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Improving the Performance of an Electrostatically Actuated MEMS by Nonlinear Control: Some Advances and Comparisons

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***Abstract*— Though certain control schemes, such as charge control and capacitive feedback, can extend the stable travelling range of electrostatically actuated micro-devices to the full gap, the transient behavior of actuators is dominated by their mechanical dynamics. Thus, the performance may be poor if the natural damping of the devices is too low or too high. The presented work aims at improving the performance of a parallel-plate electrostatic micro-actuator by nonlinear control. Three control schemes, based on differential flatness, control Lyapunov functions, and backstepping, are considered, which are capable of stabilizing the system while ensuring desired performances. The simulation results demonstrate the efficiency of the considered control schemes and provide some comparisons on their performance.**

I. Introduction

This paper addresses the problem of control of a one degree of freedom (1DOF), parallel-plate electrostatic actuator driven by a voltage source, whose scheme is shown in Fig 1. This model is widely used in the study of micro-electromechanical systems (MEMS), such as micro-mirrors, optical gratings, variable capacitors, and accelerometers, though it may be considered as a reduced-order model of an infinite dimensional micro-device, e.g. a micro-beam or micro-plate.

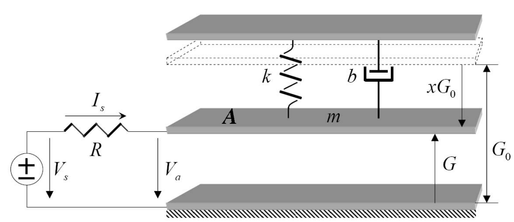

Fig. 1. 1DOF parallel-plate electrostatic actuator.

It is well known that by using a static open-loop voltage control scheme, the stable travelling range of the moveable plate in an electrostatic actuator is limited to one third of its full gap (also called *pull-in* position) due to the instability beyond that point. Extending the travelling range of the moveable plate is the primary objective in the control of such devices. Several closed-loop control schemes have been proposed for this purpose (see, e.g. [2], [6], [13]) and it has been shown that certain popular techniques for extending the travelling range of actuators, such as employing charge control scheme [8], [9] and adding series capacitances to a voltage control scheme [9], are effective implementations of nonlinear feedback control [6].

Besides stabilizing the actuator around the set-point of operations, many applications of MEMS impose also stringent requirements on the actuator transient behavior, such as settling times, overshoots, and oscillations. To illustrate the performance issue in the control of MEMS, let us consider actuators using ideal charge control scheme for which the dynamics of the electrical subsystem are much faster than the ones of the mechanical subsystem. The behavior of the actuator will then be dominated by the latter. Fig. 2 shows the responses corresponding to a deflection of 50% gap of three systems with damping ratio of 0.1, 1 and 5, respectively. Clearly, the system with high damping ratio suffers from a long settling time, while the one with low damping ratio suffers from oscillations and important overshoot.

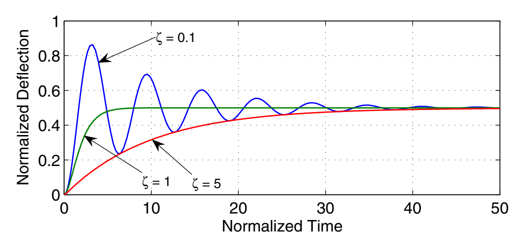

Fig. 2. Responses of parallel-plate electrostatic actuators using ideal charge control scheme.

It is recognized to be a very complex task to incorporate all the mentioned factors into the design of control algorithms in the framework of linear control theory, and compromising the optimality of the system is inevitable [2]. This motivates the application of nonlinear control techniques to improve the overall performance for MEMS devices.

In this paper three nonlinear control methods, namely flatness-based control, control Lyapunov function (CLF) synthesis, and backstepping design, will be used in the design of control system for electrostatically actuated MEMS devices. The considered problem is set-point control from any point in the gap to any other points between the electrodes.

The flatness-based control employed in this work was presented firstly in [5], which combines the techniques of trajectory planning and robust nonlinear control. More precisely, based on differential flatness [4], a feasible reference

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trajectory is constructed first in the phase plane, ensuring fast rise time while having well damped transient response. Then a robust closed-loop feedback control obtained by potential method [1], [3] is added to the control scheme, that makes the reference trajectory an attractive invariant manifold.

The application of the method of constructing a quadratic CLF based on the solution to an algebraic Riccati equation is inspired from the work presented in [7]. The CLF is used with Sontag's stabilizing feedback [12] to enhance the stability and improve the performance with respect to a linear controller. The performance tuning of such a controller can then use the intuition from classical LQR control.

It will be seen that the considered system admits the so-called lower triangular form and hence, it is suitable to use the technique of backstepping to construct stabilizing control laws. Since the formulation of backstepping can be directly applied to tracking problems, the desired performance can be specified through reference trajectories.

The rest of the paper is organized as follows. Section II presents the model of 1DOF parallel-plate electrostatic actuator. Section III, Section IV, and Section V are devoted to the construction of the above-mentioned control laws. Section VI presents a reduced order speed observer. Finally, the simulation results are reported in Section VII and Section VIII contains some concluding remarks.

II. Modeling of Electrostatic Actuator

The mechanical part of the considered actuator can be represented as a spring-mass-dashpot assembly, as shown in Fig. 1, for which m, b, and k are the mass of the moveable upper electrode, the damping coefficient, and the elastic constant, respectively. The actuator is driven by a voltage source, where $I_s(t)$, $V_s(t)$, and $V_a(t)$ are the source current, the applied voltage, and the actuation voltage, respectively. Let $Q(t)$ be the charge on the device, $G(t)$ be the air gap, G_0 be the zero voltage gap, A be the plate area, and ε be the permittivity in the gap. Then the equation of motion of the actuator is given by [10]:

$$m\ddot{G}(t)+b\dot{G}(t)+k(G(t)-G_0)=-\frac{Q^2(t)}{2\varepsilon A}. \quad (1)$$

The current through the resistor R can be obtained by applying Kirchhoff's Voltage Law and reads [10]:

$$\dot{Q}(t)=\frac{1}{R}\left(V_s(t)-\frac{Q(t)G(t)}{\varepsilon A}\right). \quad (2)$$

To make the system analysis and control design easier, we transform the system (1)-(2) into normalized coordinates by changing the time scale, $\tau=\omega_0 t$, and performing a normalization as follows [8]:

$$x=1-\frac{G}{G_0},\ q=\frac{Q}{Q_{pi}},\ u=\frac{V_s}{V_{pi}},\ i=\frac{I_s}{V_{pi}\omega_0 C_0},\ r=\omega_0 C_0 R,$$

where $C_0=\varepsilon A/G_0$ is the capacitance at rest, $V_{pi}=\sqrt{8kG_0^2/27C_0}$ the pull-in voltage, $Q_{pi}=\frac{3}{2}C_0V_{pi}$ the pull-in charge, $\omega_0=\sqrt{k/m}$ the undamped natural frequency, and $\zeta=b/2m\omega_0$ the damping ratio.

Let $v=\dot{x}$, then the system (1) and (2) can be written in the normalized coordinates as:

$$\begin{cases}\dot{x}=v\\ \dot{v}=-2\zeta v-x+\frac{1}{3}q^2\\ \dot{q}=-\frac{1}{r}q(1-x)+\frac{2}{3r}u\end{cases}, \quad (3)$$

which is defined on the state space $\chi=\{(x,v,q)\subset\mathbb{R}^3 \mid x\in[0,1],\ q\geq 0\}$.

Since in what follows we will deal only with normalized quantities, we can use t to denote the time and omit the qualifier "normalized" while not causing any confusion.

III. Flatness-Based Control

It has been shown that the system (3) is differentially flat with $y=x$ as flat output [13]. Therefore all of the remaining states, as well as the input in (3) can be obtained from x and its derivatives until an appropriate order, and it is possible to compute any trajectory of the system without integrating the corresponding differential equations [4]. We remark also that if the reference trajectory is such that y_r has a monotonic behavior then we can use x itself as a parameter instead of time. Moreover, by eliminating the time from (3), the new system:

$$\begin{cases}vv'=-2\zeta v-x+\frac{1}{3}q^2\\ vq'=-\frac{1}{r}q(1-x)+\frac{2}{3r}u\end{cases} \quad (4)$$

where $v'=\frac{dv}{dx}$ and $q'=\frac{dq}{dx}$, is also flat with respect to v (now a function of x). In fact, we have that

$$q^2=3vv'+6\zeta v+3x, \quad (5)$$

$$q'=\frac{3}{2q}(2\zeta v'+v'^2+vv''+1), \quad (6)$$

$$u=\frac{3}{2}(rvq'+q(1-x)), \quad (7)$$

are all functions of v, v' and v'', with $v''=\frac{dv'}{dx}=\frac{d^2v}{dx^2}$. Consequently, any trajectory of (4) may be expressed as a function of the flat output $x\mapsto v(x)$ and its derivatives.

It is convenient to denote by θ, ρ and μ the functions $x\mapsto v$, $x\mapsto q$, and $x\mapsto u$ respectively, to avoid confusion with the time functions. The trajectory planning problem now consists in determining the curve $x\mapsto\theta(x)$.

Since the trajectory we are looking for will connect two equilibrium points, at the initial and final position, x_i and x_f, we have:

$$v_i=v_f=0,\ \dot{v}_i=\dot{v}_f=0,\ \ddot{v}_i=\ddot{v}_f=0.$$

The desired trajectory $x\mapsto\theta(x)$ may therefore be obtained, by interpolation, as a 5th degree polynomial:

$$\theta(x)=a_0+a_1\xi(x)+a_2\xi^2(x)+a_3\xi^3(x)+a_4\xi^4(x)+a_5\xi^5(x), \quad (8)$$

where $\xi(x) = \frac{x-x_i}{X}$ with $X = x_f - x_i$. The coefficients in (8) can be obtained by applying the initial and final conditions and the results are:

$$
\begin{aligned}
a_0 &= 0,\ \ a_1 = Xv'(x_i),\ \ a_2 = \frac{1}{2}X^2v''(x_i),\\
a_3 &= -6Xv'(x_i) - \frac{3}{2}X^2v''(x_i) - 4Xv'(x_f) + \frac{1}{2}X^2v''(x_f),\\
a_4 &= 8Xv'(x_i) + \frac{3}{2}X^2v''(x_i) + 7Xv'(x_f) - X^2v''(x_f), \qquad (9)\\
a_5 &= -3Xv'(x_i) - \frac{1}{2}X^2v''(x_i) - 3Xv'(x_f) + \frac{1}{2}X^2v''(x_f).
\end{aligned}
$$

Since $\dot{v} = vv'$ and $\ddot{v} = 2vv'^2 + v^2v''$, the initial and final constraints on the trajectory will be respected as long as

$$|v'(x_i)| < \infty,\ \ |v'(x_f)| < \infty,\ \ |v''(x_i)| < \infty,\ \ |v''(x_f)| < \infty.$$

The free parameters $v'(x_i)$, $v''(x_i)$, $v'(x_f)$, and $v''(x_f)$ can be chosen, for example, to make the reference trajectory to fit performance specifications, under constraints on drive current and applied voltage.

Note that the pull-in equilibrium point is removed in this approach since we are using time-varying controls that exactly generate the required polynomial reference trajectory of $v(x)$. This remark will even be enhanced by a closed-loop synthesis that makes the reference trajectory tracking stable.

To construct the closed-loop control, we have observed that the function (θ,ρ) previously designed defines an invariant set of system (3). Indeed, for any initial condition (x,v,q) satisfying:

$$(v,q) = (\theta(x), \rho(x)), \qquad (10)$$

the input:

$$u = \mu(x) \qquad (11)$$

satisfies (3) and is such that the corresponding solution $(x(t),v(t),q(t))$ satisfies for all t:

$$(v(t),q(t)) = (\theta(x(t)), \rho(x(t))). \qquad (12)$$

Moreover, this trajectory coincides with the planned reference trajectory.

On this set, the dynamics of system (3) reduce to:

$$\dot{x} = \theta(x). \qquad (13)$$

Outside this set, we have:

$$\dot{x} = v = \theta(x) + (v - \theta(x)). \qquad (14)$$

Thus, in order that the trajectory of the dynamic system (3) converge to the planned reference trajectory, it suffices to asymptotically stabilize the invariant set $v = \theta(x)$. Such objective can be achieved by the closed-loop control proposed in [13] of the following form:

$$
\begin{aligned}
u =& \mu(x) + \frac{3r}{2}\Bigg(\frac{1}{r}(1-x)(q-\rho(x))\\
&+ \left(g_0\left(2\zeta + \theta'(x) + \frac{1}{3}g_0(\rho(x)+q)\right) + \rho'(x)\right)(v-\theta(x))\\
&+ \left(k_0 + \frac{1}{3}g_0(\rho(x)+q)\right)(\rho(x) - q - g_0(v-\theta(x)))\Bigg)
\end{aligned} \qquad (15)
$$

where

$$g_0 \geq 3\sup_{x\in(0,1]}\left(\frac{|\theta'(x)|}{\rho(x)}\right), \qquad (16)$$

such that

$$\eta(x,q) = 2\zeta + \theta'(x) + \frac{1}{3}g_0(\rho(x)+q) > 0, \qquad (17)$$

and k_0 is a positive constant. So that the dynamics of the deviation with respect to the invariant set $e_v = v - \theta(x) = 0$ becomes $\dot{e}_v = -\eta(x,q)e_v - \frac{1}{3}(\rho(x)+q)e_q$, $\dot{e}_q = -k_0e_q + \frac{1}{3}(\rho(x)+q)e_v$ with $e_q = \rho(x) - q - g_0e_v$, and is globally asymptotically stable as can be proved using the Lyapunov function $V = \frac{1}{2}(e_v^2 + e_q^2)$.

IV. CLF SYNTHESIS

Under the framework of CLF synthesis, the control problem is formulated as stabilizing the system around set-points. Consider an equilibrium point $(\bar{x},\bar{v},\bar{q},\bar{u})$, let $\xi_1 = x - \bar{x}$, $\xi_2 = v - \bar{v}$, $\xi_3 = q - \bar{q}$, and $\upsilon = u - \bar{u}$, and note that $\bar{v} \equiv 0$, the system (3) becomes

$$\dot{\xi} = f(\xi) + g(\xi)\upsilon \qquad (18)$$

where

$$f(\xi) = \begin{pmatrix} \xi_2 \\ -2\zeta\xi_2 - \xi_1 + \frac{2\bar{q}}{3}\xi_3 + \frac{1}{3}\xi_3^2 \\ \frac{1}{r}(\bar{q}\xi_1 + \xi_1\xi_3 + (\bar{x}-1)\xi_3) \end{pmatrix},\ g(\xi) = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{3r} \end{pmatrix}. \qquad (19)$$

A smooth, positive definite, and radially unbounded function $V(\xi)$ is called a control Lyapunov function for the system (18) if for all $\xi \neq 0$,

$$L_gV(\xi) = 0 \Rightarrow L_fV(\xi) < 0, \qquad (20)$$

where L_fV and L_gV are Lie derivatives of the CLF, defined as

$$L_fV = \frac{\partial V}{\partial \xi}f(\xi),\ \ L_gV = \frac{\partial V}{\partial \xi}g(\xi).$$

If a CLF is known, a particular stabilizing control law, derived from CLF, is given by Sontag's formula [12]

$$\upsilon = \begin{cases} -\left(L_fV + \sqrt{L_fV^2 + L_gV^4}\right)\Big/L_gV, & L_gV \neq 0,\\ 0, & L_gV = 0. \end{cases} \qquad (21)$$

With the notion of inverse optimal control, it has been shown in [11] that the control law (21) is optimal for the cost functional

$$J = \int_0^\infty \left(\frac{1}{2}p(\xi)L_gV^2 + \frac{1}{2p(\xi)}\upsilon^2\right)dt \qquad (22)$$

where

$$p(\xi)=\begin{cases}-\dfrac{L_fV+\sqrt{L_fV^2+L_gV^4}}{L_gV^2}, & L_gV\neq 0,\\ 0, & L_gV=0.\end{cases} \tag{23}$$

As pointed out in [11] that for an arbitrary CLF, interpreting what the optimality determined from (22) means in terms of performance is extremely difficult. This motivated the use of some well known CLFs, for example the one of the following form:

$$V=\xi^T P\xi, \tag{24}$$

where P is a positive definite matrix satisfying the algebraic Riccati equation

$$PA+A^TP+Q-PBB^TP=0 \tag{25}$$

for

$$A=\left.\frac{\partial f(\xi)}{\partial \xi}\right|_{\xi=0}, \quad B=g(0). \tag{26}$$

The CLF defined in (24) is actually the value function of an LQR optimal control problem that minimizes the cost functional

$$J=\int_0^\infty\left(\xi^TP\xi+\upsilon^2\right)dt \tag{27}$$

subject to the dynamics

$$\dot{\xi}=A\xi+B\upsilon. \tag{28}$$

So the idea is to use the Sontag's formula with the function V given in (24). There is no guarantee in general that it will work in a large domain but at least it will work on a neighborhood of the origin since there V is a CLF.

Note that the LQR formulation in the linearized coordinates is used only to construct a CLF and to help tuning the feedback control. In a neighborhood of the equilibrium point, an approximation of the expression (21) is given by

$$\upsilon_l=-\left(L_{A\xi}V+\sqrt{L_{A\xi}V^2+L_BV^4}\right)\Big/L_BV. \tag{29}$$

Note that the latter expression differs from the optimal control of (27) for the tangent system (28), namely $-B^TP\xi$. More precisely, it can be proved that υ_l is about twice its value [7].

In general, the matrix P used to construct the CLF should be adjusted for each operation point in order to guarantee the overall performance.

V. Backstepping Design

The system (3) being of lower-triangular form, it is possible to recursively construct a CLF by backstepping. In order to make use of the standard design scheme of backstepping, we impose firstly the state transformation $x_3=q^2$, and denote $x_1=x$ and $x_2=v$. The system (3) can be written as

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-2\zeta x_2-x_1+\dfrac{1}{3}x_3\\ \dot{x}_3=-\dfrac{2}{r}x_3(1-x_1)+\dfrac{4}{3r}\sqrt{x_3}u\end{cases}. \tag{30}$$

We consider again the tracking problem with $y=x_1$ as the output. Following a classical approach, we design a reference trajectory y_r for x as a function of time. Then we use backstepping to make this trajectory attractive.

In the first step of backstepping, we consider the control of the first equation in (30) with x_2 as a virtual input. Let $e_1=x_1-y_r$ be the position tracking error, and select a Lyapunov-like function:

$$V_1=\frac{1}{2}e_1^2. \tag{31}$$

Then the time derivative of V_1 along the solutions of the corresponding subsystem of (30) is

$$\dot{V}_1=e_1\dot{e}_1=e_1(x_2-\dot{y}_r)=e_1(e_2+x_{2d}-\dot{y}_r),$$

where $e_2=x_2-x_{2d}$ and x_{2d} is the desired input. By choosing

$$x_{2d}=\dot{y}_r-k_1e_1, \tag{32}$$

we have

$$\dot{V}_1=-k_1e_1^2+e_1e_2.$$

The Lyapunov-like function for the next step of backstepping can be constructed by augmenting V_1 as:

$$V_2=V_1+\frac{1}{2}e_2^2. \tag{33}$$

Now x_3 will be considered as a virtual input to the subsystem $(x_1,\ x_2)$. The time derivative of V_2 along the solutions of the corresponding subsystem is then

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+e_2\dot{e}_2=-k_1e_1^2+e_2\left(e_1+\dot{x}_2-\dot{x}_{2d}\right)\\ &=-k_1e_1^2+e_2\left(e_1-2\zeta x_2-x_1+\frac{1}{3}(e_3+x_{3d})-\dot{x}_{2d}\right)\end{aligned}$$

where $e_3=x_3-x_{3d}$ and x_{3d} is the desired input to subsystem $(x_1,\ x_2)$. If x_{3d} is chosen as

$$x_{3d}=3(-k_2e_2-e_1+2\zeta x_2+x_1+\dot{x}_{2d}), \tag{34}$$

where

$$\dot{x}_{2d}=\ddot{y}_r+k_1(\dot{y}_r-x_2), \tag{35}$$

then

$$\dot{V}_2=-k_1e_1^2-k_2e_2^2+\frac{1}{3}e_2e_3.$$

Finally, a Lyapunov function for the system (30) can be chosen as

$$V_3=V_2+\frac{1}{2}e_3^2. \tag{36}$$

Therefore, with a control given as

$$u=\frac{3r}{4\sqrt{x_3}}\left(-k_3e_3-\frac{1}{3}e_2+\frac{2}{r}x_3(1-x_1)+\dot{x}_{3d}\right), \tag{37}$$

where

$$\begin{aligned}\dot{x}_{3d}=3\Bigg(&(2\zeta-k_1-k_2)\left(-2\zeta x_2-x_1+\frac{1}{3}x_3\right)-k_1k_2x_2\\ &+y_r^{(3)}+(k_1+k_2)\ddot{y}_r+(1+k_1k_2)\dot{y}_r\Bigg),\end{aligned} \tag{38}$$

the time derivative of V_3 along the solutions of (30) becomes

$$\dot{V}_3 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2,$$

which is negative definite for all $k_i > 0$, $i = 1,2,3$.

Note that the control (37) is singular when $x_3 = 0$. However this situation happens only when the system is at rest and it is easy to see that the system (3) is stabilizing about the origin with an input $u = 0$. Therefore this control will stabilize the system (3) in all air gap except in the origin, where it suffices to remove the control signal. The actual feedback should be chosen as:

$$u = \begin{cases} \dfrac{3r}{4\sqrt{x_3}} \left(-k_3 e_3 - \dfrac{1}{3} e_2 + \dfrac{2}{r} x_3 (1 - x_1) + \dot{x}_{3d} \right), & x_3 \neq 0 \\ 0, & x_3 = 0. \end{cases} \tag{39}$$

Now we can conclude that the error dynamics related to the considered tracking problem are asymptotically stable for a feedback law given by (32), (34), (38), and (39).

Since the reference trajectory is arbitrary, we can simply choose $y_r = x_f$ and $\dot{y}_r = \ddot{y}_r = y_r^{(3)} = 0$. We will have then an ordinary backstepping design. The technique of trajectory planning described in Section III can also be used to determine reference trajectories. Since y_r is chosen as $x_r(t)$, it is obtained by integrating the differential equation $\dot{x}_r(t) = \theta(x_r(t))$. Furthermore θ is a polynomial, the solution $x_r(t)$ is smooth in a given open interval of time that we assume to contain the one considered for the reference trajectory design.

This method, referred henceforth as backstepping tracking control design, slightly differs from the flatness-based one of Section III since it is designed to track a reference trajectory which is a function of time, whereas the flatness-based one is a function of x. Therefore, one might expect a greater sensitivity of the feedback (39) to time delays.

VI. Speed Observer Design

Usually, the charge on the device and the gap between the electrodes can be deduced from the input current, the voltage across the device, and the capacitance. However, directly sensing the velocity during the normal operation of the device is extremely difficult, if not impossible. We need therefore to construct a speed observer in order to provide the estimate of v required for implementing the control schemes described in previous sections. A reduced order speed observer can be constructed as follows. We set:

$$z = v - k_v x \tag{40}$$

where k_v is an arbitrary positive real number. Differentiating (40), we get

$$\dot{z} = -((2\zeta + k_v)k_v + 1)x - (2\zeta + k_v)z + \frac{1}{3}q^2. \tag{41}$$

Thus, if we denote

$$\hat{z} = \hat{v} - k_v x$$

where $\hat{v}$ is the estimate of v and $\hat{z}$ the one of z, then

$$\dot{\hat{z}} = -((2\zeta + k_v)k_v + 1)x - (2\zeta + k_v)\hat{z} + \frac{1}{3}q^2. \tag{42}$$

Let $e = \hat{z} - z = \hat{v} - v$ be the estimation error, and note that $\frac{d}{dt}(\hat{z} - z) = \frac{d}{dt}(\hat{v} - v) = \dot{e}$. The error dynamics can be deduced from (41) and (42):

$$\dot{e} = -(2\zeta + k_v)e, \tag{43}$$

which are globally exponentially stable at the origin with a decay rate defined by k_v. This implies that

$$\hat{v} = \hat{z} + k_v x \tag{44}$$

and (42) form an exponential observer.

Due to space limitations, the stability analysis of the closed-loop system, including the above observer, is omitted and will only be demonstrated by numerical simulations.

VII. Examples and Simulation Results

In our simulation, the nominal trajectory in phase plane is shown in Fig 3.*a*. From that, the parameters $v'(x_i)$, $v''(x_i)$, $v'(x_f)$, and $v''(x_f)$ can be deduced, and the reference trajectory can be determined. The obtained reference trajectories corresponding to different deflections are shown in Fig. 3.b.

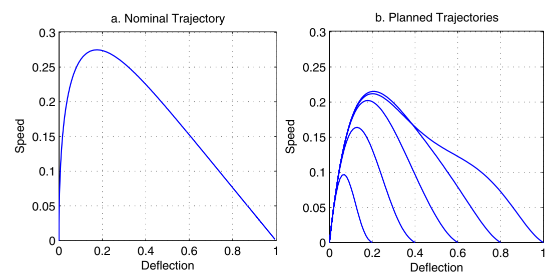

Fig. 3. Nominal and planned trajectories.

The tested system has a damping ratio of 0.1, controlled by the proposed schemes designed for a nominal system with a damping ratio of 0.5. For the purpose of comparison, a charge control scheme is also considered. The system is tested at the set-points equal to a deflection of 10%, 50%, and 100% of full gap. The normalized resistance is chosen to be 1. A pulse voltage with a width of 0.2 and an amplitude of 2 is applied to steer the operation. The actuator is supposed to be driven by a bi-polar voltage source, whose amplitude is limited to ± 3 by a saturator. The simulation results are shown in Fig. 4. It can be seen that all the presented control schemes provide a satisfactory performance in terms of damping oscillations and improving settling time, even in the presence of system parameter mismatches. The tracking control schemes in particular exhibited a better uniformity in different operation points. Nevertheless, it can be expected that a finer tuning should further improve the performance for each of the presented control schemes.

The system is also tested against measurement errors, which are modeled as independent zero-mean uniformly distributed noises with maximum amplitude equal to 5% of the value of set-point and the corresponding charge, respectively. The simulation results are shown in Fig. 5. Note that with the noise free measurements, the observer gain could be arbitrarily high. In the presence of measurement noise, however, the observer gain must be carefully tuned in order to obtain the desired performance while not destroying the stability. Consequently, the system stability is weakened, as observed in our simulation.

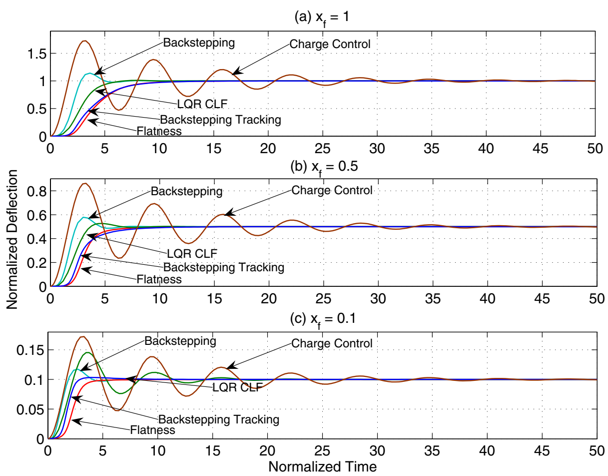

Fig. 4. Comparison of system performance for different deflections.

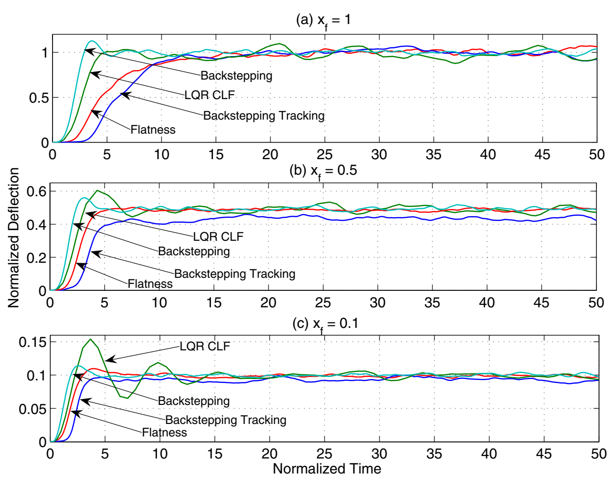

Fig. 5. System responses with noisy measurements.

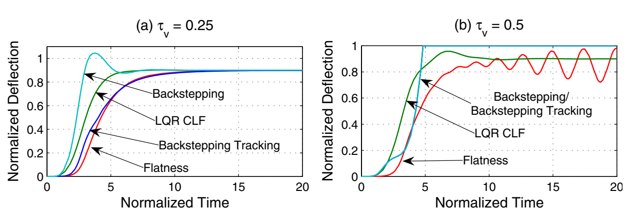

Fig. 6. System responses with control signal delays.

In the last test, we simulated a voltage source delayed by a first order system whose time constant is τ_v. Fig 6 shows the responses corresponding to different control schemes for a voltage source with time constant τ_v equal to 0.25 and 0.5. We have observed that the value of τ_v for which the system starts to diverge is about 0.25 for backstepping schemes with a deflection of 0.1, 0.35 for flatness-based control, and 1.45 for LQR-based control, respectively.

VIII. Conclusions

This paper addressed the nonlinear control of an electrostatic MEMS. It has been shown that for a tracking control scheme, the desired performance can be explicitly specified through reference trajectories. The controller tuning amounts then to obtaining the desired convergence rate to a specific trajectory and hence, it does not depend on operation points. This design is particularly simple using the system flatness, the reference trajectory being designed in the phase plane, namely time-independent. The backstepping approach also gives a reasonably simple design, but with a reference trajectory depending on time in a tracking control scheme. The CLF design, on the contrary, uses the intuition from LQR optimal control in performance tuning, which depends in general on operation points. All the control schemes show satisfactory performances for small measurement errors and delays but different sensitivities to larger ones. Their practical use may thus depend on noise and delay considerations.

IX. ACKNOWLEDGMENTS

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