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Brief Paper

Examination of the SPR Condition in Output Error Parameter Estimation*

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Key Words—Parameter estimation; adaptive control; stability–instability boundary.

Abstract—When the SPR condition is not satisfied then there exist input signals for which the considered output error algorithm is locally unstable. A condition is given which delimits the sharp stability–instability boundary in the case of slow estimation, whereas local stability properties are guaranteed by a more conservative signal-dependent average SPR condition. These conditions are also illustrated by an example.

1. *Introduction*

THE ROLE of the strictly positive real (SPR) condition of a transfer function $H(z)$ in establishing stability properties of output error estimation algorithms is well known (Ljung, 1977; Landau, 1979; Goodwin and Sin, 1984). In this paper it is shown that when this condition is not met, a sharp stability–instability boundary exists and limits the spectral content of the regressor vector ϕ. The spectrum of ϕ in the range where $\operatorname{Re} H < 0$ leads to instability. To preserve stability, it must be dominated by the spectrum in the range where $\operatorname{Re} H > 0$. The stability is assured if ϕ is such that *on average* H behaves as an SPR transfer function. The stability boundary is derived under the assumption that the estimation is slow, i.e. the update gain ε is small. This assumption allows us to focus on the drift-type instability which can occur even at extremely low values of ε. It is also assumed that the input signal u is bounded and periodic, an assumption made for clarity.

This paper extends the continuous-time results of Riedle and Kokotovic (1984, 1985). It is written as a discrete-time counterpart of Kokotovic *et al.* (1985). The discrete-time algorithm to be analyzed is an "A-class" identifier of Landau, p. 292 (1979), Goodwin and Sin, pp. 83–87 (1984), with the proportional gain zero and the integral gain εI. For a study of the effects of a non-SPR transfer function, the fixed moving average filter is deleted, i.e. let $D(q^{-1}) = 1$ in the Goodwin–Sin notation. The update law, with (′) denoting transposition, is given by

$$\hat{\theta}(k+1) = \hat{\theta}(k) - [\varepsilon\bar{\phi}(k)/1 + \varepsilon\bar{\phi}'(k)\bar{\phi}(k)](\bar{\phi}'(k)\hat{\theta}(k) - y(k+1)) \tag{1.1}$$

where $y(k)$ is the output of the plant to be identified. The identifier output $\bar{y}(k)$, adjustable parameter vector $\hat{\theta}(k)$ and regressor vector $\bar{\phi}(k)$ are

$$\bar{y}(k) = \bar{\phi}'(k-1)\hat{\theta}(k) \tag{1.2}$$

$$\hat{\theta}(k) = [-\hat{a}_1(k), -\hat{a}_2(k), \ldots, \hat{a}_n(k), \hat{b}_1(k), \ldots, \hat{b}_r(k)]' \tag{1.3}$$

$$\bar{\phi}(k) = [\bar{y}(k), \bar{y}(k-1), \ldots, \bar{y}(k+1-n), u(k), \ldots, u(k+1-r)]' \tag{1.4}$$

where $u(k)$ is the input signal. For the analysis of this implementable algorithm, its non-implementable form

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \varepsilon\bar{\phi}(k)(\bar{\phi}'(k)\hat{\theta}(k+1) - y(k+1)). \tag{1.5}$$

will also be used. Along with $\bar{\phi}(k)$, the "plant regressor",

$$\phi(k) = [y(k), y(k-1), \ldots, y(k+1-n), u(k), \ldots, u(k+1-r)]', \tag{1.6}$$

is introduced and $y(k)$ is said to be "matchable" if $\hat{\theta}(k)$ can be "tuned" to a constant value θ_* such that

$$y(k+1) = \phi'(k)\theta_*, \quad \forall k. \tag{1.7}$$

If $u(k)$ is a persistently exciting (PE) signal for the plant, then the output matchability (1.7) implies the plant–identifier transfer function matchability. However, if the identifier is of lower order than the plant, (1.7) can still be achieved, but only for a severely restricted class of inputs $u(k)$. For example, $u(k)$ can be an input containing an exact minimum number of frequencies required to be PE for the identifier and, hence, exciting only a matchable part of the plant. Local stability properties of all such "tuned regimes" $\{\bar{y}(k) = y(k),\ \hat{\theta}(k) = \theta_*\}$ are of interest, because they show the tendency of the algorithm to converge to, or move away from, an equilibrium point θ_* in the parameter space $\hat{\theta}$.

For a local analysis the variables

$$\eta(k) = \bar{y}(k) - y(k), \theta(k) = \hat{\theta}(k) - \theta_*, \psi(k) = \bar{\phi}(k) - \phi(k) \tag{1.8}$$

are introduced, representing the deviations around a tuned regime. The equation governing the output error is

$$\eta(k+1) = (\phi(k) + \psi(k))'\theta(k+1) + \psi'(k)\theta_*. \tag{1.9}$$

where a_i^* denotes the tuned value of $\hat{a}_i(k)$ and

$$\psi'(k)\theta_* = -\sum_{i=1}^{n} a_i^*\eta(k+1-i). \tag{1.10}$$

In terms of (1.8) the update law (1.5) is rewritten as

$$\theta(k+1) = \theta(k) - \varepsilon(\phi(k) + \psi(k))\eta(k+1). \tag{1.11}$$

It is clear from (1.9)–(1.10) that $\eta(k)$ is the output of the transfer function

$$H(z) = z^n/(z^n + a_1^* z^{n-1} + \cdots + a_{n-1}^* z + a_n^*) \tag{1.12}$$

when its input is $(\phi(k-1) + \psi(k-1))'\theta(k)$.

Remark 1.1. When $H(z)$ is SPR and $\bar{\phi}(k+N) = \bar{\phi}(k)$ is PE then the function $W(k) = \theta'(k)\theta(k)$ satisfies $W(k+N) - W(k) < 0$, and hence, $\theta(k) = \hat{\theta}(k) - \theta_* \to 0$ as $k \to \infty$, because

$$\sum_{i=k}^{k+N-1} \theta'(i)\bar{\phi}(i)\eta(i+1) > 0, \forall k. \tag{1.13}$$

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When $H(z)$ is not SPR, the sign of this sum depends on the spectrum of $\bar{\phi}(k)$. To make this dependence explicit $\bar{\phi}(k)$ is approximated by $\phi(k)$ and $\eta(k+1)$ by $v_0^1(k)\theta(k)$, where $v_0(k)$ is the output of $H(z)$, when its input is $\phi(k)$. Then instead of (1.13), the sum

$$\sum_{i=k}^{k+N-1} \theta'(i)\phi(i)v_0'(i)\theta(i) \tag{1.14}$$

can be used to determine the stability properties of the tuned regime when $H(z)$ is not SPR.

The stability criterion based on (1.14) is extremely simple: it is a test of the eigenvalues of the matrix

$$R = \sum_{i=k}^{k+N-1} \phi(i)v'_0(i), \tag{1.15}$$

which delimits a sharp stability–instability boundary for slow estimation.

2. *The stability–instability boundary*

For a local analysis around $\{y(k), \theta_*, \phi(k)\}$, the second order quantities $\psi'(k)\theta(k+1)$ and $\psi(k)\eta(k+1)$ are neglected and (1.9)–(1.11) are approximated by the linear time-varying system

$$\eta(k+1) = -\sum_{i=1}^{n} a_i^*\eta(k+1-i) + \phi'(k)\theta(k+1), \tag{2.1}$$

$$\theta(k+1) = \theta(k) - \varepsilon\phi(k)\eta(k+1). \tag{2.2}$$

Using $x(k) = [\eta(k), \ldots, \eta(k+1-n)]'$ as the state of $H(z)$, (2.1)–(2.2) is rewritten in the form

$$x(k+1) = Ax(k) + b\phi'(k)\theta(k+1), \tag{2.3}$$

$$\theta(k+1) = \theta(k) - \varepsilon\phi(k)[\phi'(k)\theta(k+1) + c'x(k)]. \tag{2.4}$$

where $b' = [1, 0, \ldots, 0]$, $c' = [-a_1^*, \ldots, -a_n^*]$ and A is in the corresponding canonical form, see Fig. 1. Note from these definitions that the sum in (2.1) is $c'x(k)$ and the bracketed term in (2.4) is $\eta(k+1)$. The implementable form of (2.4) is

$$\theta(k+1) = \theta(k) - \varepsilon g(k,\varepsilon)\phi(k)(\phi'(k)\theta(k) + c'x(k)), \tag{2.5}$$

where $g(k,\varepsilon) = [1 + \varepsilon\phi'(k)\phi(k)]^{-1} \leqslant 1$. In order to replace $\phi'(k)\theta(k+1)$, the input of $H(z)$, by $\phi'(k)$, the transformation

$$\xi(k) = x(k) - L(k,\varepsilon)\theta(k) \tag{2.6}$$

is used, requiring that the $n \times m$ matrix $L(k,\varepsilon)$ be a bounded N-periodic solution of

$$\begin{aligned} L(k+1,\varepsilon) = AL(k,\varepsilon) + b\phi'(k) + \varepsilon g(k,\varepsilon)(L(k+1,\varepsilon) \\ - b\phi'(k))\phi(k)(\phi'(k) + c'L(k,\varepsilon)). \end{aligned} \tag{2.7}$$

All the results of this paper are obtained under the following assumption.

Assumption 2.1. The plant to be identified is uniformly asymptotically stable (u.a.s.), that is $|\lambda(A)| < 1$, the input $u(k)$ is a bounded N-periodic sequence $u(k) = u(k+N)$, and, hence, $\phi(k) = \phi(k+N)$ is also bounded.

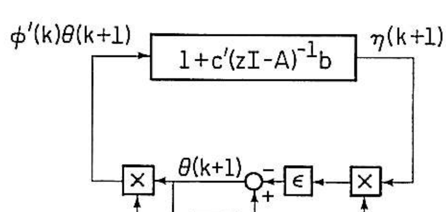

Fig. 1. Linearization of the output error algorithm.

As a consequence of this assumption, an N-periodic bounded solution $L(k,0) = L_0(k)$ of the non-linear equation (2.7) at $\varepsilon = 0$ exists and represents the steady state periodic response of the linear time-invariant system

$$L_0(k+1) = AL_0(k) + b\phi'(k), \qquad L_0(k+N) = L_0(k). \tag{2.8}$$

Noting that at $\varepsilon = 0$, $L = L_0$, the right-hand side of (2.7) is continuously differentiable with respect to both ε and L, the implicit function theorem is invoked to state the following result.

Lemma 2.1 (Existence of L). There exists $\varepsilon_L > 0$ such that for all $\varepsilon \in (-\varepsilon_L, \varepsilon_L)$ and all k, (2.8) has a bounded N-periodic solution $L(k,\varepsilon)$, which is unique and can be represented by

$$L(k,\varepsilon) = L_0(k) + \varepsilon L_1(k,\varepsilon) \tag{2.9}$$

where the norm of $L_1(k,\varepsilon)$ is bounded by a constant independent of ε.

Remark 2.1. Bounds for ε_L and $L_1(k,\varepsilon)$ can be calculated via a contraction map proof of this lemma, as in the continuous-time case (Kokotovic *et al.*, 1985).

The L-transformation separates a fast ξ-system from the slow θ-update system, that is,

$$\xi(k+1) = [A + \varepsilon g(k,\varepsilon)(L(k+1,\varepsilon) - b\phi'(k))\phi(k)c']\xi(k). \tag{2.10}$$

$$\begin{aligned} \theta(k+1) = [I - \varepsilon g(k,\varepsilon)\phi(k)v'(k,\varepsilon)]\theta(k) \\ - \varepsilon g(k,\varepsilon)\phi(k)c'\xi(k). \end{aligned} \tag{2.11}$$

where

$$v'(k,\varepsilon) = \phi'(k) + c'L(k,\varepsilon) = v_0'(k) + \varepsilon c'L_1(k,\varepsilon). \tag{2.12}$$

The last expression introduces the signal

$$v'_0(k) = \phi'(k) + c'L_0(k), \tag{2.13}$$

the output of $H(z)$ for the input $\phi'(k)$. As stated in Remark 1.1, this signal will be used to delimit the stability–instability boundary. The next result points out that this boundary is determined by the θ-update dynamics, because for ε small the ξ-system (2.10) remains u.a.s.

Lemma 2.2 (u.a.s. of ξ). There exists $\varepsilon_A > 0$ such that for all $\varepsilon \in (-\varepsilon_A, \varepsilon_A)$ the ξ-system (2.10) is u.a.s.

Remark 2.2. Whereas the proof of this lemma is by a standard perturbational argument and need not be repeated here, an important fact is that $\varepsilon_A > \varepsilon_L$ and, hence, whenever Lemma 2.1 holds then (2.10) is also u.a.s. This result is established as in the continuous-time case (Kokotovic *et al.*, 1985).

Henceforth the stability analysis focuses on the θ-system (2.11), where the bounded forcing term is not essential and can be disregarded. The state transition matrix $F(k,k_0)$ of (2.11) is defined by

$$\begin{aligned} F(k+1,k_0) &= [I - \varepsilon g(k,\varepsilon)\phi(k)v'(k,\varepsilon)]F(k,k_0), \\ F(k_0,k_0) &= I. \end{aligned} \tag{2.14}$$

The stability of this linear N-periodic system is determined by the eigenvalues $F(k,k_0)$ evaluated over any period N, say $F_N = F(N,0)$. If $|\lambda_i(F_N)| < 1$ for all $i = 1, \ldots, n+r$, then (2.14) is exponentially stable, and if $|\lambda_j(F_N)| > 1$ for some j, then (2.14) is unstable. In order to convert this condition into a practical stability criterion, F_N must be expressed in terms of some easily interpretable quantities.

Lemma 2.3 (Averaging). The matrix F_N can be expressed as

$$F_N = I - \varepsilon\left(\sum_{k=0}^{N-1} \phi(k)v'_0(k) + \varepsilon M\right) \tag{2.15}$$

and there exists $\varepsilon_M > 0$ such that for all $\varepsilon \in (-\varepsilon_M, \varepsilon_M)$ the norm of M is bounded by a constant independent of ε.

To prove this lemma the fact that $g(k,\varepsilon) = 1 + O(\varepsilon)$.

$v'(k) = v'_0(k) + O(\varepsilon)$ is used and ε_M is determined such that for all $\varepsilon \in (-\varepsilon_M, \varepsilon_M)$ a contraction property of the operator can be established.

$$U(F)(0) = I: \quad U(F)(k) = I - \varepsilon \sum_{i=0}^{k-1} g(i,\varepsilon)\phi(i)v'(i,\varepsilon)F(i), \tag{2.16}$$
$$k = 1, 2, \ldots, N.$$

The rest of the proof is the same as in the continuous-time case (Kokotovic *et al.*, 1985).

The stability of (2.19), and, hence, of the whole linear system (2.1)–(2.2), is concluded to be determined by the eigenvalues of the matrix R defined by (1.15).

Theorem 2.1 (Stability–instability boundary). Under Assumption 2.1 there exists $\varepsilon_s > 0$ such that for all $\varepsilon \in (0, \varepsilon_s)$ the θ-system (2.14) and, hence, the full system (2.1)–(2.2), are exponentially stable if for all $i = 1, \ldots, n + r$

$$\operatorname{Re}\lambda_i(R) = \operatorname{Re}\lambda_i\left(\sum_{k=0}^{N-1} \phi(k)v'_0(k)\right) > 0, \tag{2.17}$$

unstable if for some $j \in \{1, \ldots, n + r\}$

$$\operatorname{Re}\lambda_i(R) = \operatorname{Re}\lambda_j\left[\sum_{k=0}^{N-1} \phi(k)v'_0(k)\right] < 0. \tag{2.18}$$

To prove this theorem, the way the eigenvalues of $F_N = I - \varepsilon R - \varepsilon^2 M$ depend on the eigenvalues of R is examined. Let $\sigma + j\beta$ be an eigenvalue of $R + \varepsilon M$ and $\lambda(F_N)$ the corresponding eigenvalue of F_N. Then

$$|\lambda(F_N)| = [1 - 2\varepsilon\sigma + \varepsilon^2(\sigma^2 + \beta^2)]^{1/2} = 1 - \varepsilon\sigma + O(\varepsilon^2) \tag{2.19}$$

and, since $\sigma = \operatorname{Re}\lambda(R + \varepsilon M) = \operatorname{Re}\lambda(R) + o(\varepsilon)$,

$$|\lambda(F_N)| = 1 - \varepsilon \operatorname{Re}\lambda(R) + o(\varepsilon), \tag{2.20}$$

noting that, if the eigenvalues of R are distinct, $o(\varepsilon)$ can be replaced by $O(\varepsilon)$.

3. *Discussion and example*

To interpret the meaning of the above stability theorem, we recall that $v'_0(k)$ is the output of $H(z)$ for the input $\phi'(k)$, and express the regressor vector $\phi(k)$ as

$$\phi(k) = \sum_{i=-N_1}^{N_2} \alpha(i)e^{j\omega_i k}, \qquad \omega_i = 2\pi/Ni, \qquad \alpha(-i) = \overline{\alpha(i)}, \tag{3.1}$$

where $\overline{\alpha(i)}$ is the complex conjugate of $\alpha(i)$ and $N_1 \leqslant \frac{1}{2}N$, $N_2 \leqslant \frac{1}{2}(N-1)$. Then the vector $v_0(k)$, and the matrix R are given by

$$v_0(k) = \sum_{i=-N_1}^{N_2} \alpha(i)H(e^{j\omega_i})e^{j\omega_i k}. \tag{3.2}$$

$$R = \sum_{k=0}^{N-1} \phi(k)v'_0(k) = N \sum_{i=-N_1}^{N_2} H(e^{j\omega_i})\overline{\alpha(i)}\alpha'(i) \tag{3.3}$$

Corollary 3.1 (Instability). If $H(z)$ does not satisfy

$$\operatorname{Re} H(e^{j\omega}) \geqslant 0, \qquad \forall\omega \in [-\pi, \pi], \tag{3.4}$$

then there exists an N-periodic $\phi(k)$ and $\varepsilon_u > 0$ such that the model (2.1)–(2.2) is unstable for all $\varepsilon \in (0, \varepsilon_u]$.

The proof is by observing that if

$$\operatorname{trace} R = \sum_{i=-N_1}^{N_2} |\alpha(i)|^2 \operatorname{Re} H(e^{j\omega_i}) < 0 \tag{3.5}$$

then at least one eigenvalue of R satisfies the instability condition (2.18). This corollary leads to the following interpretation of the SPR property:

> if stability is required for all possible bounded N-periodic sequences $\phi(k)$, then the SPR property of $H(z)$ is necessary.

In other words

> the only way to relax the SPR requirement is to restrict the spectrum of $\phi(k)$.

In applications, a careful design of the spectrum of $\phi(k)$ is required, a conclusion reached by Ioannou and Kokotovic (1982) via a different route. Theorem 2.1 offers guidelines for a design of $\phi(k)$, that is, the choice of $u(k)$, based on some *a priori* information about $H(z)$. First note that if $u(k)$ is PE for $H(z)$, then $\operatorname{Re}\lambda(R) \neq 0$ and the stability condition (2.17) is both necessary and sufficient. As this condition is equivalent to the existence of $P = P' > 0$ such that $PR + R'P > 0$, it can also be derived by using $W_p = \theta'(k)P\theta(k)$ as a Lyapunov function. Different choices of P lead to different sufficient conditions for stability which may be more conservative than (2.17), but simpler to interpret. The simplest choice, $P = I$, has already been made in Remark 1.1 which is now reconsidered. From (2.1), (2.5) and (2.6),

$$\begin{aligned} \eta(k+1) &= c'x(k) + \phi'(k)\theta(k+1) \\ &= g(k,\varepsilon)[\phi'(\varepsilon)\theta(k) + c'x(k)] \\ &= (\phi'(k) + c'L_0(k))\theta(k) + c'\xi(k) + O(\varepsilon) \\ &= v'_0(k)\theta(k) + O(\varepsilon) \end{aligned} \tag{3.6}$$

is obtained where the last step disregarded $\xi(k)$ as an exponentially decaying term. This expression shows that the goal of approximating (1.13) by (1.14) has been achieved, not by linearization alone, but primarily due to the L-transformation and smallness of ε. The choice $P = I$, although conservative, leads to a convenient relaxation of the SPR condition.

Corollary 3.2 (Average SPR). If

$$\sum_{i=-N_1}^{N_2} \operatorname{Re} H(e^{j\omega_i}) \operatorname{Re} \alpha(i)\overline{\alpha'(i)} > 0, \tag{3.7}$$

then there exists $\varepsilon_H > 0$ such that (2.1)–(2.2) is exponentially stable for all $\varepsilon \in (0, \varepsilon_H]$.

This relaxation of the SPR condition allows $\operatorname{Re} H(e^{j\omega_i}) < 0$ for some i, as long as the sum in (3.7) is positive. In other words:

> $\phi(k)$ should be PE and have more Re H-weighted energy at the frequencies where $\operatorname{Re} H > 0$ than at those where $\operatorname{Re} H < 0$.

It may not be obvious that (3.7) is more conservative than (2.17). Its conservativeness is illustrated in Case (c) of the example below. As it is better to be conservative than risk instability, the sufficient condition (3.7) seems to be a reasonable objective to satisfy by the choice of $u(k)$, or by filtering, as suggested by Johnson *et al.* (1984).

Example. As an illustration results of simulation experiments are presented in which the output error algorithm (1.1)–(1.4) was used to estimate the parameters −1.6 and 0.8 in the transfer function

$$H(z) = \frac{z^2}{z^2 - 1.6z + 0.8}. \tag{3.8}$$

Note that $H(z)$ is stable, but *not* SPR. For the input signal

$$u(k) = \sum_{i=-N_1}^{N_2} r_i e^{j\omega_i k} \tag{3.9}$$

the matrix R is found to be

$$R = N \sum_{i=-N_1}^{N_2} |H(e^{j\omega_i})|^2 r_i^2 H(e^{j\omega_i}) \begin{vmatrix} 1 & e^{-j\omega_i} \\ e^{j\omega_i} & 1 \end{vmatrix}. \tag{3.10}$$

Case (a). Let the input $u(k)$ and the corresponding R be

$$u(k) = 0.064\cos\left(\frac{2\pi}{13}k\right), R = 13\begin{vmatrix}1.02 & -0.17\\ 1.98 & 1.02\end{vmatrix} \qquad (3.11)$$

where $u(k)$ has period $N = 13$. In this case $\lambda(R) = 13(1.02 \pm j0.58)$ and both Theorem 2.1 and Corollary 3.2 predict exponential stability. This is confirmed by simulated trajectories in the $(\hat{a}_1, \hat{a}_2)$-plane shown in Fig. 2a.

Case (b). If instead of (3.11)

$$u(k) = 1.2\cos\left(\frac{4\pi}{13}k\right), R = 13\begin{vmatrix}-1.07 & -3.19\\ 1.98 & -1.07\end{vmatrix} \qquad (3.12)$$

is used, then $\lambda(R) = 13(-1.07 \pm j2.51)$ and Theorem 2.1 predicts instability. A simulated trajectory shown in Fig. 2b "unwinds", as predicted. However, this local instability does not necessarily imply unboundedness and it appears that the trajectory in Fig. 2b has a tendency to remain bounded.

Case (c). To the input $u(k)$ in (3.12), which led to instability, a d.c. component is now added, namely,

$$u(k) = 0.11 + 1.2\cos\left(\frac{4\pi}{13}k\right), R = 13\begin{vmatrix}0.45 & -1.68\\ 3.49 & 0.45\end{vmatrix}. \qquad (3.13)$$

This final experiment is of interest because the sufficient condition of Corollary 3.2 is not satisfied, while $\lambda(R) = 13(0.45 \pm j2.42)$ satisfies the exponential stability condition of Theorem 2.1. The prediction of exponential stability is confirmed by trajectories in Fig. 2c. Near the equilibrium they are similar to the trajectories of a linear system with oscillatory but stable eigenvalues.

Most of the stable simulated trajectories show the characteristic pattern of a fast transient followed by a slow "averaged" behaviour. The same pattern, but in the reverse direction of time, is present in unstable trajectories. The local theory predicts the qualitative behaviour of the averaged parts of trajectories. When the parameter error is large, the adaptation is fast even for small values of ε, and the assumptions of local theory are not met.

4. *Concluding remarks*

The local theory presented in this paper contributes to the understanding of stability and instability mechanisms in slow adaptation with non-SPR transfer functions. For some test inputs the output of a non-matchable plant can still be matched by the output of a lower order model. If, for an algorithm, all such tuned solutions are exponentially stable, it is reasonable to expect that the algorithm is more robust than if this were not the case. A "total stability" analysis by small gain theorems or similar analytical tools can be used to estimate the stability region around exponentially stable tuned solutions.

Another useful aspect of the stability criterion presented here is a qualitative indication how to influence the frequency content of relevant signals in order to enhance the stability properties. As a rule, the signal energy in the range where $\mathrm{Re}\,H > 0$ should dominate.

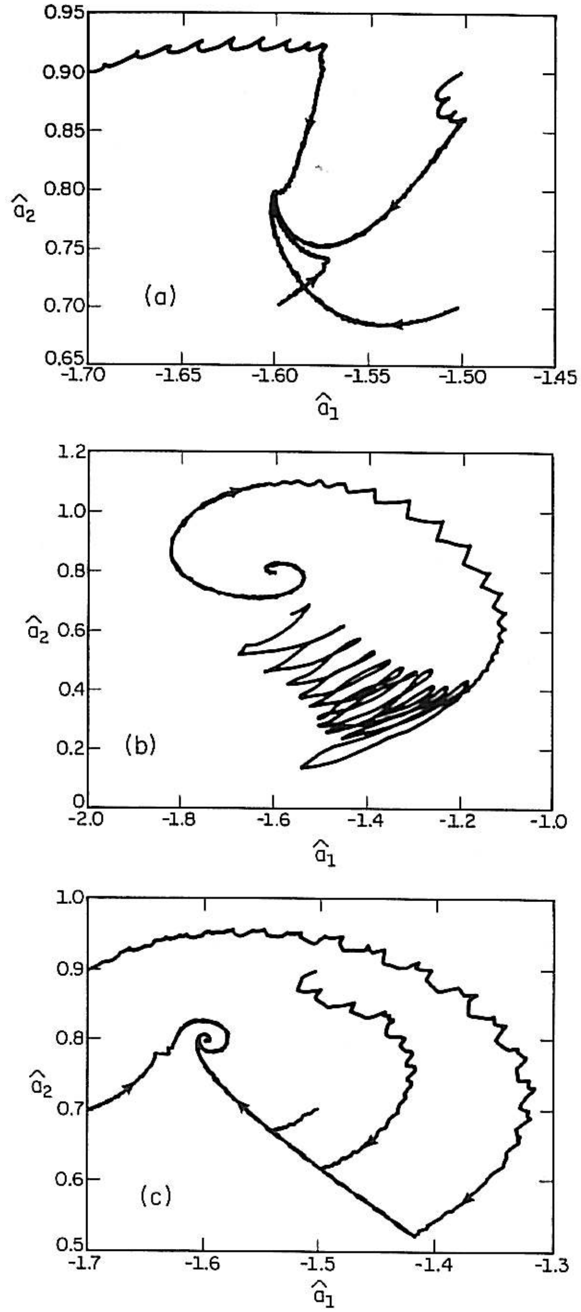

Fig. 2. Simulated trajectories for example. (a) Stable trajectories in Case (a) satisfying both stability conditions of Theorem 2.1 and Corollary 3.2. (b) Unstable trajectories in Case (b) satisfying the instability condition of Theorem 2.1. (c) Stable trajectories of Case (c) satisfying the stability condition of Theorem 2.1 but not that of Corollary 3.2.

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