APII - 1986 - 20 - 295 - 315 — **Commande des systèmes complexes technologiques**

Decentralized indirect adaptive control

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Abstract/*Résumé*

We prove that a decentralized adaptive controller based on a completely decoupled reduced order model can stabilize a complex system subject to structure uncertaintes. Our study is done for an indirect scheme assuming that the stabilizability problem is solved. The important aspects are :

— the adaptation law which incorporates robustifying mechanisms such as signal normalization and guarentees parameter boundedness,

— the techniques of proof we have used which apply to many other robustness problems of adaptive schemes.

Nous établissons qu'un contrôleur adaptatif décentralisé conçu à partir d'un modèle d'ordre réduit supposé découplé, peut stabiliser un système complexe sujet à des incertitudes de structure. Notre étude est faite pour un schéma indirect en supposant résolu le problème de la stabilisabilité. Les aspects importants sont :

— l'utilisation d'une loi d'adaptation modifiée garantissant la bornitude des paramètres adaptés et utilisant des signaux normalisés,

— les méthodes utilisées dans nos démonstrations qui s'appliquent à de nombreux autres problèmes de robustesse des schémas adaptatifs.

Keywords/*Mots clés*

Adaptive control, decentralized control, robustness, indirect schemes.

Commande adaptative, commande décentralisée, robustesse, schéma indirect.

1. Introduction

Classically, adaptive controllers are designed from the assumption that, though parametrically unknown, the plant has some known structure properties. It is now well established that such controllers, in this nominal

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situation, meet their objective (Goodwin, Sin, 1984). But one can justely question the behavior in presence of not only parametric but also structure uncertainty, i.e. possible changes for which the adaptation process is not a priori designed. For this problem to be well posed, both the adaptive controller (corresponding to the nominal structure) and the structure uncertainty have to be precised.

Such a situation is encountered in the decentralized control of interconnected systems with reduced order models. The nominal structure of the plant is that of a collection of M decoupled subsystems and the structure uncertainty is due to the actual presence of coupling terms, neglected local dynamic effects and external disturbances. Isolation and quantification of these uncertainties require a precise knowledge of the nominal structure parameters. This is a strong requirement since the operating conditions of large scale systems are often such that the parameters are poorly known. This is a typical case where adaptive schemes could be very helpful. Unfortunately, their study in this context has not received so much attention. The more complete results have been given by Ioannou and Kokotovic (1985) for the continuous time case. They have shown that classical adaptive controllers can lead to instability, and, introducing the so called σ-modification, they have derived sufficient conditions which guarentee the existence of a region of attraction to a reference signal dependent residual set. These results, however, have been obtained only if each isolated subsystem dominant part has relative degree one.

The reference signal dependence mentionned above has been more deeply understood and written as a frequency dependent positivity condition by Ortega and Kelly (1985). They have obtained this condition for a very simple controller by applying the local analysis technique introduced by Riedle and Kokotovic (1984).

To see how the results depend on the structure uncertainty, we can compare the above results with those of Gavel and Siljak (1985). They have established that unmodified adaptive controllers lead to global stability if the only structure uncertainty is due to the possible presence of coupling terms which are within the range of control variables and accessible to the measured outputs.

As far as the discrete time case is concerned, (and as far as we are aware of) no such results are available. Let us mention however the paper by Yang and Papavassilopoulos (1985) about a partially decentralized adaptive controller where all the outputs, but not the inputs, are available to each controller.

In this paper, we treat the discrete time counterpart of the problem studied by Ioannou and Kokotovic. Unfortunately, their technique is too particular to the relative degree one continuous time case and we have not been able to extend it here. We have preferred to extend the results of (Praly, 1983) for a single input-single output plant to the multi input-multi output decentralized case (see also the extensions given by Samson (1983)). In particular, besides a parameter projection somehow equivalent to the σ-modification, we have incorporated the normalization procedure used in (Praly, 1983). Though our analysis applies similarly to the direct scheme case, we present here an indirect decentralized adaptive controller based on a pole placement design. This allows us to get rid of the minimum phase assumption which is so critical for discrete time systems. However, inherent with an indirect approach, is the estimated model stabilizability problem. This question is the theme of very interesting research and some important results have been obtained by de Larminat (1984) and Lozano and Goodwin (1985). To simplify this paper, we concentrate our attention on the robustness problem and we assume the stabilizability problem to be solved.

In section 2, we design a decentralized adaptive controller from the data of a nominal structure and a pole placement objective. We study this algorithm in section 3. This motivates the assumptions about the structure uncertainty given in section 4 where our main result is presented. Finally, in section 5, we give our conclusion.

2. A decentralized adaptive controller based on pole placement design

Our objective is to design local adaptive controllers for a plant whose nominal structure is assumed to be a collection of M decoupled single input-single output time invariant linear rational subsystems. Each of them is represented by :

$$A^{\alpha}(q^{-1})\, y^{\alpha}(k) = B^{\alpha}(q^{-1})\, u^{\alpha}(k-1) \quad \alpha = 1, \dots, M, \tag{2.1}$$

where $u^{\alpha}(k)$, $y^{\alpha}(k)$ are the input and output respectively of the α'th subsystem, with zero initial conditions, and $A^{\alpha}(q^{-1})$, $B^{\alpha}(q^{-1})$ are coprime polynomials in the unit delay operator q^{-1}, with known degree m^{α} but unknown coefficients. Collecting all the coefficients of $A^{\alpha}(q^{-1})$, $B^{\alpha}(q^{-1})$ in a vector θ^{α} and introducing the vector $\Phi^{\alpha}(k)$ as :

$$\Phi^{\alpha}(k)' = (y^{\alpha}(k-1) \dots y^{\alpha}(k-m^{\alpha})\; u^{\alpha}(k-1) \dots u^{\alpha}(k-m^{\alpha}))\,, \tag{2.2}$$

we can rewrite (2.1) in :

$$y^{\alpha}(k) = \theta^{\alpha\prime}\Phi^{\alpha}(k) \quad \alpha = 1, \dots, M. \tag{2.3}$$

For each subsystem, the information available to its local controller is its own input-output signals. Its control criterion is to place the closed loop poles as defined by a polynomial $A_d^\alpha(q^{-1})$ of degree $2\,m^\alpha - 1$, and to follow " as well as possible " a reference output $y_d^\alpha(k)$.

We remark that for any vector θ^α (with the coprimeness assumption), we can define a vector ψ^α as the solution of the following linear system (equivalent to a Bezout identity) :

$$\Omega(\theta^\alpha)\,\psi^\alpha = \omega^\alpha\,, \tag{2.4}$$

where ω^α is a vector collecting the coefficients of $A_d^\alpha(q^{-1})$:

$$\omega^{\alpha\prime} = (1\ a_{d1}^\alpha \ldots a_{d2m^\alpha-1}^\alpha)\,, \tag{2.5}$$

and $\Omega(\theta^\alpha)$ is a matrix collecting the coefficients of θ^α or equivalently of $A^\alpha(q^{-1})$, $B^\alpha(q^{-1})$:

$$\Omega(\theta^\alpha) = \begin{bmatrix} 0 & & 1 & \\ b_1^\alpha & & a_1^\alpha & \\ & 0 & & 1 \\ b_{m^\alpha}^\alpha & b_1^\alpha & a_{m^\alpha}^\alpha & a_1^\alpha \\ & b_{m^\alpha}^\alpha & & a_{m^\alpha}^\alpha \end{bmatrix}. \tag{2.6}$$

With these notations, we can now introduce a decentralized adaptive controller which would achieve our control objective if the plant would actually satisfy (2.1). It is a modified version of the algorithm proposed by Goodwin and Sin (1984). In particular, it incorporates a least squares parameter estimation, with parameter projection, signal normalization and covariance matrix regularization (for more details, see (Praly, 1983)) :

$$e^\alpha(k) = y^\alpha(k) - \theta^\alpha(k-1)'\,\Phi^\alpha(k) \tag{2.7}$$

$$r^\alpha(k)^2 = \mu^2\,r^\alpha(k-1)^2 + \mathrm{Max}\,[\|\Phi^\alpha(k)\|^2, (r_0^\alpha)^2] \tag{2.8}$$

$$f^\alpha(k) = \frac{1}{r^\alpha(k)^2 + \Phi^\alpha(k)'\,P^\alpha(k-1)\,\Phi^\alpha(k)} \tag{2.9}$$

$$\underline{\theta}^\alpha(k) = \theta^\alpha(k-1) + f^\alpha(k)\,P^\alpha(k-1)\,\Phi^\alpha(k)\,e^\alpha(k) \tag{2.10}$$

$$\underline{P}^\alpha(k) = P^\alpha(k-1) - f^\alpha(k)\,P^\alpha(k-1)\,\Phi^\alpha(k)\,\Phi^\alpha(k)'\,P^\alpha(k-1) \tag{2.11}$$

$$\theta^\alpha(k) = \theta_0^\alpha + \mathrm{Min}\left[1, \frac{R^\alpha(\lambda_1^\alpha/\lambda_0^\alpha)^{1/2}}{\|\underline{\theta}^\alpha(k) - \theta_0^\alpha\|}\right](\underline{\theta}^\alpha(k) - \theta_0^\alpha) \tag{2.12}$$

$$P^\alpha(k) = \left(1 - \frac{\lambda_0}{\lambda_1}\right) \underline{P}^\alpha(k) + \lambda_0 I \tag{2.13}$$

$$\psi^\alpha(k) = \Omega(\theta^\alpha(k))^{-1}\, \omega^\alpha \tag{2.14}$$

$$\psi^\alpha(k)'\, \Phi^\alpha(k+1) = E^\alpha(k)\, A_d^\alpha(q^{-1})\, y_d^\alpha(k) \tag{2.15}$$

$$E^\alpha(k) = \begin{cases} \dfrac{1}{B^\alpha(k,1)} & \text{if } \ |B^\alpha(k,1)| > \varepsilon \\ 1 & \text{if not} \end{cases} \tag{2.16}$$

where, with $b_i^\alpha(k)$ the m^α last components of $\theta^\alpha(k)$:

$$B^\alpha(k,1) = \sum_{i=1}^{m^\alpha} b_i^\alpha(k)\,. \tag{2.17}$$

The design of this controller consists in choosing values for the following parameters (omitting the superscript α) :

m = subsystem model order, defines the number of parameters.

$0 \leqslant \mu < 1$, r_0 characterize the normalization factor $r(k)$ whose magnitude influences the speed of adaptation.

R, θ_0 are the radius and center of the projection sphere $S(R, \theta_0)$ which defines the area in which the true parameter vector is assumed to lie.

$0 < \lambda_0 < \lambda_1$ fix lower and upper bounds for the P-matrix which defines the geometrical weighing of the parameter vector and influences the speed of adaptation.

$A_d(q^{-1})$ defines the desired closed loop poles. *We choose this polynomial such that its spectral radius is strictly smaller than* μ.

$E(k)$ is chosen to obtain a unit tracking *dc*-gain. More generally E can be be any polynomial in q^{-1} with bounded time varying coefficients.

The study of the feedback system will allow us to precise the effective role of some of these parameters.

Remark : Since the matrix $\Omega(\theta^\alpha(k))$ has to be inverted, this algorithm needs some monitoring to guarentee its invertibility or equivalently to guarentee the controllability of the identified model given by $\theta^\alpha(k)$ (see de Larminat, 1984, for example). There is no difficulty if all the models given by θ in the sphere with radius $R(\lambda_1/\lambda_0)^{1/2}$ and center θ_0 are controllable. In any case, as mentionned in introduction, we assume that this problem is solved and in particular that we have :

ASSUMPTION $A1$: *For all i, j,*

$$\| \psi^\alpha(i) - \psi^\alpha(j) \| \leqslant g\, \| \theta^\alpha(i) - \theta^\alpha(j) \|\,. \tag{2.18}$$

3. Properties of the adaptive controller

In this section, we deal only with one subcontroller. To simplify the notations, we omit the superscript α.

A first property is given by the adaptation law (2.7)-(2.13) :

LEMMA 1 : *The parameter estimate satisfy, for all k :*

$$\| \theta(k) \| \leqslant T \tag{3.1}$$

$$\| \theta(k) - \theta(k-1) \| \leqslant s \frac{| e(k) |}{r(k)}, \tag{3.2}$$

with

$$s = 2 \lambda_1^{1/2} . \tag{3.3}$$

Proof : See (Praly, 1983).

COROLLARY 1 : *Under assumption A1, we have also, for all k :*

$$\| \psi(k) \| \leqslant P \tag{3.4}$$

$$\| \psi(k) - \psi(k-1) \| \leqslant gs \frac{| e(k) |}{r(k)} . \tag{3.5}$$

These very elementary properties are sufficient to prove the following lemma :

LEMMA 2 : *Under assumption* A1 :

(i) *there exist positive constants* Γ, Δ *such that, for all k and* K_0, *we have :*

$$\mu^{-2k} r(k)^2 \leqslant \Pi(K_0) \mu^{-2K_0} r(K_0)^2 + \Delta \mu^{-2k} + \Gamma \sum_{n=K_0+1}^{k-1} \mu^{-2n} e(n)^2 \tag{3.6}$$

where, with Γ_1 *a constant given in the proof, we have :*

$$\Pi(K_0) = 1 + \Gamma_1 \operatorname*{Max}_{k \in [0,K_0]} \left[\frac{e(k)}{r(k)} \right]^2 \tag{3.7}$$

(ii) *there exists a positive constant* Γ_2 *such that :*

$$| e_t(k) | \leqslant \Gamma_2 \sum_{n=0}^{m-1} | e(k-n) | \tag{3.8}$$

where $e_t(k)$ is defined as the following « tracking » error :

$$e_t(k) = A_d(q^{-1})\, y(k) - \sum_{i=1}^{m} b_i(k)\, E(k-i)\, A_d(q^{-1})\, y_d(k-i). \quad (3.9)$$

Proof : see Appendix A.

We remark that this lemma is established without any assumption on the actual plant or subsystem. Therefore, it establishes a property of the adaptive controller only.

Property ii explains why by reducing the estimation error $e(k)$, we also reduce the « tracking » error $e_t(k)$. Note that we use tracking in quotes since ideally a tracking error would involve the exact parameters b_i, E instead of the estimated ones $b_i(k)$, $E(k)$.

Property *i* proves that the $l_2(\mu)$-norm of the input-output signals on the interval $[K_0, k]$ is essentially proportionnal to the $l_2(\mu)$-norm of the estimation error on the same interval. Taking $K_0 = 0$, this can also be seen as a proof of the existence of a $(2, \mu)$-exponentially stable operator with input $e(k)$ and output $r(k)$. Referring to (Praly, 1985b), we know that boundedness will follow from a sufficient smallness in the mean of the ratio $|e(k)|/r(k)$ (the normalized estimation error).

Let us now state that this ratio is related to a normalized error obtained by using a fixed (non adapted) parameter vector : To each vector θ, we associate the error $w_\theta(k)$ defined as :

$$w_\theta(k) = y(k) - \theta' \Phi(k). \quad (3.10)$$

We have :

LEMMA 3 : *There exists a positive definite quadratic form in $\theta(k) - \theta$ denoted $V_\theta(k)$ such that for all vector θ in the sphere with radius R and center θ_0, and for all k :*

$$\left[\frac{e(k)}{r(k)}\right]^2 \leqslant V_\theta(k-1) - V_\theta(k) + \gamma \left[\frac{w_\theta(k)}{r(k)}\right]^2, \quad (3.11)$$

$$V_\theta(k) \leqslant W \quad (3.12)$$

with

$$\gamma = 1 + \lambda_1 \qquad W = 4\, R^2\, \frac{(1+\lambda_1)\,\lambda_1}{\lambda_0^2}. \quad (3.13)$$

Proof : See (Praly, 1983).

This emphasizes the adaptation property of our algorithm : the estimated parameter vector is able to do better than any fixed vector, in the sense of the following mean squares normalized estimation error criterion : for all k,

$$\limsup_{K \to \infty} \frac{1}{K} \sum_{n=k+1}^{k+K} \left[\frac{e(n)}{r(n)}\right]^2 \leqslant \gamma \underset{\theta \in S(R, \theta_0)}{\text{Min}} \limsup_{K \to \infty} \frac{1}{K} \sum_{n=k+1}^{k+K} \left[\frac{w_\theta(n)}{r(n)}\right]^2 . \tag{3.14}$$

However, we have to be very careful in interpreting this inequality. The signals, involved here, depend on the estimated parameter vector. Unfortunately it is possible that the only solution for the adaptive controller to minimize this criterion, is to create unbounded signals. This phenomenon has been very well described by Ioannou and Kokotovic (1984). Hence, to exploit this result, we need first to prove at least the boundedness of the input-output signals.

We remark also that for each subsystem which would satisfy (2.1) or (2.3), the corresponding normalized estimation error would be square summable. Consequently, as mentioned above, the corresponding input-output signals would be bounded.

4. Main result

Though our adaptive controller is designed for a collection of completely decoupled subsystems, we assume that the actual plant is multi input-multi-ouput and admits the following representation :

$$A^\alpha(q^{-1})\, y^\alpha(k) = B^\alpha(q^{-1})\, u^\alpha(k-1) + w^\alpha(k) \quad \alpha = 1, \ldots, M, \tag{4.1}$$

or equivalently

$$y^\alpha(k) = \theta^{\alpha\prime}\, \Phi^\alpha(k) + w^\alpha(k) \quad \alpha = 1, \ldots, M, \tag{4.2}$$

where $w^\alpha(k)$ results from the structure uncertainty associated with the nominal structure defined by (2.1), i.e. it represents all the effects which have not been taken into account in the models used for the controller design. We assume :

ASSUMPTION A2 : *θ^α belongs to the sphere with radius R^α and center θ_0^α.*

ASSUMPTION A3 : *Characterization of the structure uncertainty :*

$$w^\alpha(k) = v^\alpha(k) + H^\alpha[\Phi^1(i), \ldots, \Phi^M(i)\,;\, 0 \leqslant i \leqslant k, k]\,, \tag{4.3}$$

where $v^\alpha(k)$ is a bounded sequence (output disturbances + initial conditions) :

$$v^\alpha(k)^2 \leqslant v \tag{4.4}$$

and H^α is an operator with a finite $(2, \mu)$-exponential gain, i.e. :

$$| H^\alpha[\Phi^1(i), \ldots, \Phi^M(i)\,;\, 0 \leqslant i \leqslant k, k] |^2 \leqslant \varepsilon \sum_{\alpha=1}^{M} r^\alpha(k)^2 . \tag{4.5}$$

Technically this assumption is motivated by lemma 3 and the comments following lemma 2. However, it is also practically interesting since, in particular, it allows H^α to represent all the linear (or non-linear) coupling terms if they are defined by operators which are μ-exponentially stable with a gain smaller than ε.

We remark that H^α does depend on the choice of θ^α. It is interesting to think of θ^α as being given by the following minimization problem :

$$\underset{\theta^\alpha \in S(R^\alpha, \theta_0^\alpha)}{\text{Min}} \quad \underset{\{k, u^\alpha(.), y^\alpha(.) \in 1_{2e}(\mu)\}}{\text{Max}} \frac{|w_\theta^\alpha(k)|^2}{\sum_\alpha r^\alpha(k)^2} \tag{4.6}$$

Since, in a global stability analysis, we have no a priori informations about the signals, we have used the worst case in this definition. However, we know that the adaptive controller, with its adaptation property (3.14), would allow us to minimize using the actual input output signals. Since the optimal value of this criterion is nothing but an expression for ε as introduced in (4.5), we can guess that this definition (4.6) will lead to very conservative bounds. Conversely, in a local analysis, one could replace the complete $1_{2_e}(\mu)$ space by the actual signals of the solution about which the analysis is done and the criterion (4.6) being smaller than ε could be written as a signal (or frequency) dependent condition (see Ortega, Kelly, 1985).

For the decentralized adaptive controller of section 2, implemented in feedback with the plant defined by (4.1) (or (4.2)), we have the following properties :

THEOREM : *Under assumptions* A1 *to* A3, *there exists* ε_* *such that for all* ε, $0 \leqslant \varepsilon \leqslant \varepsilon_*$, *the signals* $u^\alpha(k)$, $y^\alpha(k)$ *generated by the feedback system are bounded, i.e. :*

$$\sum_{\alpha=1}^{M} r^\alpha(k)^2 \leqslant S \quad \forall k. \tag{4.7}$$

Proof : See Appendix B.

Comments :

(i) With in mind property *ii* of lemma 2, we are also interested in the behavior of the estimation error.

From (3.11), assumptions A2, A3 and this boundedness result, we obtain : for all k, α :

$$|e^\alpha(k)| \leqslant [\text{Max}(0, V_{\theta^\alpha}^\alpha(k-1) - V_{\theta^\alpha}^\alpha(k))]^{1/2} r^\alpha(k) + \\ + (1 + \lambda_1^\alpha)^{1/2} (v^{1/2} + \varepsilon^{1/2} S^{1/2}) \tag{4.8}$$

Hence, given a small δ, for each time k such that :

$$V^{\alpha}_{\theta^{\alpha}}(k-1) - V^{\alpha}_{\theta^{\alpha}}(k) \leqslant \delta \tag{4.9}$$

i.e. for each time for which $V^{\alpha}_{\theta}(k)$ is not too much decreasing, the corresponding subsystem satisfies :

$$|e^{\alpha}(k)| \leqslant (1+\lambda^{\alpha}_1)^{1/2} v^{1/2} + (\delta^{1/2} + (1+\lambda^{\alpha}_1)^{1/2} \varepsilon^{1/2}) S^{1/2}) . \tag{4.10}$$

This is exactly what could be expected from a non adaptive decentralized controller. Namely, the error is proportionnal to the external disturbances (i.e. v) and to the unmodelled dynamics effects (i.e. εS). The interesting fact is that the gain is $(1+\lambda^{\alpha}_1)^{1/2}$ and therefore decreases as λ^{α}_1 decreases, i.e. as the speed of adaptation decreases.

Unfortunately, this result is not always true since it depends on (4.9). How often is this inequality satisfied ?

Assume that for all n in $[k+1, k+K]$, we have :

$$V^{\alpha}_{\theta^{\alpha}}(n-1) - V^{\alpha}_{\theta^{\alpha}}(n) \geqslant \delta . \tag{4.11}$$

Summation in n and (3.12) yield :

$$4\, R^{\alpha 2} \frac{(1+\lambda^{\alpha}_1)\,\lambda^{\alpha}_1}{\lambda^{\alpha 2}_0} \geqslant V^{\alpha}_{\theta^{\alpha}}(k) - V^{\alpha}_{\theta^{\alpha}}(k+K) \geqslant \delta K . \tag{4.12}$$

This implies that (4.9) happens at least once for each $4\, R^{\alpha 2} \dfrac{(1+\lambda^{\alpha}_1)\,\lambda^{\alpha}_1}{\delta \lambda^{\alpha 2}_0}$ times.

This establishes that bad behavior of the error can only appear by an oscillatory phenomenon. In particular, referring to the local theory of (Praly, 1985a) and (Praly, 1985c), we can conjecture that this will be the case if the assumptions of our theorem hold but the system has only locally unstable bounded solutions whose estimated parameter vectors are stationary.

(ii) This analysis gives us the following informations about the parameters described in section 2 :

λ^{α}_1 allow us to trade off the rate of convergence *versus* the reduction of error gain.

θ^{α}_0 has to be chosen as the best a priori known model of the α'th subsystem according to the criterion (4.6). Then R^{α} allows us to trade off the confidence we have in this model and the « period » of the possible oscillations of the estimation error $e^{\alpha}(k)$.

The role of μ is more difficult to analyze as it appears only in the proof of the theorem. Our simulation experience shows that there exists some optimal value to which corresponds a smaller bound S for the signals.

5. Conclusion

We have proposed a decentralized adaptive controller based on a pole placement design and on the assumption that the plant is a collection of decoupled single input-single output time invariant rational subsystems with known order but unknown coefficients. This algorithm incorporates estimated parameter vector projection and signal normalization as the robustifying modifications proposed by Praly (1983). We have studied the stability of this controller in feedback with a plant which departs from the nominal structure described above by structure uncertainty such as the possible presence of coupling terms, neglected local dynamics or external disturbances.

The ratio of the error, induced on the subsystem equation by these unexpected effects, to an exponentially weighted l_2-norm of all the input-output signals, is used to quantify this structure uncertainty. Though accounting for a wide class of structure uncertainty effects, this ratio may lead to conservative robustness result if no extra informations are known about the signals.

We have established that if all these error to signal ratios, attached to each subsystem, are sufficiently small then all the input-output signals are bounded. Moreover, each local estimation error is proportional to its corresponding ratio, at least once for each K times. This gain increases as the speed of adaptation is increased, and K increases as the uncertainty about the subsystem model a priori parameters is increased.

Hence it is established that discrete time adaptive control can be used for the decentralized control of interconnected systems with simplified models. However this result depends very deeply on our choice of the nominal plant structure and the structure uncertainty. This choice corresponds to the minimal requirements for a decentralized controller. It is an interesting, practically motivated topic of research to extend this study to other types of nominal structure and structure uncertainty.

APPENDIX A

Proof of lemma 2

Part (i) : Let us define $\eta(k)$ as the a posteriori estimation error, i.e. :

$$\eta(k) = y(k) - \theta(k)' \Phi(k) = e(k) - (\theta(k) - \theta(k-1))' \Phi(k). \quad (A.1)$$

Clearly, since :

$$\| \Phi(k) \| \leqslant r(k), \quad (A.2)$$

we have with (3.2) :

$$|\eta(k)| \leqslant (1 + s)\,|e(k)|\,. \tag{A.3}$$

Let us decompose the vectors $\theta(k)$, $\psi(k)$ as follows :

$$\theta(k)' = (-\,a_1(k), \ldots, -\,a_m(k), b_1(k), b_2(k), \ldots, b_m(k)) \tag{A.4}$$

$$\psi(k)' = (d_0(k), \ldots, d_{m-1}(k), 1, c_1(k), \ldots, c_{m-1}(k))\,. \tag{A.5}$$

We denote by $A(k, q^{-1})$, $B(k, q^{-1})$, $C(k, q^{-1})$, $D(k, q^{-1})$ the polynomials with coefficients $a_i(k)$, $b_i(k)$, $c_i(k)$, $d_i(k)$ respectively. From (2.14), it follows that :

$$A(k, q^{-1})\, C(k, q^{-1}) + q^{-1}\, B(k, q^{-1})\, D(k, q^{-1}) = A_d(q^{-1})\,. \tag{A.6}$$

Also we can rewrite the equations (A.1), (2.15) in :

$$A(k, q^{-1})\, y(k) = B(k, q^{-1})\, u(k-1) + \eta(k) \tag{A.7}$$

$$C(k, q^{-1})\, u(k) + D(k, q^{-1})\, y(k) = z(k)\,, \tag{A.8}$$

with :

$$z(k) = E(k)\, A_d(q^{-1})\, y_d(k) \tag{A.9}$$

which is bounded :

$$|z(k)| \leqslant z\,. \tag{A.10}$$

(A.7), (A.8) can be rewritten in a state space form with $\Phi(k)$ as state vector :

$$\Phi(k+1) = F(k)\,\Phi(k) + G_\eta(k)\,\eta(k) + G_z\, z(k) \tag{A.11}$$

where :

$$F(k) = \left[\begin{array}{cccc|cccc}
-a_1(k) & . & -a_{m-1}(k) & -a_m(k) & b_1(k) & . & b_{m-1}(k) & b_m(k) \\
1 & 0 & . & 0 & 0 & & & 0 \\
0 & . & 0 & & & & & \\
. & & & . & . & & & . \\
0 & . & 1 & 0 & 0 & & . & 0 \\
\hline
\begin{array}{c} -d_1(k) \\ +d_0(k)\,a_1(k) \end{array} & . & \begin{array}{c} -d_{m-1}(k) \\ +d_0(k)\,a_{m-1}(k) \end{array} & +d_0(k)\,a_m(k) & \begin{array}{c} -c_1(k) \\ -d_0(k)\,b_1(k) \end{array} & . & \begin{array}{c} -c_{m-1}(k) \\ -d_0(k)\,b_{m-1}(k) \end{array} & -d_0(k)\,b_m(k) \\
0 & . & & 0 & 1 & 0 & . & 0 \\
 & & & & 0 & . & 0 & \\
. & & & . & . & & & . \\
0 & . & . & 0 & 0 & . & 1 & 0
\end{array}\right]$$

$$G_\eta(k)' = (1, 0, \ldots, 0 \mid -\,d_0(k), 0, \ldots, 0) \tag{A.13}$$

$$G_z' = (0, 0, \ldots, 0 \mid 1, 0, \ldots, 0)\,. \tag{A.14}$$

The identity (A.6) yields :

$$\det(I - q^{-1} F(k)) = A_d(q^{-1}) \tag{A.15}$$

and the inequalities (3.1), (3.4) implies the following boundedness result :

$$\| F(k) \| \leqslant F . \tag{A.16}$$

Hence, as a consequence of theorem 5 of (Fuchs, 1982), there exist f, ρ such that :

$$\| F(k)^n \| \leqslant f\rho^n \tag{A.17}$$

and, with the choice of $A_d(q^{-1})$, ρ can be chosen as :

$$0 \leqslant \rho < \mu . \tag{A.18}$$

Finally from (3.2), (3.5), it follows that, for all k :

$$\| F(k) - F(k-1) \| \leqslant G \frac{| e(k) |}{r(k)} . \tag{A.19}$$

Let us now apply the variation of constants formula to (A.11). Using our above inequalities, we get, with the boundedness of $G_\eta(k)$ and the zero initial conditions of $\Phi(k)$:

$$\| \Phi(k+1) \| \leqslant f \sum_{n=0}^{k} \rho^{k-n} [\| F(k) - F(n) \| \, \| \Phi(n) \| + p \, | \eta(n) | + z] . \tag{A.20}$$

From inequality (A.19), we know :

$$\| F(k) - F(n) \| \leqslant G \sum_{i=n+1}^{k} \frac{| e(i) |}{r(i)} . \tag{A.21}$$

Hence :

$$\sum_{n=0}^{k} \rho^{k-n} \| F(k) - F(n) \| \, \| \Phi(n) \| \leqslant G \sum_{i=1}^{k} \rho^{k-i+1} \frac{| e(i) |}{r(i)} \sum_{n=0}^{i-1} \rho^{i-1-n} \| \Phi(n) \| . \tag{A.22}$$

Applying the Schwarz inequality and using (2.8), we get :

$$\sum_{n=0}^{i-1} \rho^{i-1-n} \| \Phi(n) \| \leqslant \left[\sum_{n=0}^{i-1} \left[\frac{\rho}{\mu} \right]^{2(i-1-n)} \right]^{1/2} \times$$
$$\times \left[\sum_{n=0}^{i-1} \mu^{2(i-1-n)} \operatorname{Max} (\| \Phi(n) \|^2, r_0^2) \right]^{1/2} \tag{A.23}$$

$$\leqslant \frac{\mu}{(\mu^2 - \rho^2)^{1/2}} r(i-1) \tag{A.24}$$

$$\leqslant \frac{1}{(\mu^2 - \rho^2)^{1/2}} r(i) . \tag{A.25}$$

This allows us to rewrite (A.22) in :

$$\sum_{n=0}^{k} \rho^{k-n} \| F(k) - F(n) \| \| \Phi(n) \| \leqslant \frac{G}{(\mu^2 - \rho^2)^{1/2}} \sum_{i=1}^{k} \rho^{k-i+1} | e(i) | . \tag{A.26}$$

Introducing this relation in (A.20) and using (A.3) lead to :

$$\| \Phi(k+1) \| \leqslant f \sum_{n=0}^{k} \rho^{k-n} \left[\left(\frac{G\rho}{(\mu^2 - \rho^2)^{1/2}} + p(1+s) \right) | e(n) | + z \right] . \tag{A.27}$$

This proves that the operator : $e(k) \rightarrow \Phi(k)$ is $(1, \rho)$-exponentially stable.

Now from the Schwarz inequality, we have (since $\rho < \mu$) :

$$\left[\sum_{n=0}^{k} \rho^{k-n} | e(n) | \right]^2 \leqslant \frac{\mu}{\mu - \rho} \sum_{n=0}^{k} (\mu\rho)^{k-n} e(n)^2 . \tag{A.28}$$

To simplify the notations, let Γ_0, Δ_0 be the following constants :

$$\Gamma_0 = \frac{2 f^2 \mu}{\mu - \rho} \left[\frac{G\rho}{(\mu^2 - \rho^2)^{1/2}} + p(1+s) \right]^2 \tag{A.29}$$

$$\Delta_0 = 2 \left(\frac{fz}{1 - \rho} \right)^2 + r_0^2 . \tag{A.30}$$

With (A.27), (A.28) we have established :

$$\mu^{-2k} \operatorname{Max} (\| \Phi(k) \|^2, r_0^2) \leqslant \Gamma_0 \mu^{-2k} \sum_{n=0}^{k-1} (\mu\rho)^{k-1-n} e(n)^2 + \Delta_0 \mu^{-2k} . \tag{A.31}$$

But by definition of $r(k)$, we have :

$$\mu^{-2K} r(K)^2 - \mu^{-2K_0} r(K_0)^2 = \sum_{k=K_0+1}^{K} \mu^{-2k} \operatorname{Max} (\| \Phi(k) \|^2, r_0^2) . \tag{A.32}$$

Hence

$$\mu^{-2K} r(K)^2 - \mu^{-2K_0} r(K_0)^2 \leqslant$$

$$\leqslant \sum_{k=K_0+1}^{K} \Gamma_0 \mu^{-2k} \sum_{n=0}^{k-1} (\mu\rho)^{k-1-n} e(n)^2 + \sum_{k=K_0+1}^{K} \Delta_0 \mu^{-2k} \tag{A.33}$$

$$\leqslant \Gamma_0 \left[\sum_{n=K_0+1}^{K-1} (\mu\rho)^{-n-1} e(n)^2 \sum_{k=n+1}^{K} \left[\frac{\rho}{\mu}\right]^k + \right.$$

$$\left. + \sum_{n=0}^{K_0} (\mu\rho)^{-n-1} e(n)^2 \sum_{k=K_0+1}^{K} \left[\frac{\rho}{\mu}\right]^k \right] + \frac{\Delta_0}{1-\mu^2} \mu^{-2K} \tag{A.34}$$

$$\leqslant \frac{\Gamma_0}{\mu^2 - \mu\rho} \left[\sum_{n=K_0+1}^{K-1} \mu^{-2n} e(n)^2 + \left[\frac{\rho}{\mu}\right]^{K_0} \sum_{n=0}^{K_0} (\mu\rho)^{-n} e(n)^2 \right] + \frac{\Delta_0}{1-\mu^2} \mu^{-2K} \tag{A.35}$$

$$\leqslant \frac{\Gamma_0}{\mu^2 - \mu\rho} \left[\sum_{n=K_0+1}^{K-1} \mu^{-2n} e(n)^2 + \left[\frac{\rho}{\mu}\right]^{K_0} \underline{e}(K_0) \sum_{n=0}^{K_0} (\mu\rho)^{-n} r(n)^2 \right] + \frac{\Delta_0}{1-\mu^2} \mu^{-2K} \tag{A.36}$$

where $\underline{e}(K_0)$ is defined as :

$$\underline{e}(K_0) = \operatorname*{Max}_{k \in [0, K_0]} \left[\frac{e(k)}{r(k)}\right]^2 . \tag{A.37}$$

Now we remark that, since $\mu^{-2n} r(n)^2$ is increasing, we have :

$$\left[\frac{\rho}{\mu}\right]^{K_0} \sum_{n=0}^{K_0} (\mu\rho)^{-n} r(n)^2 = \sum_{n=0}^{K_0} \left[\frac{\rho}{\mu}\right]^{K_0 - n} \mu^{-2n} r(n)^2 \tag{A.38}$$

$$\leqslant \frac{\mu}{\mu - \rho} \mu^{-2K_0} r(K_0)^2 . \tag{A.39}$$

Hence we get our conclusion :

$$\mu^{-2K} r(K)^2 \leqslant \left[1 + \frac{\Gamma_0}{(\mu-\rho)^2} \underline{e}(K_0) \right] \mu^{-2K_0} r(K_0)^2 + \frac{\Delta_0}{1-\mu^2} \mu^{-2K} +$$

$$+ \frac{\Gamma_0}{\mu^2 - \mu\rho} \sum_{n=K_0+1}^{K-1} \mu^{-2n} e(n)^2 . \tag{A.40}$$

APPENDIX B

Proof of theorem

To simplify the notations, let :

$$\Delta = \operatorname*{Max}_{\alpha} \Delta^{\alpha}, \quad \Gamma = \operatorname*{Max}_{\alpha} \Gamma^{\alpha}, \quad \gamma = \operatorname*{Max}_{\alpha} \gamma^{\alpha}, \quad W = \operatorname*{Max}_{\alpha} W^{\alpha},$$
$$r_0 = \operatorname*{Min}_{\alpha} r_0^{\alpha}. \tag{B.1}$$

We proceed by induction. Let us assume that :

$$\sum_{\alpha=1}^{M} r^{\alpha}(k)^2 \leqslant S, \quad \forall k \leqslant K-1. \tag{B.2}$$

We want to prove that the same property holds for $k = K$, with an appropriate choice of S.

From assumption A.2, we can apply lemma 3 with $\theta = \theta^{\alpha}$. Hence, with assumption A.3 and induction assumption (B.2), we have for all $k \leqslant K-1$:

$$\left[\frac{e^{\alpha}(k)}{r^{\alpha}(k)}\right]^2 \leqslant V_{\theta^{\alpha}}^{\alpha}(k-1) - V_{\theta^{\alpha}}^{\alpha}(k) + 2\gamma \frac{v + \varepsilon S}{r^{\alpha}(k)^2} \tag{B.3}$$

with for all α, k :

$$0 \leqslant V_{\theta^{\alpha}}^{\alpha}(k) \leqslant W. \tag{B.4}$$

In particular, since :

$$r_0 \leqslant r^{\alpha}(k) \tag{B.5}$$

we have :

$$\left[\frac{e^{\alpha}(k)}{r^{\alpha}(k)}\right]^2 \leqslant W + 2\gamma \frac{v + \varepsilon S}{r_0^2}, \quad \forall k \leqslant K-1. \tag{B.6}$$

Hence, letting :

$$\underline{\Pi}(S, \varepsilon) = 1 + \Gamma_1 \left[W + 2\gamma \frac{v + \varepsilon S}{r_0^2}\right] \tag{B.7}$$

we obtain the following bound for $\Pi^{\alpha}(K_0)$ (defined in (3.7)) with $K_0 < K$:

$$\Pi^{\alpha}(K_0) \leqslant \underline{\Pi}(S, \varepsilon). \tag{B.8}$$

In the following we omitt the arguments S, ε when they are not necessary.

This inequality allows us to rewrite (3.6) of lemma 2 for all k in $[K_0, K]$ in :

$$\mu^{-2k} r^{\alpha}(k)^2 \leqslant \underline{\Pi}\mu^{-2K_0} r^{\alpha}(K_0)^2 + \Delta\mu^{-2k} + \Gamma \sum_{n=K_0+1}^{k-1} \left[\frac{e^{\alpha}(n)}{r^{\alpha}(n)}\right]^2 \mu^{-2n} r^{\alpha}(n)^2 . \tag{B.9}$$

Let us now choose K_0. The idea is to obtain an interval $[K_0, K]$ on which the signals (given by $r^{\alpha}(k)$) dominates the external disturbances (given by $v(k)$). We define K_0 as the largest integer, smaller than K, such that :

$$r^{\alpha}(k)^2 \leqslant \frac{v}{\delta} \tag{B.10}$$

where δ is a threshold (to be precised later) such that :

$$0 \leqslant \delta \leqslant \frac{v}{r_0^2} . \tag{B.11}$$

We meet our objective since this choice implies for all k in $[K_0 + 1, K]$:

$$\frac{v}{r^{\alpha}(k)^2} < \delta . \tag{B.12}$$

This defines one K_0 for each α. We have to distinguish three cases for K_0 :

Case 1 : $K_0^{\alpha} = K$:

Then we have :

$$r^{\alpha}(K)^2 \leqslant \frac{v}{\delta} . \tag{case.1}$$

Case 2 : $K_0^{\alpha} = K - 1$:

Then from (B.9) we have :

$$r^{\alpha}(K)^2 \leqslant \mu^2 \, \underline{\Pi} \frac{v}{\delta} + \Delta . \tag{case.2}$$

Case 3 : $K_0^{\alpha} \leqslant K - 2$:

Then (B.3) and (B.12) imply for all k in $[K_0, K - 2]$:

$$\sum_{n=k+1}^{K-1} \left[\frac{e^{\alpha}(n)}{r^{\alpha}(n)}\right]^2 \leqslant W + 2\gamma\delta\left(1 + \frac{\varepsilon S}{v}\right)(K - 1 - k) . \tag{B.13}$$

Hence referring to (Praly, 1985b), we can guess that the result should follow from the smallness of δ and ε. To establish this statement let us apply the

Bellman-Gronwall lemma (see Desoer, Vidyasagar, 1975) to the following inequality, obtained from (B.9) using (B.10) :

$$\mu^{-2k} r^\alpha(k)^2 \leqslant \underline{\Pi}\mu^{-2K_0} \frac{v}{\delta} + \Delta\mu^{-2k} + \Gamma \sum_{n=K_0+1}^{k-1} \left[\frac{e^\alpha(n)}{r^\alpha(n)}\right]^2 \mu^{-2n} r^\alpha(n)^2 . \tag{B.14}$$

We get :

$$\mu^{-2K} r^\alpha(K)^2 \leqslant \begin{cases} \underline{\Pi}\mu^{-2K_0} \frac{v}{\delta} \prod_{k=K_0+1}^{K-1} \left(1 + \Gamma\left[\frac{e^\alpha(k)}{r^\alpha(k)}\right]^2\right) \\ + \Delta\mu^{-2K} \\ + \Delta\Gamma \sum_{k=K_0+1}^{K-1} \left[\frac{e^\alpha(k)}{r^\alpha(k)}\right]^2 \mu^{-2k} \prod_{n=k+1}^{K-1} \left(1 + \Gamma\left[\frac{e^\alpha(n)}{r^\alpha(n)}\right]^2\right). \end{cases} \tag{B.15}$$

But, since :

$$1 + x \leqslant \exp x \tag{B.16}$$

we have (with (B.13)) :

$$\prod_{n=k+1}^{K-1} \left(1 + \Gamma\left[\frac{e^\alpha(n)}{r^\alpha(n)}\right]^2\right) \leqslant \exp\left(\Gamma \sum_{n=k+1}^{K-1} \left[\frac{e^\alpha(n)}{r^\alpha(n)}\right]^2\right) \tag{B.17}$$

$$\leqslant \exp \Gamma W\left[\exp\left(\Gamma\gamma\delta\left(1 + \frac{\varepsilon S}{v}\right)\right)\right]^{2(K-k-1)} . \tag{B.18}$$

For the time being, let us assume that we can find δ, ε, S such that :

$$\nu(\delta, \varepsilon, S) = \mu \exp\left(\Gamma\gamma\delta\left(1 + \frac{\varepsilon S}{v}\right)\right) < 1 . \tag{I.1}$$

With (B.15), this yields (with $U = \exp \Gamma W$) :

$$r^\alpha(K)^2 \leqslant \frac{\underline{\Pi} v \mu^2 U}{\delta \nu^2} \nu^{2(K-K_0)} + \Delta + \Gamma \Delta\mu^2 U \sum_{k=K_0+1}^{K-1} \nu^{2(K-1-k)} \left[\frac{e^\alpha(k)}{r^\alpha(k)}\right]^2 \tag{B.19}$$

or with (B.3), (B.12) and (I.1) :

$$r^\alpha(K)^2 \leqslant \begin{cases} \frac{\underline{\Pi} v \mu^2 U}{\delta \nu^2} \nu^{2(K-K_0)} + \Delta + \Delta\mu^2 U \frac{2 \operatorname{Log} \nu/\mu}{1 - \nu^2} \\ + \Gamma \Delta\mu^2 U \sum_{k=K_0+1}^{K-1} \nu^{2(K-1-k)} [V^\alpha_{\theta^\alpha}(k-1) - V^\alpha_{\theta^\alpha}(k)] \end{cases} \tag{B.20}$$

But summation by part and uniform boundedness of $V_{\theta^\alpha}^\alpha(k)$ lead to :

$$\sum_{k=K_0+1}^{K-1} \nu^{2(K-1-k)}[V_{\theta^\alpha}^\alpha(k-1) - V_{\theta^\alpha}^\alpha(k)] \leqslant W. \tag{B.21}$$

Hence, under assumption (I.1), we have obtained :

$$r^\alpha(K)^2 \leqslant \frac{\Pi(S, \varepsilon)\, v\mu^2\, U}{\delta\nu^2} + \Delta\left(1 + \Gamma W\mu^2\, U + \mu^2\, U \frac{2 \operatorname{Log} \nu/\mu}{1-\nu^2}\right). \quad \text{(case.3)}$$

Clearly, since $\mu < \nu$, this last inequality covers cases 1 and 2.

Summing in α, our result will be established if δ, ε, S satisfy also :

$$\frac{\Pi(S, \varepsilon)}{\delta} \frac{Mv\mu^2\, U}{\nu^2} + M\,\Delta\left[1 + \Gamma W\mu^2\, U + \mu^2\, U \frac{2 \operatorname{Log} \nu/\mu}{1-\nu^2}\right] \leqslant S \tag{I.2}$$

with ν the function of δ, ε, S defined in (I.1).

Therefore, our proof relies on the existence of positive δ, ε, S satisfying (I.1), (I.2). Choosing arbitrarily ν in $(\mu, 1)$, we denote by h_1, h_2, h_3 the following constants :

$$h_1 = \left(1 + \Gamma_1\left(W + 2\gamma \frac{v}{r_0^2}\right)\right) \frac{Mv\mu^2\, U}{\nu^2} \tag{B.22}$$

$$h_2 = 2 \frac{\Gamma_1\, \gamma}{r_0^2} \frac{Mv\mu^2\, U}{\nu^2} \tag{B.23}$$

$$h_3 = M\,\Delta\left(1 + \Gamma W\mu^2\, U + \mu^2\, U \frac{2 \operatorname{Log} \nu/\mu}{1-\nu^2}\right). \tag{B.24}$$

We notice that :

$$h_1 - h_2\, v > 0 \tag{B.25}$$

and that, with (B.7), (I.2) can be rewritten in :

$$\frac{h_1}{\delta} + h_2 \frac{\varepsilon S}{\delta} + h_3 \leqslant S. \tag{B.26}$$

From (I.1), we see that we can take any ε satisfying :

$$\varepsilon \leqslant \frac{\operatorname{Log} \nu/\mu - \Gamma\gamma\delta}{\Gamma\gamma\delta S}\, v. \tag{B.27}$$

And, from (I.2), it is sufficient to take S as :

$$S = \frac{h_1 - h_2\, v}{\delta} + \frac{h_2\, v \operatorname{Log} \nu/\mu}{\Gamma\gamma\delta^2} + h_3\,. \tag{B.28}$$

Therefore, let us define δ_* as an argument of the following maximization problem :

$$\operatorname*{Max}_{0 \leqslant \delta \leqslant v/r_0^2} J(\delta)$$

with

$$J(\delta) = \frac{v\delta(\operatorname{Log} \nu/\mu - \Gamma\gamma\delta)}{\Gamma\gamma h_3\, \delta^2 + \Gamma\gamma(h_1 - h_2\, v)\, \delta + h_2\, v \operatorname{Log} \nu/\mu}\,. \tag{B.29}$$

This maximum is finite since we have (B.25) and δ_* is strictly positive.

Then we can choose S as given by (B.28) with $\delta = \delta_*$ and :

$$0 \leqslant \varepsilon \leqslant \varepsilon_* = J(\delta_*)\,. \tag{B.30}$$

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