A TENTATIVE DIRECT LYAPUNOV DESIGN OF OUTPUT FEEDBACKS

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Abstract: Given a finite dimensional nonlinear control system, we study the problem of designing dynamic output feedbacks rendering the origin a globally asymptotically stable equilibrium. First we give necessary structure of an Output Control Lyapunov function. Then we exhibit necessary conditions on a given Control Lyapunov function to allow the derivation of an Output Control Lyapunov function. With the help of a minimax framework, we give also a sufficient condition for the existence of an Output Control Lyapunov function. We check that our necessary condition and sufficient condition coincide for linear systems and are equivalent to the standard necessary and sufficient conditions of stabilizability and detectability. Finally we consider specific structures, in particular "Euler-Lagrange systems" and "integral output" systems.

Keywords: nonlinear stabilization, Output Control Lyapunov function, minimax theorem

1. INTRODUCTION

We consider finite dimensional nonlinear control systems with dynamics:

$$\dot{x} = f(x) + g(x)u \quad , \qquad y = h(x) \, , \tag{1}$$

where f: $\mathbb{R}^n \to \mathbb{R}^n$, g: $\mathbb{R}^n \to \mathbb{R}^m$ and h: $\mathbb{R}^n \to \mathbb{R}^p$ are locally Lipschitz functions, x is the state, u is the control and y is the only available measurement.

The general problem under investigation in this paper is :
Find an integer q, and continuous functions u: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^m$ and v: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^q$ such that the origin is a globally asymptotically stable equilibrium of the extended system :

$$\begin{cases} \dot{x} = f(x) + g(x)u(y,z), \\ \dot{z} = v(y,z). \end{cases} \tag{2}$$

In general, the certainty equivalence principle does not apply to nonlinear systems for global asymptotic stabilization by output feedback. However, for the non global case, it is still the main motivation of most approaches (see (Tsinias, 1991) or (Teel and Praly, 1994) for instance). But, in the global case, invoking some kind of observer in the output feedback design may be fruitless, not mentioning the difficulty in finding such an observer. It is therefore attractive to investigate the possibility of a direct design of output feedback, meaning that no observer is invoked a priori. Different kind of techniques have been proposed in this direction. For example, immersion and invariance principles are used in (Astolfi and Ortega, 2003), model predictive control is derived in (Findeisen *et al.*, 2003).

Pure Lyapunov design is also possible. In this case, the a priori knowledge of a Control Lyapunov function is assumed typically. But then an extra property is needed. For instance in (Battilotti, 1999), it is a bounded input bounded output property, or in (Battilotti, 1994) it is stabilizability via output injection.

In our present study, we justify this extra property from a necessary condition for a Lyapunov function to be an output control Lyapunov function. For the general case, we give sufficient conditions to allow the design via a minimax approach. But we consider also the cases of specific structures such as those motivated by the study of Euler-Lagrange systems as in (Loria and Nijmeijer, 1998; Shishkin *et al.*, 1996), or of "integral output" systems as in (Prieur and de Halleux, 2004).

The key point in our approach is to build a so called output control Lyapunov function. That is (see also (Tsinias and Kalouptsidis, 1990))

Definition 1. Given an integer q, a positive definite, proper and C^1 function $V : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}_{\geq 0}$ is said to be an output control Lyapunov function if, there exist two continuous functions u: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^m$ and v: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^q$ such that, $\forall (y,z) \in \mathbb{R}^p \times \mathbb{R}^q$,

$$\sup_{x : y = h(x)} \mathcal{D}V(x,z,u(y,z),v(y,z)) \leq 0 , \qquad (3)$$

where $\mathcal{D}V$ is the function defined by:

$$\mathcal{D}V(x,z,u,v) = \frac{\partial V}{\partial x}(x,z)\,[f(x)+g(x)u] + \frac{\partial V}{\partial z}(x,z)v .$$

To mimic the definition of a control Lyapunov function, we would need to replace (3) by

$$\inf_{u\in\mathbb{R}^m, v\in\mathbb{R}^q} \sup_{x,\, y=h(x)} \mathcal{D}V(x,z,u,v) < 0 .$$

But getting function u and v of (y,z) from this inequality is an open problem due to the non-compactness of the set $\{x,\ y = h(x)\}$.

The paper is organized as follows. Section 2 is devoted to necessary conditions. In Paragraph 2.1 we make precise the necessary structure of an Output Control Lyapunov function. In Paragraph 2.2, we give necessary conditions on a given Control Lyapunov function for the system (1) to allow the construction of an Output Control Lyapunov function of the extended system (2). Then we investigate sufficient conditions. By a minimax approach, we give a sufficient condition for the existence of an Output Control Lyapunov function in Section 3. We observe also that our necessary condition and our sufficient condition coincide for linear systems and are equivalent to the standard necessary and sufficient conditions of stabilizability and detectability. Section 4 is devoted to two nonlinear but very specific cases where our minimax approach does allow the design of a continuous dynamic output feedback. Finally Section 5 summarizes the main contributions of the work and points out some open problems and future research directions.

Due to space limitations, we cannot give the proofs. They can be found in the extended version of this paper (Prieur and Praly, 2004).

2. NECESSARY CONDITIONS

2.1 Necessary structure of V

Assume for the time being we know continuous functions u: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^m$ and v: $\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^q$ such that the origin is a globally asymptotically stable equilibrium of the system (2). Then, from Kurzweil Theorem (Kurzweil, 1956), there exists a positive definite, proper and C^∞ function $V : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}_{\geq 0}$ whose derivative along the solutions of (2) is negative definite, i.e. $\forall (x,z) \in \mathbb{R}^n \times \mathbb{R}^q \setminus \{(0,0)\}$,

$$\mathcal{D}V(x,z,u(h(x),z),v(h(x),z)) < 0 . \qquad (4)$$

According to Definition 1, V is an output control Lyapunov function. Moreover since, for each x in $\mathbb{R}^n$, V is lower bounded and proper in z in $\mathbb{R}^q$, it admits a global minimum in z. Let $Argmin_z V(x,z)$ be the set of such minimizers. We have for each x in $\mathbb{R}^n$ and each ϕ in $Argmin_z V(x,z)$

$$\frac{\partial V}{\partial z}(x,\phi(x)) = 0 \qquad (5)$$

and there exists a function $U : \mathbb{R}^q \to \mathbb{R}_{\geq 0}$ such that

$$V(x,z) \geq V(x,\phi) = U(x),\ \forall z \in \mathbb{R}^q .$$

Also, since V admits its unique global minimum in (x,z) at the origin, we have :

$$Argmin_z V(0,z) = \{0\} ,$$

and thus

$$U(0) = 0 .$$

We have (see also (Pan *et al.*, 2001, Lemma 1)) :

Proposition 1. Assume that there exists a function ϕ : $\mathbb{R}^n \to Argmin_z V(x,z) \subset \mathbb{R}^q$ which is locally Hölder continuous of order strictly larger than $\frac{1}{2}$, then U is a Control Lyapunov function for the system (1) and there exists M: $\mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^q \times \mathbb{R}^q$ such that V can be decomposed as :

$$V(x,z) = U(x) + (z-\phi(x))^T M(x,z)(z-\phi(x)), \qquad (6)$$

where T is the transposition in $\mathbb{R}^q$.

In view of Proposition 1, we restrict our attention to positive definite, proper and C^1 function in the form :

$$V(x,z) = U(x) + \frac{1}{2}|z-\phi(x)|^2 , \qquad (7)$$

where :

- U is a given weak control Lyapunov function (CLF) for the system (1), i. e. a positive definite, proper and C^1 function such that there exists a C^0 function $k : \mathbb{R}^n \to \mathbb{R}^m$ satisfying [1], $\forall x \in \mathbb{R}^n$,

$$L_f U(x) + L_g U(x) k(x) = \mathcal{D}U(x) \leq 0 .$$

- $\phi : \mathbb{R}^n \to \mathbb{R}^q$ is a C^1 function.

In such a choice, there are two restrictions:

(1) U is given but this a priori data may not be the right one.
(2) Compared to (6), we do not introduce M. Two reasons for this :
   a. In the context of Proposition 1, z is given. In the design case, the variable z is not defined, in particular z can be transformed by any diffeomorphism.
   b. At this stage of our study, we have not been able to see what would be the interest of introducing M.

2.2 *A necessary condition on* ϕ

In (7), the two parameters we can play with are the integer q and the function ϕ. We have the following necessary conditions to help us in choosing these parameters.

Proposition 2. If V, given by (7), is an output control Lyapunov function, we have :

- For any compact subset C of $\mathbb{R}^p \times \mathbb{R}^q$, there exists a real number $c > 0$ such that, $\forall (y,z) \in C$,

$$\inf_{(u,v):|u|+|v|\leq c} \sup_{x: y=h(x)} \mathcal{D}V(x,z,u,v) \leq 0 , \quad (8)$$

- For any compact subset C of $\mathbb{R}^p \times \mathbb{R}^q$, there exists a real number $c_0 > 0$ such that, for all (y,z) in C, we have :

$$\sup_{x: y=h(x), L_g U(x) \neq 0} \frac{L_f U(x) - (z-\phi(x))^T L_f \phi(x)}{|L_g U(x) - (z-\phi(x))^T L_g \phi(x)| + |z-\phi(x)|} \leq c_0, \quad (9)$$

- For each positive real number Φ and each y in $\mathbb{R}^p$, there exists a real number c_1 satisfying :

$$\sup_{x: y=h(x), |\phi(x)|\leq \Phi, L_g U(x)\neq 0} \frac{L_f U(x)}{|L_g U(x)|} \leq c_1 . \quad (10)$$

The necessary condition (10) allows us to understand why the dynamics of the feedback should have a sufficiently large dimension q. Indeed if, as can be expected in the general case, the function $x \mapsto \frac{L_f U(x)}{|L_g U(x)|}$ is not upper bounded on $\mathbb{R}^n$ in all the directions, the only way to have (10) satisfied is if the function $x \mapsto |h(x)| + |\phi(x)|$ is proper. This would be the case if the map $x \mapsto (h(x), \phi(x))$ were a global diffeomorphism, implying $q = n - p$.

In the following section, we shall see that (9) is sufficient for linear systems.

[1] $L_f U(x)$ denotes the Lie derivative of U along the vector field f at the point x.

3. A SUFFICIENT CONDITION

In this section, we derive a sufficient condition for the existence of an Output Control Lyapunov function and apply it to the case of a linear control system.

To simplify our presentation and makes the constraint $h(x) = y$ on x trivial, let us assume that y is a part of the state coordinates. This is a restriction since it says that $h(x) = y$ is an $n-p$ differentiable manifold. In this case we decompose x in (x,y) where x, defined up to a global diffeomorphism, describes the remaining unconstrained coordinates.

With this notation we observe that, given (x,y,z),

- if

$$|L_g U(x) - (z-\phi(x))^T L_g \phi(x)| + |z-\phi(x)| \neq 0 ,$$

then

$$\inf_{(u,v)} \mathcal{D}V(x,y,z,u,v) = -\infty .$$

- and if

$$|L_g U(x) - (z-\phi(x))^T L_g \phi(x)| + |z-\phi(x)| = 0 \quad (11)$$

then

$$\inf_{(u,v)} \mathcal{D}V(x,y,z,u,v) = \mathcal{D}U(x) \leq 0 .$$

This implies, for any function V given as (7) with U a weak control Lyapunov function, we have, for all $(y,z) \in \mathbb{R}^p \times \mathbb{R}^q$,

$$\sup_x \inf_{(u,v)} \mathcal{D}V(x,y,z,u,v) \leq 0 . \quad (12)$$

But, on the other hand, we know from (8) that, if V is an output Lyapunov function, then we have, for each compact set C, the existence of $c > 0$ such that, $\forall (y,z) \in C$

$$\inf_{(u,v):|u|+|v|\leq c} \sup_x \mathcal{D}V(x,y,z,u,v) \leq 0 .$$

We conclude that we would be done if we would have a saddle point property. Specifically, we have:

Proposition 3. Assume

(1) the map U is a control Lyapunov function such that, with the output function h, the function

$$y \mapsto \sup_{x:\, y=h(x), L_gU(x)=0} L_fU(x)$$

is negative definite;

(2) the map ϕ and the coordinates x are such that

  (a) for each quadruple (y,z,u,v), the function $x \mapsto \mathcal{D}V(x,y,z,u,v)$ is concave;

  (b) for each pair (y,z), there exists a pair (u,v) satisfying

$$\lim_{|x|\to+\infty} \mathcal{D}V(x,y,z,u,v) = -\infty. \quad (13)$$

Under these conditions, there exist functions $u_e : (\mathbb{R}^p \setminus \{0\}) \times \mathbb{R}^q \to \mathbb{R}^m$ and $v_e : (\mathbb{R}^p \setminus \{0\}) \times \mathbb{R}^q \to \mathbb{R}^q$ satisfying, $\forall y \neq 0$, $\forall z$,

$$\sup_x \mathcal{D}V(x,y,z,u_e(y,z),v_e(y,z)) < 0. \quad (14)$$

Unfortunately at this stage we don't know what the smoothness properties of u_e and v_e are neither under which extra conditions they can be extended to $y=0$.

Example : As in (Mazenc *et al.*, 1994), consider the system :

$$\dot{x} = x^2 + u\,, \quad \dot{y} = x\,.$$

It can be checked that

$$U(y,x) = \frac{y^2}{2} + \frac{1}{2}(x\exp(-y)+y)^2\,.$$

is a control Lyapunov function. In particular its derivative is

$$\dot{U} = -y^2\exp(-y) + (x\exp(-y)+y)^2\exp(y) + (x\exp(-y)+y)\,u\exp(-y)\,. \quad (15)$$

So to meet Assumption 2.a of Proposition 3, it is sufficient to look for a function ϕ such that the derivative of $(z-\phi(x,y))^2$ is a concave quadratic polynomial in $X := x\exp(-y)+y$. By restricting this search to polynomials in x with coefficients depending on y, we find that

$$\phi(x,y) = ay + bx\exp(-y)$$

is appropriate. Due to (15), $\mathcal{D}V$ becomes

$$\begin{aligned}\mathcal{D}V = &-y^2\exp(-y) + X^2\exp(y) + Xu\exp(-y)\\ &+(z-(a-b)y-bX)\\ &\times(v-a(X-y)\exp(y)-bu\exp(-y))\,.\end{aligned}$$

Thus

$$\begin{aligned}\mathcal{D}V \le\ & X^2(1+ab)\exp(y)\\ &+X[(1+b^2)u\exp(-y)-bv-aby\exp(y))\\ &-a\exp(y)(z-(a-b)y)]\\ &+(z-(a-b)y)\\ &\times(v+ay\exp(y)-bu\exp(-y))\,.\end{aligned}$$

Specifically the assumptions of Proposition 3 are satisfied by picking a and b as real numbers satisfying $ab < -1$ (put $u=v=0$ to meet Assumption 2.b). This means that $\mathcal{D}V$ is made a concave quadratic polynomial in X. By choosing u and v so that its degree-one term disappears and its degree-zero term is negative, we get the globally asymptotically stabilizing output feedback :

$$\begin{cases} \dot{z} = -ay\exp(y) + bu\exp(-y) - [z-(a-b)y] \\ u = [a\exp(y)-b]\,[z-(a-b)y]\exp(y)\,. \end{cases}$$

Note that we conjecture that $\frac{1}{2}[y^2+(x+y)^2]$, which is another control Lyapunov function, cannot be used to give an output control Lyapunov function.

With this Proposition 3, we know that a possibly successful design goes along finding a function ϕ and coordinates x making $\mathcal{D}V$ concave and radially negatively unbounded. Such a procedure may appear awkward in view of the standard approaches coming from invoking the separation principle.

To illustrate it we consider the case of the linear system

$$\begin{cases} \dot{x} = Ax + By + Eu, \\ \dot{y} = Cx + Dy + Fu. \end{cases}$$

We assume it is stabilizable.

We check that Proposition 2 together with Proposition 3 almost (see below for the precise meaning) recovers the well known necessary and sufficient conditions for the existence of an output feedback, namely the stabilizability and the detectability properties (see e.g. (Dullerud and Paganini, 1999, Chapter 5))

Proposition 4. If the necessary condition (9) holds then the system is weakly detectable, i.e. there exists a matrix K, and a positive definite matrix P of appropriate dimensions such that

$$P(A+KC) + (A'+C'K')P' \le 0\,.$$

If the system is detectable, then the sufficient condition of Proposition 3 holds and we can build a stabilizing output feedback. Moreover this feedback is linear in y and z.

4. RELAXED SUFFICIENT CONDITIONS FOR SPECIFIC SYSTEMS

We investigate now if it is possible to relax the conditions of concavity and radial negative unboundedness in x of $\mathcal{D}V$. We exhibit two classes of systems (in Paragraphs 4.1 and 4.2 respectively) where our minimax approach can be applied although some conditions of Proposition 3 are not satisfied.

4.1 L_gU is the sum of the output derivative and the output integral

We restrict our attention to systems for which there exists a weak control Lyapunov function U satisfying:

$$L_fU(x) \leq L_gU(x)\ell_1(h(x)) , \qquad (16)$$

where $\ell_1 : \mathbb{R}^p \to \mathbb{R}^m$ is a continuous function. Our motivation is that, in this case, the necessary condition (10) holds whatever the function ϕ is. Also in this case, we get:

$$\begin{aligned}\mathcal{D}V(x,y,z,u,v) \leq\ & L_gU(x)[u+\ell_1(h(x))] \\ & +[z-\phi(x)][v-L_f\phi(x)-L_g\phi(x)u] .\end{aligned}$$

In the right hand side, when $y = h(x)$ is fixed, x shows up only through the functions L_gU, ϕ, $L_f\phi$, and $L_g\phi$, where ϕ is at our disposal. Then we remark that if we can choose ϕ in such a way that ϕ, $L_f\phi$, and $L_g\phi$ are functions not of x but only of $L_gV(x)$ and $h(x)$, then the maximization in x satisfying $h(x) = y$ can be replaced by the one in $L_gV(x)$. Specifically the concavity and radial negative unboundedness will be expressed in terms of $L_gV(x)$ and not x.

There is a class of systems where this program can be carried out:

Let us assume that the inequality (16) holds and $L_gU(x)$ can be expressed as

$$L_gU(x) = \ell_2(h(x)) + L_f\ell_3(h(x)) + k(x) , \quad (17)$$

where ℓ_2: $\mathbb{R}^p \to \mathbb{R}^m$ is a continuous function and ℓ_3: $\mathbb{R}^p \to \mathbb{R}^m$ is a continuously differentiable function satisfying

$$L_g\ell_3(h(x)) = 0 , \qquad (18)$$

and k: $\mathbb{R}^n \to \mathbb{R}^m$ is a continuously differentiable function satisfying :

$$L_fk(x) = \ell_4(h(x)) , \; L_gk(x) = 0, \qquad (19)$$

where ℓ_4: $\mathbb{R}^p \to \mathbb{R}^m$ is a continuous function.

In particular L_gU can be the output derivative (take $\ell_2 = k \equiv 0$ and $\ell_3(y) = y$); the output integral (take $\ell_2 = \ell_3 \equiv 0$ and $l_4(y) = y$); or, of course, the sum of the output derivative and the output integral.

Note that for nonlinear systems where L_gU is the output derivative, we have the well known results for Euler Lagrange systems see e.g. (Loria and Nijmeijer, 1998; Shishkin *et al.*, 1996). We extend these results to the case of the sum of an output derivative and an output integral as in the physical configuration of the system studied in (Prieur and de Halleux, 2004).

Let us define ϕ: $\mathbb{R}^n \to \mathbb{R}^{2m}$ as the continuous function

$$\phi(x) = (\ell_3(h(x)), k(x)) .$$

With such a choice, due to (17)-(19), we compute

$$\begin{aligned}\mathcal{D}V(x,y,z,u,v) \leq\ & L_f\ell_3(h(x))(u+\ell_1(y) \\ & +\ell_3(y)-z_1) \\ & +k(x)(u+\ell_1(y)-v_2+\ell_4(y) \\ & +\ell_3(y)-z_1) \\ & -\ell_3(y)v_1 - z_2\,\ell_4(y) + z_1v_1 + z_2v_2 \\ & +\ell_2(y)(u+\ell_1(y) .\end{aligned}$$

We get :

Proposition 5. Consider the case of a Control Lyapunov function U for (1), where $L_fU(x)$ satisfies (16) and $L_gU(x)$ is a sum of the output derivative and the output integral as in (17)-(19).

Then we can design an Output Control Lyapunov function V: $\mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}$ and

$$\left\{\begin{aligned} u &= z_1 - \ell_1(y) - \ell_3(y) , \\ \dot{z}_1 &= -z_1 + \ell_3(y) - z_2 - \ell_2(y) , \\ \dot{z}_2 &= z_1 - \ell_3(y) + \ell_4(y) \end{aligned}\right. \qquad (20)$$

is a continuous output feedback which is globally asymptotically stabilizing if all the solutions $x(t)$ of the algebro-differential system:

$$\left\{\begin{aligned} \dot{x} &= f(x) + g(x)u , \\ u &= -\ell_1(h(x)) , \\ 0 &= \ddot{\overbrace{\ell_3(h(x))}} + \dot{\overbrace{\ell_2(h(x))}} + \ell_4(h(x)) \end{aligned}\right. \qquad (21)$$

converge to the origin as t tends to infinity.

4.2 L_fU is the product of the output derivative and the output

In this section, we exhibit an other class of systems where the minimax program can been applied without applying Proposition 3.

Let us assume that $L_fU(x)$ is the product of the output derivative and the output, and that L_gU is the output. More precisely we assume that we have

$$L_fU(x) \leq \ell_2(h(x))^T\,(k(x) + \ell_1(h(x))) \qquad (22)$$

and

$$L_gU(x) = \ell_2(h(x)) , \qquad (23)$$

where ℓ_1: $\mathbb{R}^p \to \mathbb{R}^m$ and ℓ_2: $\mathbb{R}^p \to \mathbb{R}^m$ are continuous functions and k: $\mathbb{R}^n \to \mathbb{R}^m$ is a continuously differentiable function satisfying :

$$L_fk(x) = \ell_4(h(x)) \quad , \qquad L_gk(x) = 0, \quad (24)$$

where ℓ_4: $\mathbb{R}^p \to \mathbb{R}^m$ is a continuous function.

We define ϕ: $\mathbb{R}^n \to \mathbb{R}^m$ as the continuous function $\phi(x) = k(x)$. With this choice, due to (22)-(24), we compute

$$\mathcal{D}V(x,y,z,u,v) \leq k(x)^T \left(\ell_2(y) + \ell_4(y) - v\right) \\ +\ell_2(y)\ell_1(y) - z\ell_4(y) + \ell_2(y)u + zv \,.$$

We get :

Proposition 6. Consider the case of a Control Lyapunov function U for (1), where $L_f U(x)$ and $L_g U(x)$ satisfy (22), (24) and (23).

Then we can design an Output Control Lyapunov function V: $\mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}$ and

$$\begin{cases} u = -z - \ell_1(y) - \ell_2(y) \,, \\ \dot{z} = \ell_2(y) + \ell_4(y) \end{cases} \tag{25}$$

is a continuous output feedback which is globally asymptotically stabilizing if all the solutions $(x(t), z(t))$ of the algebro-differential system:

$$\begin{cases} \dot{x} = f(x) - g(x)(z + \ell_1(h(x))) \,, \\ \dot{z} = \ell_4(h(x)) \,, \\ 0 = \ell_2(h(x)) \end{cases} \tag{26}$$

converge to the origin as t tends to infinity.

5. CONCLUSION

The problem of global asymptotic stabilization of nonlinear control systems by a continuous dynamic output feedback has been addressed. In particular, we have shown that a possible design is via the solution of a minimax problem on the derivative of an appropriately chosen function called output control Lyapunov function. We have checked that this formalism allows us to recover what is known for linear systems. We have also applied it to specific systems reestablishing this way some results known for Euler-Lagrange systems.

The present study leaves several issues open. In particular the gap between our necessary condition and sufficient should be studied for general nonlinear systems. We have proved that our necessary and sufficient conditions coincide for linear control systems under the standard necessary and sufficient conditions of stabilizability and detectability. For general nonlinear systems, this gap can probably be reduced by noting that the existence of a saddle point may not exist even when there exists an Output Lyapunov Control function. More precisely the assumptions 1 and 2 of Proposition 3 could probably be relaxed.

REFERENCES

Astolfi A. and R. Ortega (2003). Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems. *IEEE Trans. Automat. Control.* **48** (4), 590–606.

Basar T. and P. Bernhard (1995). *H^∞-optimal control and related minimax design problems. A dynamic game approach.* Birkhäuser, Basel.

Battilotti S. (1994). Stabilization via dynamic output feedback for systems with output nonlinearities. *Systems Control Lett.* **23 (6)**, 411–419.

Battilotti S. (1999). Sufficient conditions for global output regulation of nonlinear interconnected systems. *Automatica J. IFAC.* **35 (5)**, 829–835.

Dullerud G. E. and F. Paganini (1999). *A course in robust control theory.* Springer, New York.

Ekeland I. and R. Temam (1976). *Convex analysis and variational problems.* North-Holland.

Findeisen R., L. Imsland, F. Allgöwer and B.A. Foss (2003). Output feedback stabilization of constrained systems with nonlinear predictive control. *Internat. J. Robust Nonlinear Control.* **13**, 211–227.

Kurzweil J. (1956). On the inversion of Ljapunov's second theorem on stability of motion. *Ann. Math. Soc. Trans. Ser. 2.* **24**, 19–77.

Loria, A. and H. Nijmeijer (1998). Bounded output feedback tracking control of fully actuated Euler-Lagrange systems. *Systems Control Lett.* **33 (3)**, 151–161.

Mazenc, F., L. Praly, and W. P. Dayawansa (1994). Global stabilization by output feedback : Examples and Counter-Examples. *Systems Control Lett.* **23**, 119-125.

Pan, Z., K. Ezal, A.J. Krenera and P.V. Kokotovic (2001). Backstepping design with local optimality matching. *IEEE Trans. Autom. Control.* **46 (7)**, 1014–1027.

Prieur C. and J. de Halleux (2004). Stabilization of a 1-D tank containing a fluid modeled by the shallow water equations. *Systems Control lett.*,**52**, **(3-4)**, 167–178.

Prieur C. and L. Praly (2004). A tentative direct Lyapunov design of output feedbacks. In preparation.

Shishkin, S., R. Ortega, D. Hill and A. Loria (1996) On output feedback stabilization of Euler-Lagrange systems with nondissipative forces. *Systems Control Lett.* **27 (5)**, 315–324.

Teel A. and L. Praly (1994). Global stabilizability and observability imply semi-global stabilizability by output feedback, *Systems Control Lett.*. **22**, 313–325.

Tsinias J. and N. Kalouptsidis (1990). Output feedback stabilization. *IEEE Trans. Autom. Control.* **35 8**, 951–954.

Tsinias J. (1991). A generalization of Vidyasagar's theorem on stabilizability using state detection. *Systems Control Letters* **17**, 37–42.