51st IEEE Conference on Decision and Control
December 10-13, 2012. Maui, Hawaii, USA

Globally Convergent Nonlinear Observer for the Sensorless Control of Surface-Mount Permanent Magnet Synchronous Machines

Jérémy Malaizé, Laurent Praly and Nicolas Henwood

***Abstract*— For a surface-mount Permanent Magnet Synchronous Motor (SM-PMSM) with currents and voltages as only measurements, we propose an observer estimating the rotor electrical phase. It comes from an appropriate choice of coordinates and exploits convexity. We prove its global convergence under the only assumption that the rotor does not stop. Experimental results assess the validity of the proposed observer.**

I. Introduction

A. The context

The problem of estimating the mechanical state components, i.e. rotor position and speed, for a SM-PMSM from the measured electrical variables has a very long history [8], rich in various and efficient methods. There are off course several technical reasons to go for sensorless control of SM-PMSM: cost reduction, wires removal and reliability improvement.

No matter what technique is considered, from a control designer point of view, it always comes down to feed an estimate of the rotor position to a state feedback, in charge of controlling the torque delivered by the machine. It is now well established that to design such a feedback, it is easier to work with a two-phase model expressed in a frame rotating with the motor. The so called dq model, that is central to the well-known field-oriented control scheme, see [7] and [8]. However, it turns out that, for the state estimation problem, a model expressed in a fixed frame is more appropriate. This remark was made in [5]. Actually, in that paper it was shown that, by immersing the standard four dimensional model leaving in $\mathbb{R}^3 \times \mathbb{S}^1$ into a five dimensional one leaving in $\mathbb{R}^5$, it is possible to describe the SM-PMSM dynamics with a triangular structure. In particular this makes possible the reconstruction of the rotor position from a simple two-dimensional subsystem, completely decoupled from the mechanical behavior of the motor. To be more specific, position and speed estimation may be carried out without any precise of the mechanical load connected to the machine shaft. There is no need for inertia, friction or load torque knowledge. This is quite an appealing characteristics, since this is usually rather complicated to accurately access these data, subject to changes when the motor is in operation.

This two-dimensional subsystem has been used as a design tool for the gradient observer proposed and studied experimentally in [3], [6] and theoretically in [4]. Unfortunately the study of the corresponding error system, closely related to an averaged approximation of the periodically forced van der Pol oscillator, is very difficult.

In the following, we suggest to modify this gradient observer by taking advantage of convexity. Convexity also has a long history in estimation and particularly in adaptive control. See [1] for instance. But in trying to exploit it, we end up facing a difficulty very similar in spirit to the one presented on [2, page 418]. As in that paper we round it by a conditional correction term.

This paper is organized as follows. The end of Section I briefly considers the dynamical system considered hereafter, and the different assumptions made throughout the paper. Section II is devoted to the design of an observer for a dynamical system, exhibiting an elementary drift and a convex output function. Under some mild assumptions, the proposed observer is proved to yield global asymptotic convergence of the estimated state to the actual state of the system. In Section III, the problem of estimating the flux of a SM-PMSM is addressed. It is first showed that a SM-PMSM falls under the scope of systems with elementary drift and output convex function. We then present the reconstruction of the position and the speed from the observed flux, and briefly sketch the implementation of a sensorless field-oriented control scheme. Some experimental results assess the relevancy of the proposed observation scheme. We wrap up the paper with some concluding remarks and future work directions in Section IV.

B. System modeling and problem statement

In the following, the equations of SM-PMSM cast in the so-called stationary frame are considered. By making use of the Faraday's and Joule's Laws, the phase-to-neutral voltages at the SM-PMSM terminals read

$$v = Ri + \dot{\Psi} \tag{1}$$

This allows us to relate the voltages v at the SM-PMSM terminals to the derivative of the total flux Ψ encompassed by the windings, and to the currents i within them, while R stands for the stator windings resistance. The quantities v, i as well as Ψ are two dimensional vectors, and, for the case of SM-PMSM, the total flux may be expressed as

$$\Psi = Li + \Phi \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2}$$

where θ is the rotor electrical phase, Φ is the flux created by the magnets and L is the inductance. Note that the flux

J. Malaizé and N. Henwood are with the Control and Systems Department, IFP New Energy, Rueil-Malmaison, FRANCE. jeremy.malaize@ifpen.fr
L. Praly is with the Centre d'Automatique et Systèmes, ParisTech, École des Mines, Fontainbleau, FRANCE. laurent.praly@mines-paristech.fr

978-1-4673-2064-1/12/$31.00 ©2012 IEEE

Ψ is constrained to evolve on a circle with radius Φ and time-varying center Li, for it is solution of:

$$|\Psi - Li|^2 - \Phi^2 = 0 \tag{3}$$

It is further assumed that the currents i are measured, while the control inputs v are known. This paper is devoted to the determination of the electrical phase θ from the only knowledge of v and i, and under a perfect knowledge of the SM-PMSM physical parameters, namely R, Φ, and L. The proposed scheme consists in first estimating the flux and then extracting the electrical phase from it.

II. An observer for a system with an elementary drift and a convex output function

In this section, we consider a broader class of dynamical systems, for which we shall build a globally convergent nonlinear state observer. The system (1) previously described obviously falls under the scope of this study. Consider a dynamical system with state x of dimension two, whose dynamics simply reads:

$$\dot{x} = u(t), \tag{4}$$

with x in $\mathbb{R}^2$, $u : \mathbb{R}_+ \to \mathbb{R}^2$ a continuous function whose value $u(t)$ is known at each time t. Assume further that u as well as its integral are bounded on $\mathbb{R}_+$. Denote by $\chi(t)$ a particular solution of the previous dynamical system, which, according to the different assumptions, is necessarily defined and bounded on $\mathbb{R}_+$.

We restrict our study to the case where the initial conditions $\chi(0)$ and the exogenous input u are such that χ runs along a closed curve which, for any given t, is described as :

$$h(x,t) = 0, \tag{5}$$

where $h : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ is a C^2 function. In the context of observer design h is considered as an output function whose measurement is zero for all time. Namely, as in [5], we rewrite (5) as :

$$y(t) = h(x,t),$$

with y identically zero.

Further assumptions on the function h are also required:

A1 For any positive scalar c, one may find $H_0(c)$, $H_1(c)$ and $H_2(c)$ such that :

$$\forall (x,t) \ : \ |x| \leq c,\ t \geq 0\ , \quad |h(x,t)| \leq H_0(c)\ ,$$
$$\left|\frac{\partial h}{\partial x}(x,t)\right| \leq H_1(c)\ , \quad \left|\frac{\partial h}{\partial t}(x,t)\right| \leq H_2(c)\ .$$

A2 There exists a strictly positive scalar Δ such that the Hessian matrix of h satisfies :

$$\frac{\partial^2 h}{\partial x^2}(x,t) \geq 2\Delta I, \quad \forall x,\ t \geq 0.$$

A3 For any solution of (4), and any unit vector e in $\mathbb{R}^2$, there exists a sequence t_n going to infinity as n grows indefinitely such that:

$$\frac{\partial h}{\partial x}(\chi(t_n), t_n)\ .\ e > 0.$$

Remark 1:

1. A1 simply requires that the functions $x \mapsto h(x,t)$ and $x \mapsto \dfrac{\partial h}{\partial x}(x,t)$ have to be bounded, uniformly in t, on any compact subset of $\mathbb{R}^2$.
2. According to A2, the function $x \mapsto h(x,t)$ is convex and coercive uniformly in t.
3. If A3 is fulfilled, this means that the gradient of h evaluated along any solution χ of (4) indefinitely points in all the directions of $\mathbb{R}^2$. This may be viewed a persistency of excitation condition, which will be central in the study of the error system generated by our observer.

Since we know $u(t)$ and $h(\cdot,t)$ in the system (4), with output $y = h(x,t)$, we propose the following as an observer for the state x

$$\dot{\hat{x}} = u(t) - \mu\ \frac{\partial h}{\partial x}(\hat{x},t)\ \max\{0,\ h(\hat{x},t)\} \tag{6}$$

where μ is an arbitrary strictly positive real number. It is an algorithm of a gradient type but with a correction term which is "on" only when $h(\hat{x},t)$ is non negative.

Proposition 1: Under assumptions A1 to A3, the observer (6) makes the zero error set $\mathscr{Z} = \left\{(x,\hat{x}) \in \mathbb{R}^2 : x = \hat{x}\right\}$ globally and asymptotically stable.

Proof: Let us start by noting that, with the help of A2, Taylor's Formula with integral remainder gives, for any $t > 0$ and (x_1,x_2) in $\mathbb{R}^2$:

$$\begin{aligned} h(x_2,t) \quad &= \quad h(x_1,t) + \frac{\partial h}{\partial x}(x_1,t)[x_2 - x_1] \\ &+ \frac{1}{2}[x_2 - x_1]^\top \left[\int_0^1 (1-s)\frac{\partial^2 h}{\partial x^2}(x_2 + s(x_2 - x_1),t)ds\right][x_2 - x_1] \\ &\geq \quad h(x_1,t) + \frac{\partial h}{\partial x}(x_1,t)[x_2 - x_1] + \Delta|x_1 - x_2|^2. \end{aligned}$$

Provided $h(x_1,t) \geq h(x_2,t)$, using the previous relation leads to the following remarkable inequality, valid at any time $t > 0$:

$$\frac{\partial h}{\partial x}(x_1,t)\,[x_1 - x_2] \geq \Delta\,|x_1 - x_2|^2\,,\ \forall (x_1,x_2) \ : \ h(x_1,t) \geq h(x_2,t) \tag{7}$$

Now, the error dynamics is obtained by combining (4) and (6), and simply reads:

$$\dot{\tilde{x}} = -\,\mu\ \frac{\partial h}{\partial x}(\hat{x},t)\ \max\{0,\ h(\hat{x},t)\}\,. \tag{8}$$

Let us consider $V = \frac{1}{2}\tilde{x}^\top\tilde{x}$ as a Lyapunov function candidate and compute its time derivative:

$$\dot{V} = -\,\mu\ \max\{0,\ h(\hat{x},t)\}\ .\ \frac{\partial h}{\partial x}(\hat{x},t)\ (\hat{x} - x)\,.$$

Making use of (7) allows us to conclude that:

$$\dot{V} \leq -2\mu\,\Delta\max\{0,\ h(\hat{x},t)\}\ .\ V\ .$$

It follows that the zero error set $\mathscr{Z}$ is globally stable. Moreover, the evaluation of V at time t which we denote $V(t)$ satisfies:

$$V(t) \leq \exp\left(-2\mu\Delta\int_0^t \max\{0, h(\hat{x}(s),s)\}\,ds\right)V(0).$$

The attractivity of the set $\mathscr{X}$ follows when the following integral diverges :

$$I(t) = \int_0^t \max\{0, h(\widehat{x}(s), s)\}\, ds.$$

Let us establish this attractivity by contradiction. We assume that V does not converge to 0. This implies that I converges. Since $\mathscr{X}$ is globally stable, and the function $t \mapsto \chi(t)$ is bounded, the function $t \mapsto \widehat{x}(t)$ is bounded too. There exists a positive scalar c, such that $|\widehat{x}(t)| \leq c$, and according to A1 and (8):

$$|\dot{\widetilde{x}}| \leq \mu H_1(c) \max\{0, h(\widehat{x}, t)\}.$$

This yields for any $0 \leq t_1 \leq t_2$,

$$|\widetilde{x}(t_2) - \widetilde{x}(t_1)| \leq \int_{t_1}^{t_2} |\dot{\widetilde{x}}(s)| ds \leq \mu H_1(c) \, |I(t_2) - I(t_1)|.$$

The convergence of $I(t)$ implies that, for any sequence t_n tending to infinity, $I(t_n)$ is a Cauchy sequence, and so is $\widetilde{x}(t_n)$ as a consequence of the previous inequality. It follows that $\widetilde{x}(t_n)$ admits a limit, denoted $\widetilde{x}_\star$, which is different from zero by assumption. Rewrite $\widetilde{x}$ as :

$$\widetilde{x}(t) = \widetilde{x}_\star + \varepsilon_1(t),$$

with $\varepsilon_1 : \mathbb{R}_+ \to \mathbb{R}^2$ tending to zero.

Moreover, according to A1 and the fact that $\dot{\widehat{x}}$ is bounded, the function $t \mapsto h(\widehat{x}, t)$ is uniformly continuous, and the same conclusion applies to the function $t \mapsto \max\{0, h(\widehat{x}(t), t)\}$. For I converges, Barbalat's Lemma may be invoked to come up with:

$$\lim_{t \to \infty} \max\{0, h(\widehat{x}, t\} = 0.$$

This allows us to define the vanishing function $\varepsilon_2 : \mathbb{R}_+ \to \mathbb{R}^2$ as:

$$\varepsilon_2(t) = \max\{0, h(\widehat{x}(t), t\} \qquad (\geq h(\widehat{x}(t), t)\ ).$$

From the definition of $\widetilde{x}$ and ε_1, this translates into:

$$\varepsilon_2(t) \geq h(\chi(t) + \widetilde{x}_\star + \varepsilon_1(t), t).$$

Let us now make use of (7) together with the fact that $h(\chi(t), t) = 0$ to come up with:

$$\varepsilon_2(t) \ \geq \ \frac{\partial h}{\partial x}(\chi(t), t)\, (\widetilde{x}_\star + \varepsilon_1(t)) + \Delta |\widetilde{x}_\star + \varepsilon_1(t)|^2$$

Since h satisfies A1, $t \mapsto \chi(t)$ is bounded, $\widetilde{x}_\star$ is non-zero and ε_1 and ε_2 tend to zero. There exists a time $T > 0$ such that:

$$0 \geq \frac{\partial h}{\partial x}(\chi(t), t)\, \widetilde{x}_\star + \Delta |\widetilde{x}_\star|^2 \qquad \forall t \geq T.$$

This is in contradiction with $A3$. So the set $\mathscr{X}$ must be globally attractive. ∎

III. Sensorless control of SM-PMSM

A. Assumptions validation

Let us move back to the system described through (1), (2) and (3). It is straightforward to check that it falls under the scope of the context exposed in section III. Let first denote $u = v - Ri$, $x = \Psi$, and finally

$$h(x,t) = |x - Li(t)|^2 - \Phi^2.$$

The assumptions A1 through A3 still have to be verified:

A1 This condition is fulfilled as soon as the currents i are bounded uniformly in t, which is necessary the case from a practical point of view, to avoid deteriorating the SM-PMSM. Actually, we have

$$|h(x,t)| \leq |x|^2 + L^2 |i|^2 + \Phi^2,$$

and

$$\left| \frac{\partial h}{\partial x}(x,t) \right| \leq 2|x| + 2L|i|$$

while the boundedness of $\dfrac{\partial h}{\partial t}$ is directly linked to the existence of physical bounds on the voltages v and the currents i, see (1).

A2 This assumption is readily satisfied, since

$$\frac{\partial^2 h}{\partial x^2}(x,t) = 2\Delta I,$$

with $\Delta = 1$.

A3 In the case of SM-PMSM, the particular solution $\chi(t)$ of (1) turns out to be given by (2) for some positive function $\theta : \mathbb{R}_+ \to \mathbb{R}$. When h is defined according to (3), its gradient evaluated along this specific solution χ simply reads:

$$\frac{\partial h}{\partial x}(\chi(t), t) = 2\Phi \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}.$$

Any unit vector e in $\mathbb{R}^2$ with argument φ is expressed as $e = \begin{pmatrix} \cos\varphi & \sin\varphi \end{pmatrix}$, and

$$\frac{\partial h}{\partial x}(\chi(t), t)\,.\, e = 2\Phi \cos(\theta - \varphi).$$

It follows that Assumption A3 is satisfied as soon as the rotor electrical phase does not freeze at some constant value φ and so, in particular, when the motor rotates indefinitely in the same direction, i.e. clockwise or anti-clockwise.

B. Flux and position observer

In view of the elements previously reported, we claim that the following observer :

$$\dot{\widehat{\Psi}} = -Ri + v - \mu\ .\ \max\left\{0, |\widehat{\Psi} - Li|^2 - \Phi^2\right\}\ .\ \left(\widehat{\Psi} - Li\right) \quad (9)$$

allows to asymptotically observe the total flux in the SM-PMSM. The knowledge of the electrical phase is critical to its accurate control, and this information has to be retrieved from the observed flux. For $\widehat{\Psi}$ does not necessary lie on the

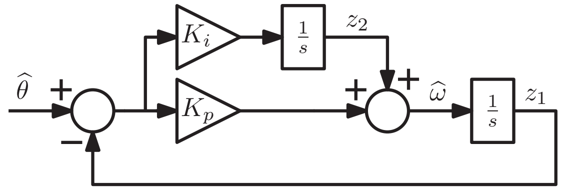


Fig. 1. Electrical pulsation estimation scheme from the phase estimate.

circle with radius Φ and center Li in presence of measurements uncertainties and during transients, the estimation of θ from $\widehat{\Psi}$ may be cast into the following optimization problem:

$$\widehat{\theta} = \arg \min_{0 \leq \theta < 2\pi} \left| \widehat{\Psi} - Li - \Phi \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \right|^2$$

By canceling the gradient of this criterion, and checking the positivity of the Hessian, one may trivially conclude that the electrical phase θ may be estimated through :

$$\begin{pmatrix} \cos\widehat{\theta} \\ \sin\widehat{\theta} \end{pmatrix} = \frac{\widehat{\Psi} - Li}{|\widehat{\Psi} - Li|}. \tag{10}$$

(10) is off course valid provided its denominator is non-zero. Practically speaking, if this situation occurs, or if $\left|\widehat{\Psi} - Li\right|$ gets close to zero, one may freeze the reconstruction of θ as long as it takes for $\left|\widehat{\Psi} - Li\right|$ to move away from zero.

Because $\widehat{\Psi}(t) - Li(t) - \Phi \begin{pmatrix} \cos\theta(t) \\ \sin\theta(t) \end{pmatrix}$ converges to 0 as t goes to infinity, $\hat{\theta}(t) - \theta(t)$ goes also to 0.

C. Speed estimation

To ensure a proper torque control in the flux weakening area [7], an estimate of the speed is highly desirable. Speed information may also be of core interest for speed tracking, though this is not the case in this paper. As in [3], a PLL-type speed estimator, as represented in Figure 1, is used. It may be regarded as a simple PI control loop tracking the input signal $\widehat{\theta}$. Provided the PI gains K_p and K_i are properly set, and that the incoming phase estimate lies within the resulting closed loop bandwidth, one may estimate the electric pulsation $\widehat{\omega}$ by doing.

D. Sensorless torque control scheme

All these elements allow to design a complete position sensorless torque control scheme, as depicted in Figure 2, not requiring any dedicated sensor provided the motor does not stop, see the validation of A3 for SM-PMSM. It is quite common to deal with a SM-PMSM featuring 3 electrical phases, usually named a, b and c. Applying the so-called Concordia Transform, to the vectors of currents and voltages, enables to derive the vector i and v of (1). From the estimated position $\widehat{\theta}$, a rotation of angle $-\theta$, i.e. the Park Transform, is applied to i to get the direct and quadrature currents, i_d and i_q respectively. A hierarchical control scheme is then used, as commonly acknowledged [7]:

- The torque to be delivered by the SM-PMSM, i.e. $\tau^\star$, is translated into direct and quadrature currents references $i_d^\star$ and $i_q^\star$.
- Two local PI controllers are in charge of making the quantities i_d and i_q (obtained from $\widehat{\theta}$) track the references $i_d^\star$ and $i_q^\star$ respectively.
- The outputs of these PI controllers are the direct and quadrature voltages, cast into voltages in the stationary frame by an inverse Park Transform, relying on the estimated electrical phase.

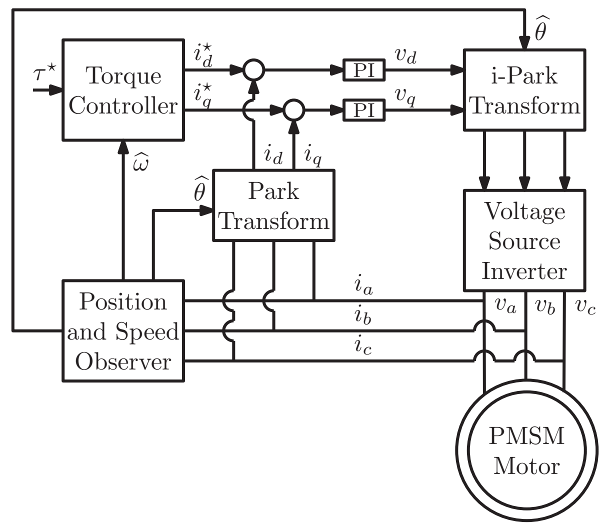
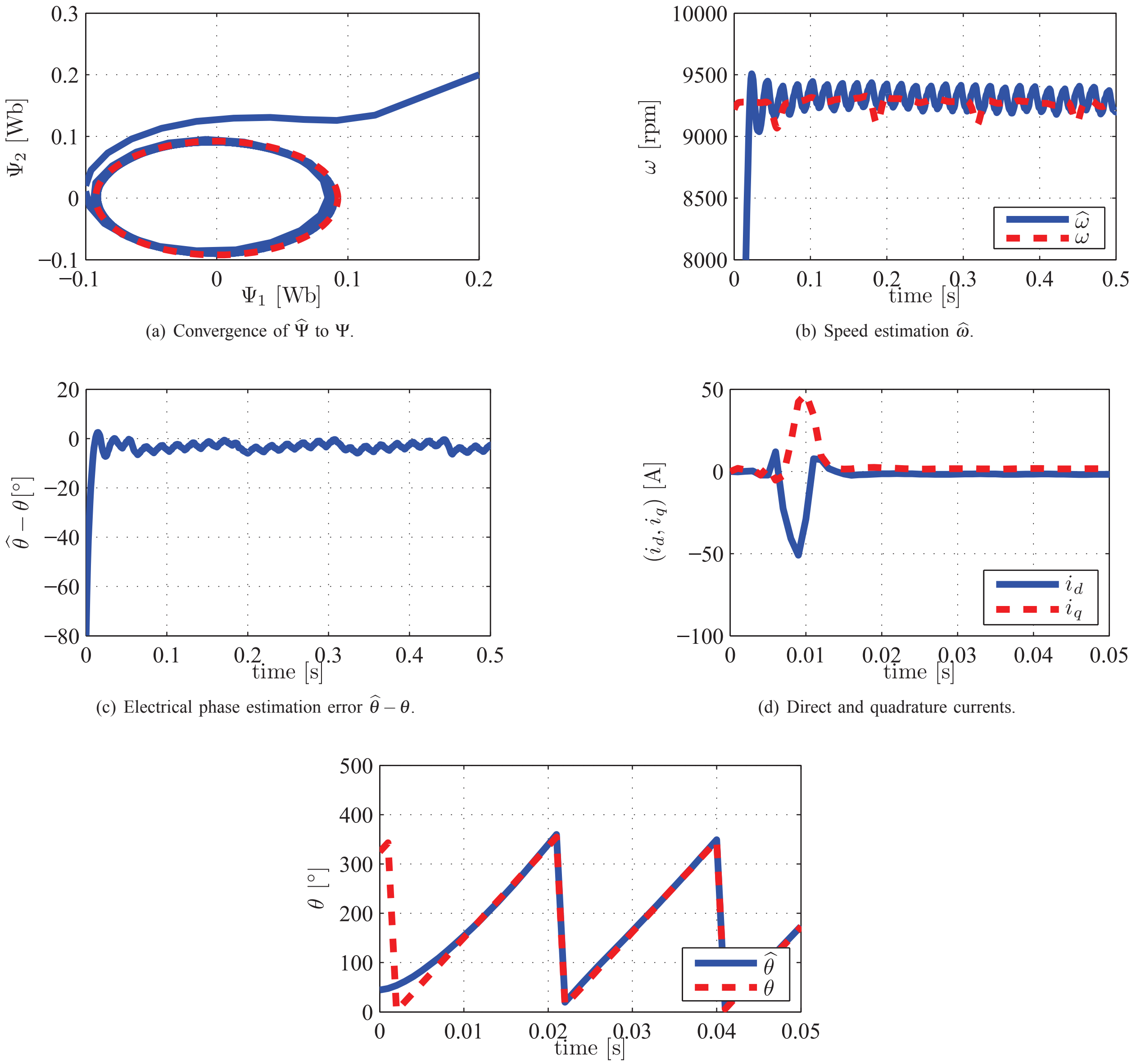


Fig. 2. Sensorless control scheme for SM-PMSM.

E. Experimental setup

This section presents the test bench used to experimentally validate the performances of the previously designed control scheme. The testbed is composed of two PMSM, connected through a shaft featuring a torque sensor delivering accurate torque measurements. This setup is illustrated in Figure 3. These two drives have respective rated power of 1.7kW and 2.2kW, and similar rated speed of 6000rpm. They are intended to deliver a desired torque for the first one, and control the rotary speed of the shaft for the latter. The control strategy previously discussed is implemented in a

Fig. 3. Experimental setup.



(a) Convergence of $\widehat{\Psi}$ to Ψ.

(b) Speed estimation $\widehat{\omega}$.

(c) Electrical phase estimation error $\widehat{\theta}-\theta$.

(d) Direct and quadrature currents.

(e) Convergence of $\widehat{\theta}$ towards θ.

Fig. 4. Experimental results of the proposed sensorless FOC scheme, at 1000 rpm.

voltage inverter connected to the first machine. Also note that this machine is equipped with a fine position sensor used to compare the relevancy of the estimated position and speed to the actually values of these quantities in operation. For information, the machine to be controlled exhibits the physical parameters reported in Table I.

stator resistance R [Ω]	0.25
windings inductance L [mH]	0.77
magnets flux Φ [Wb]	0.075
number of poles pairs p [-]	3

TABLE I
MAIN PHYSICAL PARAMETERS OF THE TESTED SM-PMSM.

F. Experimental results

The experimental results are those given in Figure 4, for the previously mentioned SM-PMSM operated at 1000rpm, and required to deliver $i_d^\star = -2$A and $i_q^\star = 2$A. At $t = 0$, while the motor is rotating, both the control and the observation schemes are turned on.

Note that no direct measurements of the actual flux are at our disposal. The best to be done is to compare the estimated position and speed to the measured values. Still, with the

Figure 4(a), one may check that $\widehat{\Psi} - Li$ does asymptotically reach the neighborhood of a circle of radius Φ (whose value is given in Table I). As expected, a fairly accurate estimation $\widehat{\theta}$ of the electrical phase θ is provided. This is first illustrated in Figure 4(e) that allows to come up to the conclusion that it takes less than an electrical revolution of the estimation error to become fewer than a couple of degrees. Figure 4(c) gives an even more informative aspect of the observation error between 0° and -5°. Looking at Figure 4(c), there are clearly two components in the observation error:

- A low frequency term, around 10Hz, which is solely caused by aliasing in the acquisition of the actual position. This components is not caused by the proposed control scheme.
- A high frequency component, with smaller magnitude, and which has to be attributed to the observation scheme. We shall comment further on this in the concluding section. Simply note that this frequency exactly equals the second harmonics with pulsation 2ω.

We deliberately set the adaptation gain μ of the observer to a rather small value. As a consequence, the convergence of the estimated position to the value of the actual one could be significantly sped up. The relative slowness of the observer causes a poor behavior of the control loop during the transients, see Figure 4(d). Especially between $t = 0$ and $t = 0.02s$, the actual currents are far from their desired values. This points is definitely set once the flux observer has reached the circle, after $t = 0.2$s. Finally, the estimation of the speed is reliable in Figure 4(b). However, for the same reasons as earlier, there is a high frequency term with pulsation 2ω, and it may be seen that the measurement of the electrical pulsation ω by numerical differentiation of θ does feature the same low frequency component at 10Hz. This definitely shows that this components is only linked to the acquisition system, and has nothing to do with the estimation scheme.

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, we proposed a nonlinear observer for the problem of observing the flux of SM-PMSM. It turns out to be a slight modification of a previous scheme, proposed in [4] and experimentally validated in [3]. In view of the complexity of the proof reported [4], the modified gradient search exposed in this paper enjoys some remarkable properties, on top of which the global and asymptotic convergence of the estimated flux to the actual one, provided the motor does not stop. As a consequence, this theoretically guarantees an accurate determination of both the electrical phase and pulsation of SM-PMSM, in order to design a sensorless FOC controller.

B. Future Work

As briefly addressed in the experimental results section, when the motor is rotating at electrical pulsation ω, the observation error, see Figures 4(c) and 4(b), exhibits a component with pulsation 2ω. This is actually due to a slight saliency on the rotor side. Saliency changes the previous expression of the flux (2) into:

$$\Psi = Li + L_1 \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} i + \Phi \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix},$$

and adds a second harmonic to the flux. If neglected in the observation, it does induce an observation error at 2ω. From a practical point of view, vibrations, together with undesirable noise, will occur, which is of course unacceptable. In view of this, future work consists in addressing the sensorless control, using the same framework, i.e. a flux observer in the same coordinates, but for salient-pole PMSM. This point is more challenging, as it requires to derive an implicit expression, similar to (3), but for the flux such as expressed in this section. Furthermore, it may be required to modify the assumptions A1 to A3, as the relationship to derive for the flux might not fulfill them, as stated so far.

REFERENCES

[1] A. Fradkov, Speed gradient scheme and its applications in adaptive control problem. Automation and Remote Control. Vol. 40, pp. 1333-1342, 1979

[2] A. Fradkov, R. Ortega, G. Bastin, Semi-adaptive control of convexly parameterized systems with applications to temperature regulation of chemical reactors. International Journal of Adaptive Control and Signal Processing. Vol. 15, 4, Jpp. 415-426, June 2001

[3] J. Lee, J. Hong, K. Nam, R. Ortega, L.Praly, A. Astolfi, Sensorless control of surface-mount permanent magnet synchronous motors based on a nonlinear observer. IEEE Transactions on Power Electronics, Vol. 25 , N. 2, February 2010 , Pages 290 - 297

[4] R. Ortega, L. Praly, A. Astolfi, J. Lee, K. Nam, Estimation of Rotor Position and Speed of Permanent Magnet Synchronous Motors With Guaranteed Stability. IEEE Transactions on Control Systems Technology, Issue: 99, 1 - 13, 2010

[5] F. Poulain, L. Praly, R. Ortega. An Observer for Permanent Magnet Synchronous Motors with Currents and Voltages as only Measurements. Proceedings of the 47th IEEE Conference on Decision and Control, December 2008

[6] W. Dib, R. Ortega, J. Malaizé. Sensorless control of permanent-magnet synchronous motor in automotive applications: Estimation of the angular position . Proceedings of the 37th Annual Conference on IEEE Industrial Electronics Society, November 2012

[7] J. Chiasson, Modeling and High-Performance Control of Electric Machines. IEEE Press, 2005

[8] P. Vas, Sensorless Vector and Direct Torque Control. Oxford Science Publication, 1998