Proceedings of the 33rd
Conference on Decision and Control
Lake Buena Vista, FL - December 1994

ON PASSIVITY-BASED OUTPUT FEEDBACK GLOBAL STABILIZATION OF EULER-LAGRANGE SYSTEMS*

Romeo Ortega†, Antonio Loría†, Rafael Kelly* and Laurent Praly‡

†Université de Compiègne
URA C.N.R.S. 817 , BP 649
60206 Compiègne
rortega@hds.univ-compiegne.fr
FRANCE

*Div. de Física Aplicada
CICESE, AP 2615, Adm. 1
Ensenada, BC, 22800
rkelly@cicese.mx
MEXICO

‡Ecole National Sup. des Mines
35 rue Saint Honoré
77350, Fontainebleau
praly@caia.ensmp.fr
FRANCE

Abstract

It is well known that in systems described by Euler-Lagrange equations the stability of the equilibria is determined by the potential energy function. Further, these equilibria are asymptotically stable if suitable damping is present in the system. These properties motivated the development of a *passivity-based* controller design methodology which aims at modifying the potential energy of the closed loop and the addition of the required dissipation. To achieve the latter objective measurement of the generalized velocities is typically required. Our main contribution in this paper is the proof that damping injection *without* velocity measurement is possible via the inclusion of a *dynamic extension* provided the system satisfies a *dissipation propagation* condition. This allows us to determine a class of Euler-Lagrange systems that can be globally asymptotically stabilized with dynamic output feedback. We illustrate this result with the problem of setpoint control of elastic joints robots. Our research contributes, if modestly, to the development of a theory for stabilization of nonlinear systems with physical structures which effectively exploits its energy dissipation properties.

1 Problem Formulation

We consider in this paper plants described by Euler-Lagrange equations (in short, EL systems) of the form

$$\frac{d}{dt}[\frac{\partial \mathcal{L}_p(q_p,\dot q_p)}{\partial \dot q_p}] - \frac{\partial \mathcal{L}_p(q_p,\dot q_p)}{\partial q_p} = Q_p \qquad (1.1)$$

where $q_p, Q_p \in \mathcal{R}^{n_p}$ are the generalized coordinates and the external forces respectively, $\mathcal{L}_p(q_p,\dot q_p) := T_p(q_p,\dot q_p) - V_p(q_p)$ is the Lagrangian function, $T_p(q_p,\dot q_p)$ is the kinetic energy, which we assume to be of the form $T_p(q_p,\dot q_p) = \frac{1}{2}\dot q_p^T D_p(q_p)\dot q_p$, $D_p(q_p) = D_p^T(q_p) > 0$, and $V_p(q_p)$ is the potential energy which we assume is twice differentiable and bounded from below, that is $V_p(q_p) + c \geq 0$ for some $c \in \mathcal{R}$. The external forces consist of *dissipative* and *control* action terms $Q_p := M_p u_p - \frac{\partial \mathcal{F}_p(\dot q_p)}{\partial \dot q_p}$, with the control signals $u_p \in \mathcal{R}^{m_p}$, $m_p \leq n_p$, and $M_p \in \mathcal{R}^{n_p \times m_p}$ full column rank. $\mathcal{F}_p(\dot q_p)$ is the Rayleigh dissipation function which defines a memoryless passive (resp., input strictly passive) operator $\dot q_p \mapsto \frac{\partial \mathcal{F}_p(\dot q_p)}{\partial \dot q_p}$, that is,

$$\dot q_p^T \frac{\partial \mathcal{F}_p(\dot q_p)}{\partial \dot q_p} \geq \alpha \|\dot q_p\|^2 \qquad (1.2)$$

holds for all $\dot q_p \in \mathcal{R}^{n_p}$ and $\alpha \geq 0$ (resp., $\alpha > 0$). Furthermore, we assume the points with zero generalized velocities ($\dot q_p = 0$) are equilibria of (1.1), that is, $\frac{\partial \mathcal{F}_p(\dot q_p)}{\partial \dot q_p}|_{\dot q_p = 0} = 0$.

To simplify the presentation we will assume, without loss of generality, M_p of the form $M_p = [0 \mid I_{m_p}]^T$, and introduce the following partition of q_p into nonactuated and actuated components[1] respectively

$$q_{p1} := M_p^{\perp} q_p = [\ I_{n_p - m_p} \mid \ 0\ ]\, q_p,\ q_{p2} := M_p q_p$$

Furthermore, we will assume the actuated variables are available for measurement and the nonactuated variables to be the regulated coordinates.

The problem we study in this paper is formulated as follows:

*This work was supported in part by the Comission of European Communities under contract ERB CHRX CT 93-0380. A. Loría's work was supported by CONACyT, Mexico.

[1] That is, generalized coordinates whose corresponding row in the input matrix contains a zero (resp., nonzero) entry M_p.

Output feedback global stabilization problem. Consider the EL system (1.1) with *measurable outputs* q_{p2} and *regulated outputs* q_{p1} with *constant* desired value q_{p1d}. Then, design a controller $q_{p2} \mapsto u_p$ that makes the closed loop system *globally asymptotically stable* (GAS) at an equilibrium point $(\bar{\cdot})$ such that $\bar{q}_{p1} = q_{p1d}$.

It is well known [12] that EL systems define passive operators. Since passive systems enjoy some usefull robustness properties, and passivity is invariant under feedback interconnection, it is of some interest to consider the utilisation of passive controllers to stabilize EL plants. On the other hand, in EL systems the stability of the equilibria is determined by the potential energy function. Further, these equilibria are asymptotically stable if suitable damping is present in the system. These input-output and internal stability properties of EL systems motivated the development of the *passivity-based energy shaping plus damping injection* controller design methodology [14], [19] [12], [13] (see also [10] for an interesting historical review of this idea). This technique aims at modifying the potential energy of the system and the injection of the required damping with a controller that preserves the passivity in the closed loop. The utilisation of this technique is stymied in some applications by the fact that measurement of the generalized velocities is typically required to add the damping. The main contribution of this paper is to prove that, for a class of EL systems, damping injection is still possible with *only output feedback* via a *dynamic extension.* The class of plants is characterized by a *dissipation propagation* condition.

The present research was motivated by the results on output feedback stabilization of robots with flexible joints of [1] and [8], which extend to the output feedback case the controllers of [2], [20]. Other efforts aimed in the direction of our research have been reported in [15], [11], [4], [5].

The organization of this paper is the following[2]. In Section 2 we recall some input-output and internal stability properties of EL systems which are relevant for control purposes. In particular we derive here the key dissipation propagation condition for asymptotic stability of underdamped systems. In Section 3 we consider EL controllers and define a class of EL systems for which the passivity-based methodology yields a GAS closed loop. In Section 4 we apply this result to the flexible joint robot stabilization problems. Finally, we give some concluding remarks in Section 5.

Notation $\|\cdot\|$ - Euclidean norm; $\mathcal{L}_2^n$, $\mathcal{L}_{2e}^n$ - spaces of n-dimensional square integrable functions and its extension; $\|\cdot\|_2$, - $\mathcal{L}_2^n$ norm; $\langle\cdot|\cdot\rangle$ - inner product in $\mathcal{L}_2^n$. A state-space system $\dot{x} = f(x)$, $x \in \mathcal{R}^n$ is *zero-state detectable* from the output $y = h(x)$, if for all initial conditions $x(0) \in \mathcal{R}^n$ we have $(y(t) \equiv 0 \Rightarrow \lim_{t\to\infty} x(t) \to 0)$.

[2]Due to space limitations we give here an abridged version of the full paper, which is available upon request to the first author.

2 Properties of EL Systems

In this section we will present some input-output and internal stability properties of EL systems. At this point notice that an EL system with generalized coordinates $q \in \mathcal{R}^n$ and input $u \in \mathcal{R}^m$ is fully characterized, via an equation of the form (1.1) by the quadruplet $\{T(q,\dot{q}), V(q), \mathcal{F}(\dot{q}), M\}$. In the sequel we will refer to the latter set as *EL parameters* of the EL system.

2.1 Input–Output Properties

It is well known [12] that EL systems have some nice *energy dissipation properties.* In particular we have the following:

Proposition 2.1 *(Passivity)*
An EL system defines a *passive operator* [7] from the inputs u to the actuated generalized velocities $M^T\dot{q}$. That is, there exists $\beta \in \mathcal{R}$ such that

$$\langle u|M^T\dot{q}\rangle \geq \beta$$

for all $u \in \mathcal{L}_{2e}^m$. Further, this property is strenghtened to *output strict passivity* if the Rayleigh dissipation function defines an input strictly passive operator. In this case

$$\langle u|M^T\dot{q}\rangle \geq \alpha\|M^T\dot{q}\|_2^2 + \beta$$

for some $\alpha > 0$, $\beta \in \mathcal{R}$ and all $u \in \mathcal{L}_{2e}^m$.

Proof. The property can be easily established taking the time derivative of the Lagrangian function and using the EL equations to get

$$\frac{d}{dt}[\frac{\partial\mathcal{L}(q,\dot{q})}{\partial\dot{q}}\dot{q} - \mathcal{L}(q,\dot{q})] = \dot{q}^T[Mu - \frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}}]$$

Now, noting that the term in brackets in the left hand side coincides with the systems total energy $H(q,\dot{q}) := T(q,\dot{q}) + V(q)$ and integrating from 0 to t we establish the key energy balance equation

$$\underbrace{H[q(t),\dot{q}(t)] - H[q(0),\dot{q}(0)]}_{stored\ energy} + \underbrace{\int_0^t \dot{q}^T\frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}}ds}_{dissipated} =$$

$$= \underbrace{\int_0^t \dot{q}^T Mu ds}_{supplied} \qquad (2.1)$$

The proof is completed using the conditions on the Rayleigh dissipation function and the fact that $V(q)$ is bounded from below.

□□□

Remark 2.1 From (2.1) we see that one way of stabilizing an EL system, insuring its total energy is *stricly decreasing*, is selecting a compensator that defines a strictly passive operator $-M^T\dot{q} \mapsto u$. This is easily achieved via proportional feedback of the generalized velocities [12]. However, when the latter are not available for measurement the energy has to be dissipated in another dynamical system as we show in the next section.

Remark 2.2 Notice that the operator $u \mapsto M^T\dot{q}$ may be output strictly passive even if energy is *not dissipated "in all directions"*. Namely, it is enough to insure $\dot{q}^T\frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}} \geq \alpha\|M^T\dot{q}\|^2$. This feature will be exploited below to achieve partial damping injection for asymptotic stabilization.

2.2 Lyapunov Stability Properties

Fully Damped Systems

The proposition below establishes conditions for *internal stability* of EL systems.

Proposition 2.2 *(GAS with full damping)*
The equilibria of an EL system with $u = 0$ are $(q,\dot{q}) = (\bar{q},0)$ where $\bar{q}$ is the solution of $\frac{\partial V(q)}{\partial q} = 0$. The equilibrium is ***unique and stable*** if the potential energy is an strictly convex function, that is if $\frac{\partial^2 V(q)}{\partial q^2} \geq \epsilon I_n > 0$. Further, this equilibrium is GAS if the Rayleigh dissipation function is ***input strictly passive***.

Proof. See [12].

□□□

Remark 2.3 It is important to underscore the fact that the passivity properties of propostion 2.1 are *independent* of the shape of the potential energy function. This fact, together with proposition 2.2, allows us to naturally split the controller tasks into (potential) energy shaping for stabilization at the desired equilibrium and damping injection to make this equilibrium attractive.

Underdamped Systems

In the following proposition we show that asymptotic stability can still be insured even when energy is not dissipated "in all directions" provided the inertia matrix $D(q)$ has a certain block diagonal structure, and the dissipation is suitably propagated. This result, though extremely simple, will be fundamental for our output feedback stabilization problem where damping will be injected only in some of the generalized coordinates. To distinguish between the *damped* and *undamped* coordinates we introduce the following partition of q:

$$q_c := [0 \mid I_{n_c}]q,\ q_p := [I_{n_p} \mid 0]q,\ n = n_p + n_c$$

which in the following section will denote the controller and plant generalized coordinates respectively.

Proposition 2.3 *(GAS with partial damping)*
The equilibrium of an EL system with $u = 0$ and strictly convex potential energy function is GAS if

i) $D(q) := \begin{bmatrix} D_p(q_p) & 0 \\ 0 & D_c(q_c) \end{bmatrix}$, where $D_c(q_c) \in \mathcal{R}^{n_c \times n_c}$,

ii) $\dot{q}^T\frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}} \geq \alpha\|\dot{q}_c\|^2$ for some $\alpha > 0$,

iii) For each q_c, the function $\frac{\partial V(q)}{\partial q_c} = 0$ has only *isolated zeros* in q_p.

Proof. From proposition 2.1 we have that the system is passive with storage function $H_1(q,\dot{q}) := H(q,\dot{q}) - V(\bar{q})$, which under the assumptions on the potential energy is positive definite and proper. Now, using condition **ii)** we get the dissipation inequality

$$H_1[q(t),\dot{q}(t)] - H_1[q(0),\dot{q}(0)] \leq -\alpha\int_0^t \|\dot{q}_c(\tau)\|^2 d\tau \leq 0 \quad (2.2)$$

Using the arguments of theorem 3.2 of [6] we also have that along the trajectories of the ω-limit set the left hand side in (2.2) is zero, thus $\dot{q}_c(t) \equiv 0$. From the structure of $D(q)$ it is easy to prove [18] that the EL equations with $u = 0$ are of the form

$$D_p(q_p)\ddot{q}_p + C_p(q_p,\dot{q}_p)\dot{q}_p + \frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}_p} + \frac{\partial V(q)}{\partial q_p} = 0 \quad (2.3)$$

$$D_c(q_c)\ddot{q}_c + C_c(q_c,\dot{q}_c)\dot{q}_c + \frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}_c} + \frac{\partial V(q)}{\partial q_c} = 0 \quad (2.4)$$

where $C_c(q_c,\dot{q}_c)$, $C_p(q_p,\dot{q}_p)$ are suitably defined matrices. From (2.4), and the fact that $\frac{\partial\mathcal{F}(\dot{q})}{\partial\dot{q}_c}|_{\dot{q}_c=0} = 0$, it is clear that $\dot{q}_c \equiv 0 \Rightarrow \frac{\partial V(q)}{\partial q_c} \equiv 0$. The proof is completed using continuity of q_p, condition **iii)** and proceeding as in theorem 3.2 of [6].

□□□

Remark 2.4. In the next section we will use proposition 2.3 for asymptotic stabilization via partial damping injection with a dynamic extension. In this case the conditions of block diagonal structure of $D(q)$ and partial dissipation **ii)** will be satisfied by design. Henceforth, the only relevant condition is the damping propagation **iii)**.

Remark 2.5. Condition **iii)** may be replaced by the assumption that the system is zero-state detectable from the "output" $\frac{\partial V(q)}{\partial q_c}$. Further, we can remove the restriction of block diagonal structure of $D(q)$ if instead of **iii)** we impose detectability from the output $\dot{q}_c$.

3 Stabilization of Euler-Lagrange Systems

We now use the input–output and internal stability properties established above to solve the output feedback global stabilization problem for a class of EL systems. That is, we want to define a class of controllers which, *preserving the EL structure*, suitably modifies the potential energy and dissipation properties of the EL plant.

3.1 Damping Injection via Dynamic Extension

To this end, we first define the desired closed-loop system EL parameters [3] $\{T(q,\dot{q}), V(q), \mathcal{F}(\dot{q}), 0\}$ as $T(q,\dot{q}) := T_p(q_p,\dot{q}_p) + T_c(q_c,\dot{q}_c)$, $V(q) := V_p(q_p) + V_c(q_c,q_{p2})$, $\mathcal{F}(\dot{q}) := \mathcal{F}_p(\dot{q}_p) + \mathcal{F}_c(\dot{q}_c)$ where $q = [q_p^T, q_c^T]^T$, $q_c \in \mathcal{R}^{n_c}$ are the generalized coordinates of the *EL controller* with EL parameters $\{T_c(q_c,\dot{q}_c), V_c(q_c,q_{p2}), \mathcal{F}_c(\dot{q}_c), 0\}$. That is, the controller dynamics are given by

$$D_c(q_c)\ddot{q}_c + \dot{D}_c(q_c)\dot{q}_c - \frac{\partial T_c(q_c,\dot{q}_c)}{\partial q_c} + \\ + \frac{\partial V_c(q_c,q_{p2})}{\partial q_c} + \frac{\partial \mathcal{F}_c(\dot{q}_c)}{\partial \dot{q}_c} = 0 \qquad (3.1)$$

where we have chosen the potential energy of the controller dependent on only the measurable output q_{p2}. In this way, q_{p2} enters into the controller via the term $\frac{\partial V_c(q_c,q_{p2})}{\partial q_c}$ while the *feedback interconnection* between plant and controller is naturally established by

$$u_p = -\frac{\partial V_c(q_c,q_{p2})}{\partial q_{p2}} \qquad (3.2)$$

It is clear that the dynamic extension we just introduced injects damping through the controllers $\mathcal{F}_c(\dot{q}_c)$, while $V_c(q_c,q_{p2})$ shapes the systems potential energy.

The resulting feedback system may be depicted as shown in Fig. 1, where the dynamic equations of the plant

$$D_p(q_p)\ddot{q}_p + C_p(q_p,\dot{q}_p)\dot{q}_p + \frac{\partial \mathcal{F}_p(\dot{q}_p)}{\partial \dot{q}_p} + \frac{\partial V_p(q_p)}{\partial q_p} = M_p u_p \qquad (3.3)$$

define the operator $\Sigma_p : u_p \mapsto q_{p2}$, and the operator $\Sigma_c : q_{p2} \mapsto u_p$ is determined by (3.2), (3.1).

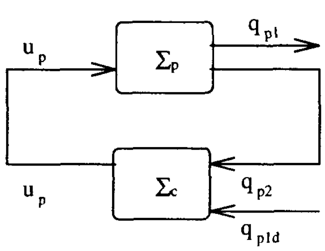

Fig. 1. Feedback System.

3.2 Main Result

From the results presented in the previous section we see that to attain the GAS objective $V(q)$ must have a global minimum at the desired equilibrium, $\mathcal{F}(\dot{q})$ must satisfy some suitable passivity properties and the system must verify a dissipation propagation condition. These requirements are summarized in the proposition below whose proof follows *mutatis mutandi* from the derivations above and proposition 2.3.

Theorem 3.1 *(Output feedback stabilization)* Consider the *EL plant* (3.3) and *assume*:

A.1 *(Dissipation propagation)* The following implication holds ($u_p \equiv$ const and $\dot{q}_{p2} \equiv 0$) $\Rightarrow$ $\lim_{t\to\infty} \dot{q}_{p1} \to 0$,

A.2 *(Energy shaping)*
We can find a function $V_{c2}(q_{p2}) : \mathcal{R}^{m_p} \to \mathcal{R}_+$ such that $\frac{\partial^2 V_1(q_p)}{\partial q_p^2} \geq \epsilon I_{n_p} > 0$, where

$$V_1(q_p) := V_p(q_p) + V_{c2}(q_{p2})$$

and $\frac{\partial V_1(q_p)}{\partial q_p}|_{\bar{q}_p} = 0$ with $q_{p1d} = [I_{n_p-m_p}\ |0]\bar{q}_p$
Under these conditions, the *EL controller* (3.1), (3.2) where $\dot{q}_c^T \frac{\partial \mathcal{F}_c(\dot{q}_c)}{\partial \dot{q}_c} \geq \alpha\|\dot{q}_c\|^2$ for some $\alpha > 0$, and

$$V_c(q_c,q_{p2}) := \tfrac{1}{2}(q_c + A_1 q_{p2})^T A_2 (q_c + A_1 q_{p2}) + \\ + V_{c2}(q_{p2}) \qquad (3.4)$$

with A_1 a full rank matrix and $A_2 > 0$, solves the *output feedback global stabilization* problem.

Remark 3.1. It is important to underscore the fact that the dissipation propagation condition given here is *independent* of the controller. Hence, the success of our controller design hinges only on the ability to find a function $V_{c2}(q_{p2})$ that achieves the energy shaping[4]. However, looking back at the argument to build the proof we see that $V_{c2}(q_{p2})$ could be used to relax **A.2** since u_p depends on this term.

Remark 3.2. **A.1** may be replaced by the (stronger) assumption of zero-state detectability from $\dot{q}_{p2}$. Further, if $D_p(q_p)$ is block diagonal then we can use the (weaker) condition that $\frac{\partial V_1(q_{p2})}{\partial q_{p2}} = 0$ defines a bijection $q_{p1} \mapsto q_{p2}$.

Remark 3.3. The action of the controller above has the following nice *passivity interpretation*. First, notice that the EL system (3.3) in closed loop with the control signal (3.2) defines

[3] Since we are dealing here with a regulation and not a tracking problem there are no external inputs to the plant, which explains our choice of 0 as the "input matrix".

[4] Some structural conditions for the solvability of this problem are given in [12].

a passive operator $-\frac{\partial V_c(q_{p2},q_c)}{\partial q_{p2}} \mapsto \dot{q}_{p2}$ with storage function $T_p(\dot{q}_p, q_p) + V_p(q_p)$. On the other hand, the controller (3.1) defines a passive operator $\dot{q}_{p2} \mapsto \frac{\partial V_c(q_{p2},q_c)}{\partial q_{p2}}$ with storage function $T_c(\dot{q}_c, q_c) + V_c(q_{p2}, q_c)$. These properties follow, of course, from the passivity of EL systems established in proposition 2.1.

3.3 Reduced Order "Dirty Derivative" Controller

It is clear from the theorem above that the kinetic energy of the controller plays no role on the stabilization task. Furthermore, the conditions on the Rayleigh dissipation function of the theorem are satisfied with $\mathcal{F}_c(\dot{q}_c) = \frac{1}{2}\|\dot{q}_c\|^2$. Thus, with this choice of $\mathcal{F}_c(\dot{q}_c)$, and setting $T_c(\dot{q}_c, q_c) = 0$ we obtain the following corollary.

Corollary 3.1 The controller

$$\begin{aligned} \dot{q}_c &= -A_2(q_c + A_1 q_{p2}) \quad (3.5) \\ u_p &= -A_1 A_2(q_c + A_1 q_{p2}) - \frac{\partial V_{c2}(q_{p2})}{\partial q_{p2}} \end{aligned}$$

solves the *output feedback global stabilization* problem for EL systems verifying **A.1** provided $V_{c2}(q_{p2})$ satisfies **A.2** above.

□□□

Remark 3.4. Notice that (3.5) may be written as

$$u_p = -[A_1(pI_{m_p} + A_2)^{-1}p]q_{p2} - \frac{\partial V_{c2}(q_{p2})}{\partial q_{p2}}$$

with $p := \frac{d}{dt}$. Choosing A_1, A_2 diagonal we see that the first right hand term is the "dirty derivative" of q_{p2}, thus providing a theoretical justification to the common practice of using this technique to estimate velocities [8]. A similar result has been shown in [16] for general system-controller structures using high gains.

4 Example: Flexible Joint Robots

If we assume that joint flexibility can be modelled by a linear spring we obtain an EL system with generalized coordinates $q_p := [q_{p1}^T, q_{p2}^T]^T$, $q_{p1}, q_{p2} \in \mathcal{R}^{\frac{n_p}{2}}$ being the link and motor shaft angles respectively. The control variables are the torques at the shafts, thus $m_p = \frac{n_p}{2}$ and $M_p := [0|I_{m_p}]^T$. We are interested in set-point control of the link angles (to a given *constant* value q_{p1d}), and we assume only the motor shaft angles are measurable.

The *kinetic and potential energies* of a flexible joint robot are given by[5]

$$T_p := \frac{1}{2}\dot{q}_p^T D_p \dot{q}_p, \; V_p := \frac{1}{2}q_p^T \mathcal{K}_p q_p + V_g(q_{p1}) \quad (4.1)$$

where

$$\mathcal{K}_p := \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}, D_p := \begin{bmatrix} D_{11}(q_{p1}) & D_{12}(q_{p1}) \\ D_{12}^T(q_{p1}) & J \end{bmatrix}$$

with $D_{12}(q_{p1})$ strict upper triangular, $D_p(q_{p1}) = D_p^T(q_{p1}) > 0$ is the robot inertia matrix, $J \in \mathcal{R}^{m_p \times m_p}$ is a diagonal matrix of actuator inertias reflected to the link side, K is a diagonal matrix containing the joint stiffness coefficients, and $V_g(q_{p1})$ is the potential energy due to the gravitational forces.

Assuming zero damping, that is, $\mathcal{F}_p(\dot{q}_p) = 0$ we get the *dynamic equations* of the flexible joint robot as

$$D_p(q_{p1})\ddot{q}_p + C_p(q_{p1}, \dot{q}_p)\dot{q}_p + g_p(q_{p1}) + \mathcal{K}_p q_p = M_p u_p \quad (4.2)$$

where $g_p(q_{p1}) := [g_{p1}^T(q_{p1}), 0]^T = \frac{\partial V_g(q_{p1})}{\partial q_{p1}}$ and $C_p(q_{p1}, \dot{q}_p)$ is the Coriolis matrix.

We consider the controller[6] (3.5) with $A_1 = I_{m_p}$ and

$$V_{c2}(q_{p2}) = \frac{1}{2}(q_{p2} - \delta)^T K_1 (q_{p2} - \delta)$$

where K_1, A_2 are symmetric positive definite matrices, and δ is a constant vector that we choose below to satisfy the conditions of theorem 3.1.

• *(Dissipation propagation)*
Condition **A.1** is verified since setting $q_{p2} \equiv$ const and $u_p \equiv$ const implies, using the structure of $D_{12}(q_{p1}), C_p(q_{p1}, \dot{q}_p)$ [20], that $q_{p1} \equiv$ const.

• *(Damping injection)*
From the definition of $\mathcal{F}_c(\dot{q}_c)$, it is clear that the damping condition (1.2) is satisfied with $\alpha = 1$.

• *(Energy shaping)*
To verify **A.2** notice that setting $\frac{\partial V_1(q_p)}{\partial q_p}|_{\bar{q}_p} = 0$ yields

$$\begin{bmatrix} K & -K \\ -K & K + K_1 \end{bmatrix} \begin{bmatrix} \bar{q}_{p1} \\ \bar{q}_{p2} \end{bmatrix} + \begin{bmatrix} g_{p1}(\bar{q}_{p1}) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ K_1\delta \end{bmatrix}$$

which has a (unique) solution of the required form $\bar{q}_p = [q_{p1d}^T, *]^T$ with

$$\delta = q_{p1d} + (K^{-1} + K_1^{-1})g_{p1}(q_{p1d})$$

The second part of condition **A.2** is

$$\frac{\partial^2 V_1(q_p)}{\partial q_p^2} = \begin{bmatrix} K + \frac{\partial g_{p1}(q_{p1})}{\partial q_{p1}} & -K \\ -K & K + K_1 \end{bmatrix} \geq \epsilon I_{n_p} > 0$$

[5]For further details on this model see, e.g., [20]. Notice that the model we consider here contains, as a particular case, the model of [18] where $D_p(q_{p1})$ is assumed to be block diagonal.

[6]This controller was reported in [8] for the case of diagonal K_1, A_2.

In [1] the fact that $||\frac{\partial g_{p1}(q_{p1})}{\partial q_{p1}}|| < \beta$ is used to show that the Hessian matrix is positive definite if $\frac{1}{2}K_1, K > \frac{3+\sqrt{5}}{2}\beta I_n$.

5 Conclusions

We have given in this paper conditions for output feedback global stabilization of EL systems. The controller, which we choose to be also an EL system, is designed using the energy shaping plus damping injection ideas of the passivity-based approach. Our main contribution is the proof that damping injection *without* velocity measurement is possible via the inclusion of a *dynamic extension* provided the system satisfies a *dissipation propagation* condition. This condition, rules out the possibility of having wandering trajectories for the nonactuated variables $q_{p1}(t)$ when the control signal u_p and the actuated variables q_{p2} are constant. As pointed out in remark 3.1 further investigation is required to exploit the dependence of u_p on $V_{c2}(q_{p2})$ to get a better *–system theoretic–* understanding of the class of EL systems that satisfy this assumption.

One potential drawback of our design technique is that to achieve the energy shaping exact knowledge of the systems potential energy is required. See the definition of δ in the example. In a recent report [17] we managed to relax this assumption for the rigid robot control problem. Current research is under way to extend this result to a wider class of EL systems.

6 Acknowledgements

The first author would like to express his gratitude to Peter Caines for some useful discussions.

7 References

[1] A. Ailon and R.Ortega, An Observer-Based Set-Point Controller for Robot Manipulators with Flexible Joints, *Systems and Control Letters*, Vol.21, pp.329-335, 1993.

[2] S. Arimoto and F. Miyasaki, Stability and Robustness of PD feedback control with gravity compensation for robot manipulator, ASME *Winter Meeting*, Anaheim, CA (DEC, 1986) pp. 67-72.

[3] V. Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag, NY, 1989.

[4] H. Berghuis, and H. Nijmeijer, "Global regulation of robots using only position measurements", *Systems and Control Letters*, Vol.21, 1993.

[5] H. Berghuis and H. Nijmeijer, "A passivity approach to controller-observer design for robots", *IEEE Trans. Robot. Aut.*, vol. 9, no. 6, pp. 790–755, 1993.

[6] C. Byrnes, A. Isidori, J. Willems, Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems, *IEEE Trans Aut. Cont.*, Vol.36, no.11, pp. 1228-1240, 1976

[7] C.A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, Academic Press, NY, 1975.

[8] R. Kelly, R. Ortega, A.Ailon, A. Loria, Global Regulation of Flexible Joints Robots Using Approximate Differentiation, *IEEE Trans. Aut. Control*, vol. 39, no. 6, pp. 1222–1224, 1994.

[9] H. Goldstein, *Classical Mecanics*, Addison Wesley, 1974.

[10] D. E. Koditschek, "Robot Planning and Control via Potential Functions", in O. Khatib, J.J. Craig, T. Perez-Lozano, *Robotics Review 1*, MIT Press, 1989.

[11] Lanari, L. and J. Wen, "Asymptotically Stable Set Point Control Laws for Flexible Robots", Systems and Control Letters 19, (1992).

[12] H. Nijmeijer and A. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer-Verlag, NY, 1990.

[13] R. Ortega and G. Espinosa, Torque Regulation of Induction Motors, *Automatica*, Vol 29, No. 3, pp. 621-633, 1993.

[14] R. Ortega and M. Spong, Adaptive motion control of rigid robots: A tutorial, *Automatica*, Vol. 25, No.6, pp. 877-888, 1989.

[15] G. K. Pozharitskii, "On Asymptotic Stability of Equilibria and Stationary Motions of Mechanical Systems with Partial Dissipation", *-Prikl. Mat. i Mekh.*, vol. 25, 1961, 657-667. Engl. transl. in *J. Appl. Math. Mech.*, vol. 25, 1962.

[16] A. Teel and Praly L., "Tools for semiglobal stabilization by partial state and output feedback", *SIAM J. Contr. Opt.*, 1994.

[17] R. Ortega, A. Loria and Rafael Kelly, "A Semiglobally Stable Output Feedback PI^2D Regulator for Robot Manipulators", UTC, URA CNRS 817, Internal report, Compiegne, France, 1994.

[18] M. Spong, and M. Vidyasagar, *"Robot Dynamics and Control"*, John Wiley & Sons, 1989.

[19] M. Takegaki, and S. Arimoto, A new feedback method for dynamic control of manipulators, *ASME J. Dyn Syst. Meas. Cont.*, Vol. 102, 1981.

[20] P. Tomei, A simple PD controller for robots with elastic joints, *IEEE Trans Aut. Cont.*, Vol.36, no.10, pp.1208-1213, 1991.