \\ T_{wy}(s) &= (B_1(P_0F_0 - Q_0E_0 + T^{-1}K(B_cF_0 + AE_0)) + E_0)\, B_d \\ T_{vy}(s) &= I - B_1 Q_0 + B_1 T^{-1} K A \end{aligned} \tag{15}$$

It is then obvious that one can use the degrees of freedom K, to place some regulation zeros, the poles (given by the roots of the determinant of $T(s)$) being unchanged. The next sections will be devoted to the stabilization of a double inverted pendulum, using the previous results.

3. THE DOUBLE INVERTED PENDULUM, COMPUTATION OF THE CLOSED-LOOP TRANSFER FUNCTIONS

We consider a mechanical system, made of a double inverted pendulum, fixed on a carriage moving on an horizontal bench. A motor applies a force u to the carriage. We measure the position x of the carriage on the bench and the angle θ_1

that the lower stick makes with the perpendicular axis to the bench. The two sticks are joined by a flat spring, see Figure 1.

Using Euler-Lagrange method, we obtain the dynamical equations of the system.

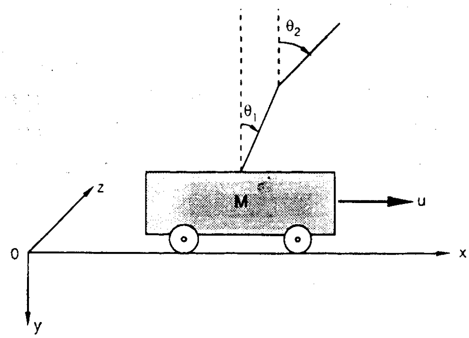

Fig. 1. The double inverted pendulum.

Considering the tangent linearization around the unstable vertical equilibrium, we deduce the following linear dynamical system:

$$\dot{X} = FX + Gu$$
$$Y = HX \tag{16}$$

with

$$F = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a & b & 0 & 0 & 0 \\ 0 & c & d & 0 & 0 & 0 \\ 0 & e & f & 0 & 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 \\ 0 \\ 0 \\ p \\ q \\ r \end{pmatrix} \tag{17}$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The coefficients of F and G are constants, depending on the physical parameters of the system. In particular, a, b, c, d, e, f depend linearly on the coefficient λ of flexibility of the flat spring when p, q and r are independent of λ (see [1]).

(16)–(17) is minimal, except for particular values of λ (see [1, 3]). We have built a Luenberger observer of the entire state. Consequently, the separation principle allows us to place independently the n poles of $F - GC$ and the n poles of $F - LH$, where $C_{m \times n}$ (resp. $L_{n \times p}$) are the gains of the controller (resp. the observer). Then, in our case, the number n_d of degrees of freedom is:

$$n_d = n(m + p - 2) = 6\,. \tag{18}$$

These degrees will be used to place some regulation zeros. There are many kinds of perturbations on the system: measurements noises, the slope of the bench which is unknown, the motor's dissymetries...

To be complete, we consider an n-vector w of dynamical perturbations and a p-vector v of measurements noises:

$$w = (0\ 0\ 0\ w_1\ w_2\ w_3)', \quad v = (dx\ d\theta)' \tag{19}$$

Applying general results of Section 1, we have to compute some polynomial matrices. We obtain directly from (16)–(17) the polynomial observer form:

$$\begin{aligned}
A(s) &= \begin{pmatrix} ds^2 & -(ad + b(s^2 - c)) \\ 0 & \alpha_1(s) \end{pmatrix} \\
B_c(s) &= \begin{pmatrix} pd - bq \\ \alpha_2(s) \end{pmatrix} \\
B_d(s) &= \begin{pmatrix} d & -b & 0 \\ 0 & (s^2 \quad f) & d \end{pmatrix}
\end{aligned} \tag{20}$$

with

$$\begin{aligned}
\alpha_1(s) &= -ed + (s^2 - f)(s^2 - c) \\
\alpha_2(s) &= rd + (s^2 - f)\,q
\end{aligned} \tag{21}$$

$A(s)$ and $B_c(s)$ being left-prime, the polynomial controller form is obtained by solving (2), which gives:

$$\begin{aligned}
A_1(s) &= \alpha(s) = ds^2\,\alpha_1(s) \\
B_1(s) &= \begin{pmatrix} \beta_x(s) = (pd - bq)\,\alpha_1(s) + (ad + b(s^2 - c))\,\alpha_2(s) \\ \beta_\theta = ds^2\,\alpha_2(s) \end{pmatrix}
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
E_0(s) &= \begin{pmatrix} \dfrac{ps^2 + d_3}{dd_1} & \dfrac{pqd_1 + d_3d_4 + pd_4s^2}{dd_1d_2} \\[2ex] \dfrac{-\alpha_2(s)}{dd_1} & \dfrac{q_2d_1 + (rd - fq)\,d_4 + qd_4s^2}{dd_1d_2} \end{pmatrix} \\
F_0(s) &= \begin{pmatrix} \dfrac{\alpha_1(s)}{dd_1} \\[2ex] \dfrac{(rd + qc - qs^2)\,d_1 - d_4\,\alpha_1(s)}{dd_1d_2} \end{pmatrix}
\end{aligned} \tag{23}$$

where the constants d_i have the following expressions:

$$\begin{aligned}
d_1 &= p(fc - ed) + q(be - af) + r(ad - bc) \\
d_2 &= q(rc - eq) + r(rd - fq) \\
d_3 &= (rb - pf) + (aq - pc) \\
d_4 &= r(pd - bq) + q(pc - aq)
\end{aligned} \tag{24}$$

The most general controller is of the form:

$$p(s)\,u + q_x(s)\,y_1 + q_\theta(s)\,y_2 = r = q_r(s)\,\bar{r} \tag{25}$$

where $\bar{r}$ is a set-point vector and q_x, q_θ and q_r have a strictly less degree than the degree of $p(s)$. As we are interested by the real values x and θ_1 of the system, using (9) and:

$$\begin{aligned} y_1 &= x + dx \\ y_2 &= \theta_1 + d\theta \end{aligned} \tag{26}$$

if we denote $\bar{y}$ the vector $\begin{pmatrix} x \\ \theta_1 \end{pmatrix}$, we obtain the different closed-loop transfer functions:

$$\bar{y} = B_1 T^{-1} r + (B_1 T^{-1}(PF_0 - QE_0) + E_0)\,B_d w - B_1 T^{-1} Q v \tag{27}$$

which gives more precisely:

$$\begin{aligned} T_{r\bar{y}}(s) &= \frac{1}{T(s)} \begin{pmatrix} \beta_x(s) \\ \beta_\theta(s) \end{pmatrix} \\ T_{v\bar{y}}(s) &= \frac{1}{T(s)} \begin{pmatrix} -q_x(s)\,\beta_x(s) & -q_\theta(s)\,\beta_x(s) \\ -q_x(s)\,\beta_\theta(s) & -q_\theta(s)\,\beta_\theta(s) \end{pmatrix} \\ T_{w\bar{y}}(s) &= \frac{1}{T(s)} \begin{pmatrix} t_{11}(s) & t_{12}(s) & t_{13}(s) \\ t_{21}(s) & t_{22}(s) & t_{23}(s) \end{pmatrix} \end{aligned} \tag{28}$$

where the polynomials $t_{ij}(s)$ have the following form:

$$\begin{aligned} t_{11}(s) &= p(s)\,h_1(s) + q_\theta(s)\,h_2(s) \\ t_{12}(s) &= p(s)\,h_3(s) - q_\theta(s)\,h_4(s) \\ t_{13}(s) &= p(s)\,h_5(s) - q_\theta(s)\,d(pd - bq) \\ t_{21}(s) &= -q_x(s)\,h_2(s) \\ t_{22}(s) &= p(s)\,ds^2(s^2 - f) + q_x(s)\,h_4(s) \\ t_{23}(s) &= p(s)\,d^2 s^2 + q_x(s)\,d(pd - bq) \end{aligned} \tag{29}$$

with the polynomials $h_i(s)$ given by:

$$\begin{aligned} h_1(s) &= d\,\alpha_1(s) \\ h_2(s) &= d\,\alpha_2(s) \\ h_3(s) &= d(eb + a(s^2 - f)) \\ h_4(s) &= d(rb + p(s^2 - f)) \\ h_5(s) &= d(ad + b(s^2 - c)) \end{aligned} \tag{30}$$

If we compute the Smith-McMillan forms of the above closed-loop transfers, we deduce (see [1]) that the zeros of $p(s)$ are also present in $T_{w\bar{y}}(s)$ and $T_{vy}(s)$. The only tracking zeros that can appear after feedback are those of the possible precompensator $q_r(s)$. Let us now parametrize the controller as explained in Section 2.

4. AFFINE PARAMETRIZATION OF THE CONTROLLER

The closed-loop poles, given by the roots of $T(s)$, being placed, we have to find $p(s)$, $q_x(s)$, $q_\theta(s)$ satisfying:

$$p(s)\,\alpha(s) + q_x(s)\,\beta_x(s) + q_\theta(s)\,\beta_\theta(s) = T(s) \tag{31}$$

Let p_0, q_{x_0} and q_{θ_0} be a particular solution, then we have:

$$\alpha(p_0 - p) = \beta_x(q_x - q_{x_0}) + \beta_\theta(q_\theta - q_{\theta_0}) \tag{32}$$

(16)–(17) being controllable and observable, $A_1(s)$ and $B_1(s)$ given by (22) are right-prime, consequently $\alpha_1(s)$ and $\alpha_2(s)$ are prime, and $\beta_x(0)$ being not zero, we deduce that β_x and β_θ given by (22) are also prime. Then there exist unique polynomials r_x and r_θ satisfying:

$$\begin{aligned} &r_x\beta_x + r_\theta\beta_\theta = -\alpha \\ &\deg(r_x) \leqq 3 \\ &\deg(r_\theta) \leqq 3 \end{aligned} \tag{33}$$

In fact, β_x, β_θ, α having no term in s^i, i odd, (33) implies that the degree of r_x and r_θ is less or equal to 2. More precisely, *denoting by a^i the coefficient of s^i in a polynomial* $a(s)$, we can show that (see [1]):

$$\begin{aligned} &r_x(s) = r_x^2 s^2 \\ &r_\theta(s) = r_\theta^0 + r_\theta^2 s^2 \\ &\deg(r_x) = \deg(r_\theta) = 2 \end{aligned} \tag{34}$$

Multiplying (33) by $(p - p_0)$ we obtain:

$$\alpha(p_0 - p) = r_x\beta_x(p - p_0) + r_\theta\beta_\theta(p - p_0) \tag{35}$$

(32) and (35) can be viewed as Bezout equations of which $(q_x - q_{x_0})$ and $(q_\theta - q_{\theta_0})$ on one hand, and $r_x(p - p_0)$ and $r_\theta(p - p_0)$ on the other hand, are solutions. Therefore there exist polynomials k and l such that:

$$\begin{aligned} p &= p_0 + l \\ q_x &= q_{x_0} + l r_x + k\beta_\theta \\ q_\theta &= q_{\theta_0} + l r_\theta - k\beta_x \end{aligned} \tag{36}$$

To obtain a particular solution we can apply the division algorithm:

$$\begin{aligned} &T = \alpha p_0 + r_0 \\ &\deg(r_0) < \deg(\alpha) = 6 \end{aligned} \tag{37}$$

We have then to solve:

$$r_0 = \beta_x q_{x_0} + \beta_\theta q_{\theta_0} \tag{38}$$

The unique solution satisfies:

$$\begin{aligned} &\deg(q_{x_0}(s)) < \deg(\beta_\theta(s)) = 4 \\ &\deg(q_{\theta_0}(s)) < \deg(\beta_x(s)) = 4 \end{aligned} \tag{39}$$

Then using (34), (36), (37) and (39) and denoting by $\bar{n}$ the degree of $T(s)$, we have:

$$\begin{aligned} &\deg(q_x) = \deg(q_\theta) = \operatorname{Max}(\deg(l) + 2, \deg(k) + 4, 3) \\ &\deg(p) = \operatorname{Max}(\deg(p_0), \deg(l)) = \operatorname{Max}(\bar{n} - 6, \deg(l)) \end{aligned} \tag{40}$$

But, l and k must satisfy the strict causality constraint of the controller, which can be written:

$$\begin{aligned} &\deg(p) > \deg(q_x) \\ &\deg(p) > \deg(q_\theta) \end{aligned} \tag{41}$$

If l and k are chosen independent, we obtain from (40) and (41) the following inequalities:

$$\begin{aligned} &\bar{n} \geqq 10 \\ &\deg(l) \leqq \bar{n} - 9 \\ &\deg(k) \leqq \bar{n} - 11 \end{aligned} \tag{42}$$

To solve (31) it is then sufficient to place 10 poles. This is quite natural since there are 2 outputs and 1 input. Consequently, we could place only 10 poles: 6 for the controller and 4 for the observer. Let us now examine how many degrees of freedom are left if $\bar{n}$ poles are placed. Using (42) we deduce the following proposition:

Proposition 1. If $\bar{n}$ poles are placed for system (16)–(17), $\bar{n}$ being greater or equal to 10, then $2\bar{n} - 18$ degrees of freedom are left.

Proof. (42) implies that:

$$n_d = \deg\max(l) + 1 + \deg\max(k) + 1 = 2\bar{n} - 18 \tag{43}$$

□

In our application we have placed 12 poles, using the classical observer-regulator synthesis and 6 degrees of freedom have been left, as it was expected in (18). Let us now use these degrees of freedom to place some regulation zeros.

5. REGULATION ZEROS PLACEMENT WITH FIXED POLES

5.1. Asymptotic rejection of constant perturbations

Using (28)–(29), for the static gains to be zero from the w_i, $i = 1, \ldots 3$, to the position x of the carriage, it is sufficient that:

$$\begin{aligned} &p(0) = 0 \\ &q_\theta(0) = 0 \end{aligned} \tag{44}$$

In this case, the static gain from $d\theta$ to x is also zero but that from dx to x is equal to -1 which means that a bias on the measure of x cannot be rejected on x. If $q_x(0)$ would be zero, all the static gains from the w_i to θ_1 would be zero. But from (31) and (44) we have:

$$T(0) = q_x(0)\,\beta_x(0)\,, \quad \beta_x(0) \neq 0 \tag{45}$$

$q_x(0)$ must be not zero to guarantee the stability of the closed-loop system. On the other hand, the fact that $\beta_\theta(0)$ is zero implies that the static gains from dx and $d\theta$ to θ_1 are zero. It means that with no dynamic perturbation, the only equilibrium position of the pendulum is the vertical one, corresponding to $\theta_1 = 0$. Let us now summarize the conditions of Asymptotic Rejection of Constant Perturbations, denoted ARCP:

$$\begin{aligned} p(0) &= 0 \\ q_\theta(0) &= 0 \\ q_x(0) &\neq 0 \end{aligned} \tag{46}$$

Using the general parametrization (36) these ARCP conditions are equivalent to choose:

$$\begin{aligned} l^0 &= -p_0^0 \\ k^0 &= \frac{q^0_{\theta 0} - p^0_0 r^0_\theta}{\beta^0_x} \end{aligned} \tag{47}$$

We can check afterwards that $q_{x_0}(0)$ is not zero. Two degrees of freedom have been use to realize ARCP conditions. Taking the four remaining degrees of freedom to zero (namely k^1, l^1, l^2, l^3), we have observed that every measurement bias on θ_1 or every motor's dissymetry was asymptotically rejected on x (see Fig. 2), as well as an unknown constant slope of the bench (see Fig. 3).

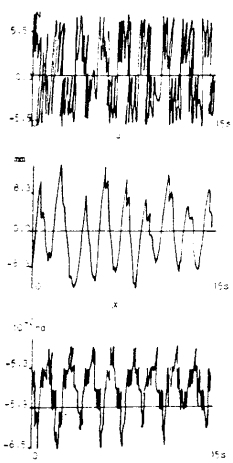

Fig. 2. Polynomial controller with ARCP.

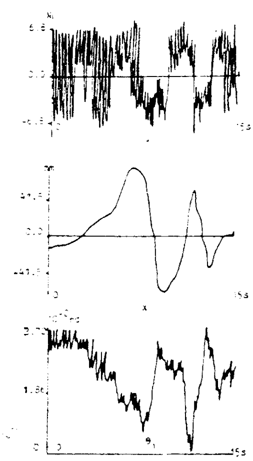

Fig. 3. Polynomial controller with ARCP.

5.2. Asymptotic rejection of ramp perturbations

We are able to reject on x, a constant slope γ of the bench. But this slope γ can be introduced as a ramp by the public with a button driving a pneumatic jack. Due to the form (19) of the dynamical perturbation vector w, this ramp perturbation can be written:

$$w_1 = \frac{g\gamma}{s^2} \tag{48}$$

Using (28) and denoting by $F_{\gamma x}(s)$ the transfer from γ to x we obtain:

$$F_{\gamma x}(s) = \frac{t_{11}(s)}{T(s)} = \frac{p(s)\, h_1(s) + q_\theta(s)\, h_2(s)}{T(s)} \tag{49}$$

$h_1(s)$ and $h_2(s)$ given by (29) and (30). If the ARCP conditions are supposed to be satisfied, $p(0)$ and $q_\theta(0)$ are zero and we deduce:

$$t_{11}(0) = 0 = F_{\gamma x}(0) \tag{50}$$

To reject asymptotically the ramp $g\gamma/s^2$ we have to satisfy moreover:

$$F'_{\gamma x}(0) = \frac{t'_{11}(0)\, T(0) - t_{11}(0)\, T'(0)}{T^2(0)} = 0 \tag{51}$$

$t_{11}(0)$ being zero, we have then to solve:

$$t'_{11}(0) = 0 = p'(0)\, h_1(0) + q'_\theta(0)\, h_2(0) \tag{52}$$

A solution can be obtained by taking:

$$p'(0) = q'_\theta(0) = 0 \tag{53}$$

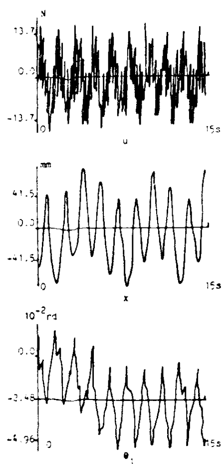

Fig. 4. Polynomial controller with ARCP and ramp rejection.

Noticing that β_x^1 and r_θ^1 are zero, (53) implies the following choice for l^1 and k^1:

$$\begin{aligned} l^1 &= -p_0^1 \\ k^1 &= \frac{q_{\theta_0}^1 - p_0^1 r_\theta^0}{\beta_x^0} \end{aligned} \tag{54}$$

Taking the two remaining degrees of freedom l^2 and l^3 to zero, we have implemented this controller k^0 and l^0 given by (47) and l^1 and k^1 by (54). We observe (see Fig. 4) that the ramp has no influence on x, but the behavior around the equilibrium position is worse than in Figure 3.

5.3. Rejection of frequencies

The oscillations of the carriage and the pendulum are amplified if the sliding friction of the carriage on the bench is not well compensated. The sliding friction $\bar{f}$ can be viewed as an additive perturbation to u. After some computations, we obtain the different transfers from $\bar{f}$ to x and θ_1:

$$\begin{aligned} F_{\bar{f}x}(s) &= \frac{p(s)\, n_1(s)}{T(s)} \\ F_{\bar{f}\theta_1}(s) &= \frac{p(s)\, n_2(s)}{T(s)} \end{aligned} \tag{55}$$

where the polynomials $n_1(s)$ and $n_2(s)$ depend on the physical system (see [1, 2]). Consequently, if we want to cancel a frequency ω due to $\bar{f}$, it is sufficient to choose:

$$p(\mathrm{i}\omega) = 0 \tag{56}$$

If ARCP conditions are satisfied, (56) implies:

$$\begin{aligned} l^1 - l^3\omega^2 &= A = -p_0^1 + p_0^3\omega^2 - p_0^5\omega^4 \\ l^2 &= -p_0^2 + p_0^4\omega^2 - p_0^6\omega^4 \end{aligned} \tag{57}$$

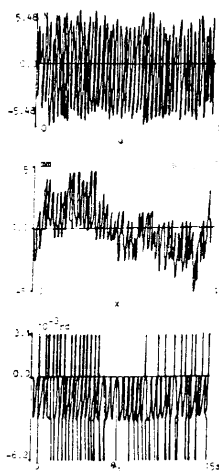

Fig. 5. Polynomial controller with ARCP and frequency rejection.

A solution which minimizes for example $(l^1)^2 + (l^3)^2$ is the following:

$$\begin{aligned} l^1 &= \frac{A}{1+\omega^4} \\ l^3 &= -\frac{A\omega^2}{1+\omega^4} \end{aligned} \tag{58}$$

We have implemented this controller as follows:
- we have measured the frequency ω in Figure 2,
- we have chosen l^0, k^0 to realize ARCP conditions and l^1, l^2, l^3 given by (57)–(58), k^1 being zero.

The frequency ω has disappeared as shown in Figure 5, but a new frequency has appeared, which is a multiple of ω. Morrover, the behavior around the equilibrium is quite satisfying, the oscillations of x and θ_1 being of very small amplitude.

6. CONCLUSION

To conclude, one can say that the polynomial approach allowed us quite easily to parametrize a controller with some degrees of freedom, used to place some regulation zeros with fixed poles. All the results obtained on the real system point out the importance of a suitable choice of the regulation zeros.

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Dr. Brigitte d'Andréa-Novel, Dr. L. Praly, CAI-Automatique, Ecole des Mines de Paris, 35 rue Saint Honoré, 77305 Fontainebleau Cedex, France.