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Constrained Control for a Pressurized Water Reactor

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Abstract

One of the main objective in controlling a Pressurized Water Reactor (PWR) of a nuclear power plant is to regulate the temperature of the water of the primary circuit while satisfying many constraints on both input and measured variables. We describe two approaches to deal with such constraints. The first approach is a classical 2-step design procedure where the synthesis is oblivious of the constraints and then an anti-windup compensation is added. The second approach is a 1-step design procedure, reminiscent to predictive control, where the constraints are taken into account in the synthesis. Comparative simulations results are presented.

Keywords: anti-windup bumpless transfer, conditioning techniques, input constraints, integral control, pressurized water reactor.

1 Introduction

Most control systems in industry are subject to constraints. Of common practice are systems with control input constraints due to actuator saturation. Typically, a limited number of selected variables are forced to lie within allowable limits. This situation is even more critical with nuclear power plants. Safety specifications require basically that any meaningful physical variable lies in a pre-specified domain. Thus, it makes the control problem mainly driven by constraints. This paper investigates two approaches to deal with such constraints. A typical approach is to consider a 2-step design procedure where the synthesis is oblivious of the constraints. That is, the controller synthesis is completed as if there were no constraints. Then, an appropriate anti-windup scheme is added on top of the controller to deal with the actual constraints. Another approach is to incorporate *a priori* knowledge of the constraints into the synthesis. This is done with an on-line path planning as in predictive control.

Most nuclear power plants utilize pressurized water reactors (PWR) to drive the steam generation in the system. This study is based on previous work on the modelling and control design for the pressurized water reactor. In particular, initial work on system identification, and subsequent model reduction, for the PWR are reported in [1] and [2] are used in this work for control design. The dynamics of the PWR change significantly over the operating range of the plant (i.e. power level), and the work in [1] showed that a single linear time-invariant (LTI) $\mathcal{H}_\infty$ controller achieves decent closed-loop performance over a reasonably wide operating range. The control techniques used so far for the PWR, do not account for constraints. The aim of this paper is to present a more complete controller design for the primary circuit of the PWR. Such control system should account for saturation nonlinearities in the input signals. In this study, we will focus on constraints in both rate of motion and position of control rods.

The paper is organized as follows. Section 2 is devoted to presentation of the plant, the problem statement and the modelling of the PWR. In Section 3, we present our two-step design: we first review the linear fractional transformation (LFT) paradigm and $\mathcal{H}_\infty$ synthesis used for the linear robust control design. We then present an anti-windup bumpless transfer (AWBT) conditioning technique to cope with the constraints. In Section 4, we present our one step design: we first observe that the constraints are such that a static model is sufficient. The constraints are therefore easily taken into account through a minimization procedure. The resulting closed loop behaviors with both approaches are examined in Section 5, and concluding remarks are given in Section 6.

2 Description, Control objective and Modelling of the PWR

2.1 Description of the primary circuit

In this paper, we consider the primary circuit associated with a PWR of a nuclear power plant. The pressurized water circulating in this circuit transmits the heat from the PWR to the steam generator. There are mainly three components

- The PWR itself, where the water is heated from the energy provided by the nuclear reaction. This reaction is regulated by two sets of control rods whose effect is to capture neutrons. One set is of higher absorbing capability, and is denoted by its position v_1, while the other set is represented by its position v_2. The more rods are present in the reactor, the lesser is the energy production. Because the control rods enter the top of the reactor, the rate of reaction is always higher at the bottom of the reactor. The axial offset, denoted below AO, is defined as the difference in power generated between the top and bottom of the PWR. Safety specifications require minimizing this axial offset.
- The pressurizer, which maintains the water pressure constant.
- The steam generator, where the heat provided by the water of the primary circuit is used to convert to steam the water of the secondary circuit. This steam is then used to drive a turbo-alternator to generate electricity.

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2.2 Control Objective

The main objective in controlling a PWR is to satisfy the power demand while respecting certain constraints. As the power demand increases so does the steam flow in the secondary circuit. This makes the temperature in the primary circuit to decrease. So from the primary circuit standpoint, one natural control objective is to track a temperature reference derived from the steam flow.

For such a control objective and from a control standpoint, the plant has three inputs and three outputs:

- Two inputs are controls. They are the rod positions v_1 and v_2. Actuation of these positions introduces amplitude and rate constraints:
  at each sampling time k, we must have
$$v_1(k+1) - v_1(k) \in \{1, 1/2, 0, -1/2, -1\} \quad (1)$$
$$v_2(k+1) - v_2(k) \in \{1, 0, -1\}\ , \quad (2)$$
$$v_1(k+1) \in [v_{1\,\min}, v_{1\,\max}] \quad (3)$$
$$v_2(k+1) \in [v_{2\,\min}, v_{2\,\max}]\ . \quad (4)$$
  Note the rates of movement have been normalized.
  This defines the mandatory constraints but, for increasing the lifetime of the fuel and reducing operating costs, there is a (more restrictive) preferred region of evolution for each control.
- Another input is the steam flow demand, denoted d_f. It is measured.
- The three outputs are the mean temperature T_m, the axial offset AO, and the primary power P_I. The control specification is that T_m follows a reference temperature T_{ref} depending on $\mathcal{P}$. But this has to be made while maintaining AO within specified bounds. Here again, we have a mandatory and a preferred region of evolution. The latter is described as follows
$$|T_m - T_{ref}(\mathcal{P})| \leq \varepsilon_{T_m}(\mathcal{P})$$
$$|AO - AO_{ref}(\mathcal{P})| \leq \varepsilon_{AO}(\mathcal{P})$$
  where $\varepsilon_{T_m}(\mathcal{P})$ and $\varepsilon_{AO}(\mathcal{P})$ denote thresholds characterizing a prescribed error, $T_{ref}(\mathcal{P})$ is a prescribed specification and $AO_{ref}(\mathcal{P})$ has been obtained by an optimization process in order to meet all the static constraints at equilibrium.

2.3 Modelling the PWR Dynamics

For control design, previous work resulted in the following parameter-dependent discrete-time first-order system (see [1] [2])

$$\begin{bmatrix} x(k+1) \\ \partial T_m(k) \\ \partial A0(k) \\ \partial P_I(k) \end{bmatrix} = \begin{bmatrix} a & b_1 & b_2 & \beta \\ c_{Tm} & d_{Tm1} & d_{Tm2} & d_{Tm} \\ c_{A0} & d_{A01} & d_{A02} & d_{A0} \\ c_{PI} & d_{PI1} & d_{PI2} & d_{PI} \end{bmatrix} \begin{bmatrix} x(k) \\ \partial v_1(k) \\ \partial v_{2\varphi}(k) \\ \partial d_f(k) \end{bmatrix} \quad (5)$$

where

- All coefficients in (5) are functions depending on the operating power $\mathcal{P}$ and the age of the fuel ϖ. For simplicity purpose, the parameter-dependency has not been reflected in the notation.
- ∂d_f, ∂v_1, ∂v_2, ∂T_m, ∂AO and ∂P_I are deviations related to the actual signals by
$$\left\{ \begin{array}{rcl} \partial d_f &=& d_f - d_{f0}(\mathcal{P})\ , \\ \partial v_1 &=& v_1 - v_{10}(\mathcal{P})\ , \\ \partial v_2 &=& v_2 - v_{20}(\mathcal{P})\ , \\ \partial T_m &=& T_m - T_{m0}(\mathcal{P})\ , \\ \partial AO &=& AO - AO_0(\mathcal{P})\ , \\ \partial P_I &=& P_I - P_{I0}(\mathcal{P}) \end{array} \right. \quad (6)$$
  with
  - $d_{f0}(\mathcal{P})$, $v_{10}(\mathcal{P})$, $v_{20}(\mathcal{P})$, $T_{m0}(\mathcal{P})$, $AO_0(\mathcal{P})$ and $P_{I0}(\mathcal{P})$ corresponding to optimized values at an equilibrium corresponding to an operating power $\mathcal{P}$.
  - $\partial v_{2\varphi}$ related to ∂v_2 by $\partial v_{2\varphi} = \partial v_2/\varphi(\mathcal{P})$. This transformation is introduced to take into account that the effectiveness of the second set of rods depends on $\mathcal{P}$.

Two fundamentals properties of this model are as follows : *Whatever $\mathcal{P}(k)$ and $\varpi(k)$ are,*

1. *the system is open loop exponentially stable,*
2. *the system with v_1 and v_2 as inputs, and T_m and AO as outputs, is invertible and its inverse is also an exponentially stable first-order system.*

To have more compact notations, it is appropriate to write the parameter-dependent state-space model as

$$G(z, \mathcal{P}, \varpi) = \left[\begin{array}{c|c} A(\mathcal{P}, \varpi) & B(\mathcal{P}, \varpi) \\ \hline C(\mathcal{P}, \varpi) & D(\mathcal{P}, \varpi) \end{array} \right] \quad (7)$$

where z denotes the $\mathcal{Z}$-transform variable.
From the primary circuit control standpoint, we can assimilate the operating power $\mathcal{P}$ to the power in the secondary circuit, which is reflected in the steam flow demand, d_f. Therefore, we let $\mathcal{P} \equiv d_f$. However, we will keep both notations to remind us of the physical interpretation.

3 Two-step design procedure: Robust linear control + conditioning

In this paper, the $\mathcal{H}_\infty$ controller developed in [1], will be completed with an anti-windup scheme. In this case, the controls are the rate u of the rod position, not the positions v themselves.

3.1 Review of standard background in robust control

Linear Fractional Transformations (LFT) are widely used in robust control to describe uncertainty and parameter variations in complex systems; see [3].

3.1.1 Linear Fractional Transformations: To describe and analyze system uncertainty and parameter variations in a mathematical framework, we use the LFT paradigm (see [3] for a comprehensive presentation). In general, Δ represents uncertainty, or a dynamic element, and $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is a realization of the map $\Delta \star M$, with $\Delta \star M = D + C\Delta(I - A\Delta)^{-1}B$ subject to the condition that $(I - A\Delta)$ is invertible. In this paper, we will consider only unstructured uncertainty, that is the parametric uncertainty is covered by full complex blocks. Hence, we assume Δ lies in a prescribed set, $\boldsymbol{\Delta} = \{\mathrm{diag}[\Delta_1, \ldots, \Delta_f] : \Delta_i \in \mathbb{C}^{n_i \times n_i}\}$

3.1.2 $\mathcal{H}_\infty$ Synthesis: Consider the standard feedback system shown in Figure 1 (left), where K is the controller and P is the generalized plant. w is a vector signal of exogenous inputs; z is the vector signal of quantities we wish to minimize; u and y are the controls and measurements, respectively.

The objective is to find a stabilizing controller K which minimizes the transfer function from w to z, denoted $P \star K$, in the sense of making the maximal energy captured by

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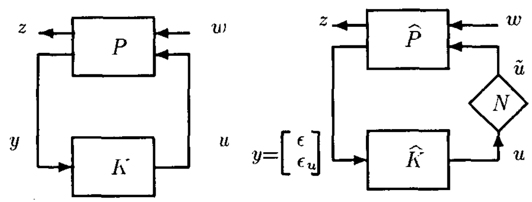

Figure 1: Standard feedback system - AWBT feedback system.

$P \star K$ small. The actual synthesis procedure is sub-optimal in the sense that a controller K is found such that for some pre-specified γ, $\|P \star K\|_\infty < \gamma$ where $\| \bullet \|_\infty$ denotes the ∞-norm.

3.2 Anti-Windup Bumpless Transfer (AWBT) conditioning technique

All control systems in industry must deal with constraints. It often happens that the actual input of a controlled process is temporarily different from the controller output. The mismatch between the actual process input and controller output can also be due to limitations or switching. As a result , the controller output does not drive the plant, thus leading to controller *windup*. Recall, the adverse effect of an integral windup is in the form of large overshoots in the output and sometimes even instability. Any controller with relatively slow or unstable modes will lead to a similar deterioration of the closed-loop performance [4]. Hence, windup can be interpreted as a lack of consistency between the states of the controller and the actual plant input. Consistency can be restored using the conditioning principle defined in [5].

So far, we considered the idealized linear design given in terms of the standard feedback problem shown in Figure 1 (left). The general AWBT problem is based on Figure 1 (right), where the nonlinear block N represents the input limitation and switching mechanism. The interconnection $\widehat{P}$ is obtained from P by providing an additional output $\epsilon_u = \hat{u} - u$, where $\hat{u}$ is the measured or estimated value of the actual plant input $\tilde{u}$. The objective is to find a stabilizing controller $\widehat{K}$ that meets the closed loop linear performance when $N \equiv I$, and exhibits graceful performance degradation when $N \not\equiv I$.

Given the linear controller

$$K(\mathbf{z}) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \tag{8}$$

The state of the controller, ν, is driven by the error signal, ϵ thus leading to significant windup during saturation whenever A_K includes slow dynamics. We can construct a conditioned controller that avoids windup by adding and subtracting two quantities that are equal in the linear case.

$$\nu(k+1) = A_K\nu(k) + B_K\epsilon(k) + H(\hat{u} - u) \tag{9}$$

$$u(k) = C_K\nu(k) + D_K\epsilon(k) \tag{10}$$

$$\hat{u}(k) = \mathbf{sat(u(k))} \tag{11}$$

The equation (9) can be rewritten as follows

$$\nu(k+1) = (A_K - HC_K)\nu(k) + (B_K - HD_K)\epsilon(k) + H\hat{u}$$

Provided D_K is left invertible, it is clear that windup is avoided by selecting $H = B_K D_K^\dagger$, where $\dagger$ denotes the pseudo-inverse operation for non square matrices. Hence the error ϵ has no effect on the state of the controller. Instead, ν is updated based on the actual plant input $\tilde{u}$ or its estimate $\hat{u}$. This parameterization is exactly the *conditioned* controller introduced by Hanus [5], even though the interpretation is different. Furthermore, it is a special case of the general AWBT framework proposed in [6] and [7].

In the general case where D_K is singular, we can still achieve a controller with anti-windup properties. Following Aström [8], H can be selected to insure that $A_K - HC_K$ has all of its eigenvalues in the open unit disc. In fact since (A_K, C_K) is observable, we can *arbitrary* assign these eigenvalues and make the dynamics driven by the error as fast as desired.

The classical approach of *turning off* error integration during saturation can be understood using Figure 2 where the anti-windup block R removes the integral action from K, and is obtained from (9)-(11) after some loop manipulations. In this *classical* formulation, the anti-windup block used in *Hanus conditioned* controller is as follows

$$R(\mathbf{z}) = \mathbf{K}(\mathbf{z})\mathbf{D}_\mathbf{K}^\dagger - \mathbf{I}$$

Thus the *Hanus conditioned* controller in the *standard* formulation (see Figure 1(right)) can be written

$$\widehat{K}(\mathbf{z}) = \left[\begin{array}{c|cc} A_K & B_K & B_K D_K^\dagger \\ \hline C_K & D_K & D_K D_K^\dagger - I \end{array}\right] \tag{12}$$

The proposed windup compensation scheme was originally dedicated to single-input single-output (SISO) systems. The state space approach allows for an extension to multi-input multi-output (MIMO) systems. However, in MIMO systems the plant gain is a function of the input direction, and it has been shown [6] that the nonlinearity changes this direction. In fact, the nonlinear operator acts on u element by element as diagonal input disturbances. Furthermore, some plant and controller interconnections can experience severe performance degradations in the presence of such disturbances. The directionality problem can be eliminated by keeping u and $\hat{u}$ aligned when one of the elements in u saturates. This can be achieved by scaling back the controller outputs until all controls are within allowable limits. This operation consists in replacing a diagonal saturation operator by a scalar times identity operator (see [6] for further details).

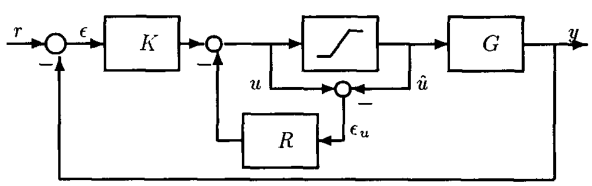

Figure 2: Classical feedback system with saturation compensation.

3.3 AWBT $\mathcal{H}_\infty$ Controller for the PWR

The idealized linear $\mathcal{H}_\infty$ control design has been first presented in [1]. Due to space limitation, it will not be included in this paper.

In our 2-step design procedure, we add an anti-windup compensation to the linear $\mathcal{H}_\infty$ controller. Since this controller satisfies $D_K \neq 0$ and we can measure quite accurately the actual input to the plant, that is $\hat{u} \equiv \tilde{u}$, we can use the

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Hanus conditioned controller given in (12). In the PWR control problem, both control inputs (i.e. the rates of motion) are restricted to lie within ± 1 (see (1)-(2)). The position of the control rods are limited as well (see (3)-(4)); hence whenever they reach the allowable limits, the control is forced to zero.

Recall, that this AWBT compensation relies on availability of the actual control and existence of the inverse of D_K. If one of these condition is not verified, we need to use a more general solution as suggested in Section 3.2. Regarding the directionality problem, the PWR control system has not found to be sensitive to diagonal input disturbances. Furthermore, if one of the control rods is at its maximum position, the other should be used instead. In fact, such use would be prohibited by the suggested alignment compensation. Hence, no specific action has been undertaken to preserve the directionality.

4 One-step design procedure: Constrained integral control

In this section, we present another controller whose linear counterpart is as simple as possible but which incorporates the constraints in its design. It is a low gain constrained integral control.

4.1 Introduction of a low gain integral control.

Our motivation for considering a low gain integral control is twofold :

1. Since the open loop system is exponentially stable and its static gain is non singular, it is known (for instance see [9] or [10] and the references therein) that, a low gain integral control is sufficient to guarantee set point regulation even in the case of control constraints (see [11]).

2. The effect of input rate limitation is very similar to that of a low pass filter. As a consequence, we only observe the system to be controlled in the low frequency range. This leads us to consider the following as design model

$$\begin{bmatrix} \partial T_m(k) \\ \partial A0(k) \end{bmatrix} = \mathcal{G}(\mathcal{P}(k), \varpi(k)) \left(\begin{bmatrix} \partial v_1(k) \\ \partial v_2(k) \end{bmatrix} + \delta(k-1) \right) + \mathcal{G}_{d_f}(\mathcal{P}(k), \varpi(k))\, \partial d_f(k) \qquad (13)$$

where

$$\mathcal{G}(\mathcal{P}, \varpi) = \frac{\begin{bmatrix} c_{Tm}(\mathcal{P}, \varpi) \\ c_{A0}(\mathcal{P}, \varpi) \end{bmatrix} \begin{bmatrix} b_1(\mathcal{P}, \varpi) & b_2(\mathcal{P})/\varphi(\mathcal{P}) \end{bmatrix}}{1 - a(\mathcal{P}, \varpi)} + \begin{bmatrix} d_{Tm1}(\mathcal{P}, \varpi) & d_{Tm2}(\mathcal{P}, \varpi)/\varphi(\mathcal{P}) \\ d_{A01}(\mathcal{P}, \varpi) & d_{A02}(\mathcal{P}, \varpi)/\varphi(\mathcal{P}) \end{bmatrix}$$

and

$$\mathcal{G}_{d_f}(\mathcal{P}, \varpi) = \frac{\begin{bmatrix} c_{Tm}(\mathcal{P}, \varpi) \\ c_{A0}(\mathcal{P}, \varpi) \end{bmatrix} \beta(\mathcal{P}, \varpi)}{1 - a(\mathcal{P}, \varpi)} + \begin{bmatrix} d_{Tm}(\mathcal{P}, \varpi) \\ d_{A0}(\mathcal{P}, \varpi) \end{bmatrix}$$

This is a completely static system with δ a bias vector introduced to cope with the fact that we deal with a variation model and that the response of a linear system to a ramp is that of static system but with a bias. To check that such a very simple model is appropriate, we show on Figure 3 the four step responses corresponding to a step from $.7\mathcal{P}_n$ to $.8\mathcal{P}_n$, where $\mathcal{P}_n$ denotes the maximal operating power of the plant. The system outputs are plotted in solid lines whereas the outputs of (13) with an online estimation of the bias are shown in mixed lines, i.e. the outputs $(\partial T_{mM}, \partial A0_M)$ of the system :

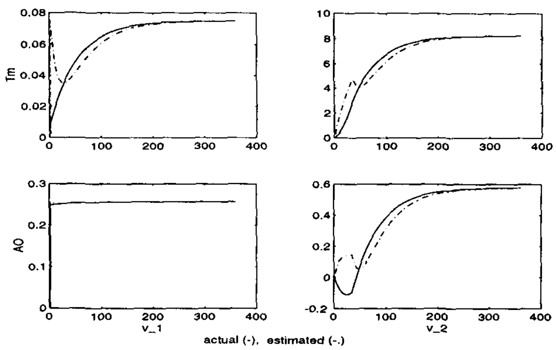

Figure 3: Step responses corresponding to a step from $.7\mathcal{P}_n$ to $.8\mathcal{P}_n$.

$$\begin{bmatrix} \partial T_{mM}(k) \\ \partial A0_M(k) \end{bmatrix} = \mathcal{G}(\mathcal{P}(k), \varpi(k)) \left(\begin{bmatrix} \partial v_1(k) \\ \partial v_2(k) \end{bmatrix} + \delta(k-1) \right) + \mathcal{G}_{d_f}(\mathcal{P}(k), \varpi(k))\, \partial d_f(k) \qquad (14)$$

$$\delta(k) = \delta(k-1) + \gamma(\mathcal{P}(k), \varpi(k)) \Big[H(\mathcal{P}(k), \varpi(k)) \times \left(\begin{bmatrix} \partial T_m(k) \\ \partial A0(k) \end{bmatrix} - \mathcal{G}_{d_f}(\mathcal{P}(k), \varpi(k)) \partial d_f(k) \right) - \begin{bmatrix} \partial v_1(k) \\ \partial v_2(k) \end{bmatrix} - \delta(k-1) \Big] \qquad (15)$$

with $H(\mathcal{P}, \varpi) = \mathcal{G}(\mathcal{P}, \varpi)^{-1}$.

Without constraints, an elementary control law for (14)-(15) is :

$$\begin{bmatrix} \partial v_1(k) \\ \partial v_2(k) \end{bmatrix} = H(\mathcal{P}(k), \varpi(k)) \left(\begin{bmatrix} \partial T_{ref}(\mathcal{P}(k)) \\ \partial AO_{ref}(\mathcal{P}(k)) \end{bmatrix} - \mathcal{G}_{d_f}(\mathcal{P}(k), \varpi(k)) \partial d_f(k) \right) - \delta(k-1)$$

Together with (15), this provides an integral control for the system to be controlled.

4.2 A low gain constrained integral control.

To take into account the constraints, we observe that the minimal time control problem :
Given $v(0)$, $\min K$ under the constraints

$$|v(k) - v(k-1)| \le v_{\max} \quad , \qquad \mathcal{G}\, v(K) = y_{ref} \qquad (16)$$

is solved by choosing, at each time k, the control $v(k)$ as the solution of :

$$\min_{|v(k) - v(k-1)| \le v_{\max}} \left| v(k) - \mathcal{G}^{-1} y_{ref} \right| \qquad (17)$$

When there is also a position constraint, we modify this latter problem into

$$\min_{v(k) \in \mathcal{V}_{ad}(k)} \left| v(k) - \mathcal{G}^{-1} y_{ref} \right| \qquad (18)$$

where $\mathcal{V}_{ad}(k)$ describes the set of possible controls at time k. This leads us to propose the following constrained integral control

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$$\left\{\begin{array}{l}\partial T_m(k)=T_m(k)-T_{m0}(\mathcal{P}(k))\\ \partial AO(k)=AO(k)-AO_0(\mathcal{P}(k))\\ \partial d_f(k)=d_f(k)-d_{f_0}(\mathcal{P}(k))\\ \partial T_{ref}(k+1)=T_{ref}(\mathcal{P}(k+1))-T_{m0}(\mathcal{P}(k+1))\\ \partial AO_{ref}(k+1)=AO_{ref}(\mathcal{P}(k+1))-AO_0(\mathcal{P}(k+1))\end{array}\right. \quad (19)$$

$$\begin{aligned}\delta(k) &= \delta(k-1)+\gamma(\mathcal{P}(k),\varpi(k))\\ &\times\left[H(\mathcal{P}(k),\varpi(k))\left(\begin{bmatrix}\partial T_m(k)\\ \partial A0(k)\end{bmatrix}-\mathcal{G}_{d_f}(\mathcal{P}(k),\varpi(k))\partial d_f(k)\right)\right.\\ &\left.-\begin{bmatrix}\partial v_1(k)\\ \partial v_2(k)\end{bmatrix}-\delta(k-1)\right]\end{aligned}$$

$$\begin{aligned}\begin{bmatrix}\partial v_{1ref}(k)\\ \partial v_{2ref}(k)\end{bmatrix} &= H(\mathcal{P}(k),\varpi(k))\left(\begin{bmatrix}\partial T_{ref}(k)\\ \partial AO_{ref}(k)\end{bmatrix}\right.\\ &\left.-\ \mathcal{G}_{d_f}(\mathcal{P}(k),\varpi(k))\partial d_f(k)\right)-\delta(k-1)\end{aligned}$$

$$\min_{\partial v_1(k),\partial v_2(k)\in\mathcal{V}_{adk}}(\partial v_1(k)-\partial v_{1ref}(k))^2+(\partial v_2k(k)-\partial v_{2ref}(k))^2$$

$$\left\{\begin{array}{l}v_1(k)=v_{10}(\mathcal{P}(k))+\partial v_1(k)\\ v_2(k)=v_{20}(\mathcal{P}(k))+\partial v_2(k)\end{array}\right. \quad (20)$$

where $\mathcal{V}_{ad}(k)$ is the set of pairs $(\partial v_1, \partial v_2)$ satisfying

$$\left\{\begin{array}{l}\partial v_1-\partial v_1(k-1)\in\{1,1/2,0,-1/2,-1\},\\ \partial v_1+v_{10}(q(k))\in[v_{1\min},v_{1\max}]\\ \partial v_2-\partial v_2(k-1)\in\{1,0,-1\},\\ \partial v_2+v_{20}(q(k))\in[v_{2\min},v_{2\max}]\end{array}\right.$$

Such a control law involves many functions of $\mathcal{P}$: $\gamma(\mathcal{P})$, $H(\mathcal{P})$, $\mathcal{G}_{d_f}(\mathcal{P})$, $T_{m0}(\mathcal{P})$, $AO_0(\mathcal{P})$, $q_0(\mathcal{P})$, $v_{10}(\mathcal{P})$, $v_{20}(\mathcal{P})$ which are all to be identified.

- The last 5 ones describe a family of optimized operating equilibrium points. Thanks to the integral action we can take only rough approximations. We have found sufficient to take $T_{m0}(\mathcal{P})$, $AO_0(\mathcal{P})$, $d_{f_0}(\mathcal{P})$, $v_{10}(\mathcal{P})$, $v_{20}(\mathcal{P})$ constant as the mean value of their allowed range.

- $\mathcal{G}_{d_f}(\mathcal{P})$ being multiplied by ∂d_f, an error on this quantity can also be absorbed by the integral action. We have chosen $\mathcal{G}_{d_f}$ constant as the mean value of $\mathcal{G}_{d_f}(\mathcal{P},\varpi)$ for $\mathcal{P}$ in $[0.5\mathcal{P}_n,\mathcal{P}_n]$ and ϖ in $[0.5,1]$.

- The first 2 are more directly related to the control dynamics and influence the closed-loop stability. If $\mathcal{P}$ were constant, we know from [9] that it is sufficient to choose $\gamma(\mathcal{P},\varpi)$ positive and small enough and $H(\mathcal{P},\varpi)$ such that the matrix $H(\mathcal{P},\varpi)\mathcal{G}(\mathcal{P},\varpi)$ has all its eigenvalues with strictly positive real part. When $\mathcal{P}$ is time varying, we can expect that the exponential stability will be preserved if these time variations are slow enough in the mean. This can be made precise as follows :

Consider the system :

$$\begin{bmatrix}x(k+1)\\ \chi(k+1)\end{bmatrix}=\begin{bmatrix}a(k) & -B(k)'\\ \gamma HC(k) & I-\gamma HD(k)\end{bmatrix}\begin{bmatrix}x(k)\\ \chi(k)\end{bmatrix} \quad (21)$$

where the sequences a, B, C and D are bounded, x is in $\mathbb{R}$ *and* χ *in* $\mathbb{R}^\kappa$. *By applying the state space decomposition of [12, Section 3.3.3] and the averaging result [12, Theorem 3.1], we can prove :*

Proposition 1 *If there exist an integer K and a positive definite symmetric matrix P such that, for all l, we have :*

$$\sum_{j=l+1}^{l+K}\left(P\,S(j)+S(j)'P\right)\geq\varepsilon I>0 \quad (22)$$

where

$$S(k)=H\left[D(k)+\frac{C(k)B(k)'}{1-a(k)}+C(k)\sum_{l=1}^{k}\left(\prod_{j=l}^{k-1}a(j)\right)\times\left(\frac{B(l-1)'}{1-a(l-1)}-\frac{B(l)'}{1-a(l)}\right)\right]$$

then there exists $\gamma^\star>0$ *such that, for all* γ *in* $(0,\gamma^\star]$, *the system (21) is exponentially stable.*

For our problem, we have found that, by taking for H the mean value of $\mathcal{G}(\mathcal{P},\varpi)^{-1}$ for $(\mathcal{P},\varpi)$ in $[0.5\mathcal{P}_n\,,\,\mathcal{P}_n]\times[0.5,1]$, we get

$$\begin{aligned}H\,\mathcal{G}(\mathcal{P},\varpi)+\mathcal{G}(\mathcal{P},\varpi)'H' &\geq 0.6\,I\\ \forall\,(\mathcal{P},\varpi) &\in [0.5\mathcal{P}_n\,,\,\mathcal{P}_n]\times[0.5,1]\end{aligned}$$

It follows that, with such a constant matrix H and with a constant gain γ (= 0.1 in our simulations), our controller (19)-(20) guarantees asymptotic set point regulation as long as the variation of $\mathcal{P}$ are slow enough so that

$$\left\|\frac{\left(b_1(\mathcal{P},\varpi(l-1))\ b_2(\mathcal{P},\varpi(l-1))\right)}{1-a(l-1)}-\frac{\left(b_1(\mathcal{P},\varpi(l))\ b_2(\mathcal{P},\varpi(l))\right)}{1-a(l)}\right\|$$

is sufficiently small in the mean. We have observed that such a condition is satisfied in the various scenarii we have to investigate.

5 Simulation Results

To evaluate the performance of our controllers, we have used a realistic simulator of a PWR. Indeed, it is a nonlinear dynamical system based on first principles models. It includes models for the pressurizer, steam generator, and the turbine, but not the alternator. The nuclear fuel is assumed to be at half of expected lifetime, that is $\varpi = 1$.

The closed-loop responses using both the AWBT-$\mathcal{H}_\infty$ controller and the constrained integral control are shown in Figures 4 and 5, respectively. The scenario used for the comparison incorporates the main features given in the specifications (look at the dashed line in the top plot). In both figures, except for the plots of P_I, the solid line represents the achieved signal and the vertical interval defined by the dashed lines represents, at each time, the preferred region of evolution for the corresponding signal, as given by the specifications. (Note that, in the case of v_2, these limits are piecewise constant functions, that are sometimes hard to satisfy.) In the plot of AO, we have an extra dashed line representing the reference axial offset AO_{ref}. In the plots of v_1 and v_2, the horizontal limits of the plots represent the mandatory limits. In the plots of P_I, the dashed line represents d_f which reflects the power demand (noted by P_{II} in the Figures) and the solid line represents the primary power P_I. It is worth noting that at steady state, P_I is always slightly higher than P_{II} to account for energy losses in the heat exchange.

When comparing the two sets of plots, the first observation is that we get similar behaviors. The controllers being very different in nature, this emphasizes that the constraints are such that they do not leave much degrees of freedom. For both controllers, the behavior of T_m is the desired one except in the response to a drop of the power demand from $\mathcal{P}_n$ to $0.5\mathcal{P}_n$. Indeed, in this case, the rate limitation is such that it is impossible to follow the reference. For the axial offset, the constrained integral controller gives a better response than the AWBT-$\mathcal{H}_\infty$ controller. Both controllers give the same behavior for P_I, this although the constrained

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integral controller does not use this signal at all. Concerning the inputs, the position v_2 exhibits very similar behaviors. But for the position v_1, the AWBT-$\mathcal{H}_\infty$ controller is able to keep it in the middle of its preferred region of evolution whereas the constrained integral controller gives more freedom to this control. This difference is related to the one concerning AO. In particular, the $\mathcal{H}_\infty$ controller has been designed with as primary objective to keep all controlled signals (particularly position of both control rods) away from saturations. Whenever this is satisfied, secondary objective is to minimize the axial offset.

6 Concluding Remarks

The AWBT compensation scheme based on *Hanus conditioned controller* allowed the initial linear robust control design to comply with all physical constraints provided they can be translated in terms of a nonlinear operation on the plant inputs. The proposed compensation scheme holds also for switching and provides a bumpless transfer (even if this has not been shown in this paper). Clearly, a more general formulation of the AWBT compensation scheme could alternatively be used, where some important issues like the directionality and the stability of the AWBT can be addressed more precisely.

The constrained integral controller is definitely a controller fitted to the application. However, the synthesis involves a simple structure with few parameters. The controller design follows from general ideas: 1) In presence of input rate limitations , we mainly observe the low frequency behavior of the plant. 2) When a system is exponentially stable with an invertible static gain, an integral control is sufficient for guaranteeing set point regulation. 3) For linear systems, control under linear constraints is, at each sampling time, a problem of linear programming for which very efficient algorithms are available.

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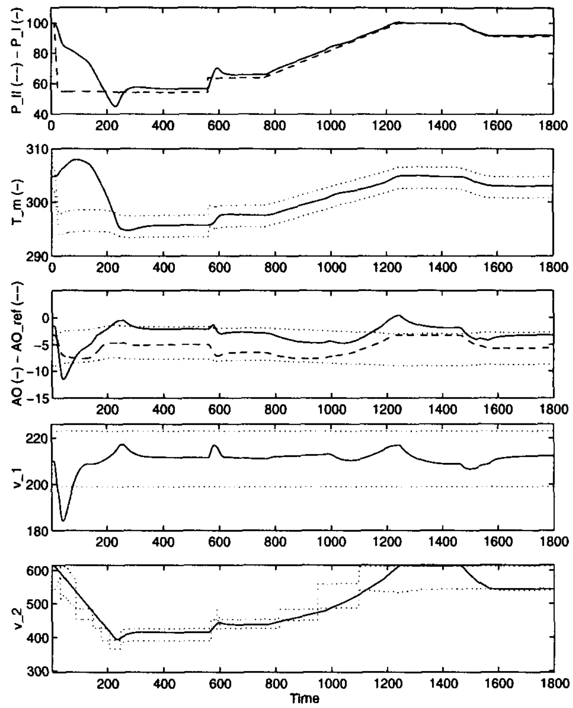

Figure 4: AWBT $\mathcal{H}_\infty$ controller.

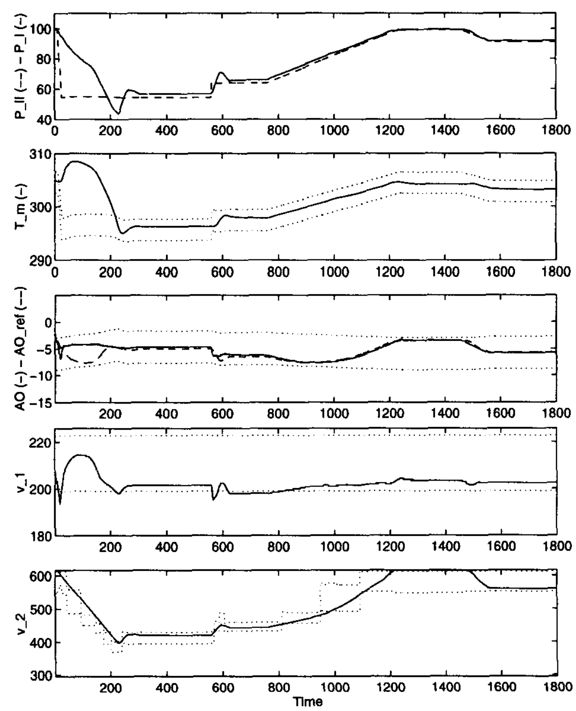

Figure 5: Constrained integral control.

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