Proceedings of the 33rd
Conference on Decision and Control
Lake Buena Vista, FL - December 1994

Small-gain theorem, gain assignment and applications

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Abstract

We introduce a concept of input-to-output practical stability (IOpS) which is a natural generalization of input-to-state stability proposed by Sontag. It allows us to establish two important results. The first one states that the general interconnection of two IOpS systems is again an IOpS system if an appropriate composition of the gain functions of the components is smaller than the identity function. The second one shows an example of gain function assignment by feedback. Applications to the problem of global stabilization via partial-state feedback and output feedback are also considered. The proofs can be found in [6].

1 Introduction

In this paper we introduce some new design tools which, when combined together, allow us to address the problem of stabilizing systems with intricate structure. Among the tools, the main contributions are to set up a small nonlinear gain theorem and a gain assignment theorem.

Notation

- $|\cdot|$ stands for the Euclidean norm, and Id denotes the identity function.
- For any measurable function $u : \mathbb{R}_+ \to \mathbb{R}^m$, $\|u\|$ denotes ess.sup.$\{|u(t)|, t \geq 0\}$. And for any $T \geq 0$, u_T is the usual truncated function.
- A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class K if it is continuous, strictly increasing and is zero at zero. It is of class K_∞ if, in addition, it is unbounded.
- A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class KL if, for each fixed t, the function $\beta(\cdot, t)$ is of class K and, for each fixed s, the function $\beta(s, \cdot)$ decreases to zero at infinity.
- GAS stands for globally asymptotically stable and LES stands for locally exponentially stable.
- UO (resp. SUO) stands for (resp. strong) unboundedness observability (see Definition 2.1).
- ISS stands for input-to-state stable and IOpS stands for input-to-output practically stable (see Definition 2.2).

2 Definitions and main results

2.1 Input-to-output practical stability

We begin with some definitions relative to the control system having x as state, u as input and y as output :

$$\begin{aligned} \dot{x} &= f(x,u)\,, \quad x \in \mathbb{R}^n\,,\ u \in \mathbb{R}^m \\ y &= h(x,u)\,, \quad y \in \mathbb{R}^p \end{aligned} \tag{1}$$

where f and h are smooth.

Definition 2.1 The system (1) is said to have the property of :

1. Unboundedness Observability (UO) if there exist a function α^o of class K and a nonnegative constant D^o such that, for each measurable control $u(t)$ defined on $[0,T)$ with $0 < T \leq +\infty$, the right maximal solution $x(t)$ of (1) defined on $[0,T')$ $(0 < T' \leq T)$ satisfies :

$$|x(t)| \leq \alpha^o\left(|x(0)| + \|(u_t, y_t)\|\right) + D^o \tag{2}$$

for all t in $[0,T')$.

2. Strong Unboundedness Observability (SUO) if, moreover, there exist a function β^o of class KL, a function γ^o of class K and a nonnegative constant d^o such that :

$$|x(t)| \leq \beta^o(|x(0)|, t) + \gamma^o\left(\|(u_t, y_t)\|\right) + d^o\,. \tag{3}$$

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Definition 2.2 The system (1) is *input-to-output practically stable (IOpS)* if there exist a function β of class KL, a function γ of class K and a nonnegative constant d such that, for any initial condition $x(0)$, and three functions $x(t)$, $u(t)$ and $y(t)$ defined on $[0,T)$ which satisfy the system (1), we have the following property :

$$|y(t)| \leq \beta(|x(0)|,t)+\gamma(\|u_t\|)+d\,,\ \text{a.e. } t\in[0,T) \quad (4)$$

When (4) is satisfied with $d=0$, the system (1) is said to be *input-to-output stable (IOS)*.

In the sequel, any function γ which satisfies (4) and is C^0, *nondecreasing*, zero at zero will be called a *(nonlinear) gain from input to output.*

Remark 2.1 The notions of UO (or SUO) and IOS introduced here differ slightly from the *strong observability* and IOS properties introduced by Sontag in [16, eq. (38)] and resp. [16, eq. (10)] in that dependence on the initial condition of the particular state space representation (1) is made explicit. In addition, the offsets D^o, d^o have been introduced. When $y=x$ in (1), IOpS is called *input-to-state practical stability (ISpS)*. In this case, if $d=0$ in (4) then IOpS becomes *input-to-state stability (ISS)* proposed by Sontag in [16, 17].

Remark 2.2 If a system has the UO property and is IOpS then the system has the "bounded input bounded state" property. If a system has the UO property and is IOS then, in addition, the system has the "converging input converging output" property. If a system has the UO property with $D^o=0$ and is IOS then, in addition, it is stable in the sense of Lyapunov when $u=0$. See Section 3 for additional properties of IOpS systems.

Associated with a detectability property, the UO and IOpS properties imply global asymptotic stability (GAS). To state such a result, we recall the following definition :

Definition 2.3 Let $\Phi(t,x,u)$ be the flow of the system (1) at time t starting from the point x under the input u. The system (1) is said to be *weakly zero-state detectable* if, for all $x\in\mathbb{R}^n$,

$$\{u\equiv 0\,,\ y(t)=0 \quad \forall t\geq 0\} \quad \Rightarrow \quad \lim_{t\to\infty}\Phi(t,x,0)=0$$

Proposition 2.1 *Assume the system (1) has the UO property with $D^o=0$ and is IOS. Under this condition, the origin of (1) is GAS when $u=0$ if and only if (1) is weakly zero-state detectable.*

2.2 Main results

Consider now the general interconnected system :

$$\begin{aligned}\dot{x}_1 &= f_1(x_1,y_2,u_1)\,, \quad y_1 = h_1(x_1,y_2,u_1) \quad (5)\\ \dot{x}_2 &= f_2(x_2,y_1,u_2)\,, \quad y_2 = h_2(x_2,y_1,u_2) \quad (6)\end{aligned}$$

where, for $i=1,2$, $x_i\in\mathbb{R}^{n_i}$, $u_i\in\mathbb{R}^{m_i}$ and $y_i\in\mathbb{R}^{p_i}$. The functions f_1, f_2, h_1 and h_2 are smooth and such that, for all x_i and $u_i\,(i=1,2)$, $y_1=h_1(x_1,h_2(x_2,y_1,u_2),u_1)$ and $y_2=h_2(x_2,h_1(x_1,y_2,u_1),u_2)$ have a unique smooth solution y_1 and y_2. Our first main result is :

Theorem 2.1 (Generalized small-gain theorem) *Suppose (5) and (6) are IOpS in the sense that :*

$$\begin{aligned}|y_1(t)| &\leq \beta_1(|x_1(0)|,t)+\gamma_1^y(\|y_2\|)+\gamma_1^u(\|u_1\|)+d_1\\ |y_2(t)| &\leq \beta_2(|x_2(0)|,t)+\gamma_2^y(\|y_1\|)+\gamma_2^u(\|u_2\|)+d_2\end{aligned} \quad (7)$$

Also, suppose that (5) and (6) have the UO property with couple (α_1^o, D_1^o) (resp. (α_2^o, D_2^o)). If there exist two functions ρ_1 and ρ_2 of class K_∞ and a nonnegative real number s_ℓ satisfying :

$$\begin{aligned}(\mathrm{Id}+\rho_2)\circ\gamma_2^y\circ(\mathrm{Id}+\rho_1)\circ\gamma_1^y(s) &\leq s\\ (\mathrm{Id}+\rho_1)\circ\gamma_1^y\circ(\mathrm{Id}+\rho_2)\circ\gamma_2^y(s) &\leq s\end{aligned} \quad (8)$$

$\forall s\geq s_\ell$, then the system (5)-(6) with (u_1,u_2) as input and (y_1,y_2) as output and (x_1,x_2) as state is IOpS and has the UO property (is IOS and has the UO property with $D^o=0$ when $s_\ell=d_i=D_i^o=0\,(i=1,2)$).

More specifically, for each pair of class K_∞ functions (r_3,ρ_3), there exist a function β of class KL and a nonnegative constant d (equal to zero when $s_\ell=d_i=D_i^o=0\,(i=1,2)$) such that the system (5)-(6) is IOpS with the triple $(\beta, r_1+r_2+r_3, d)$ where

$$\begin{aligned}r_1(s) &= (\mathrm{Id}+\rho_1^{-1})\circ(\mathrm{Id}+\rho_3)^2\circ[\gamma_1^u\\ &\quad +\gamma_1^y\circ(\mathrm{Id}+\rho_2^{-1})\circ(\mathrm{Id}+\rho_3)^2\circ\gamma_2^u](s) \quad (9)\\ r_2(s) &= (\mathrm{Id}+\rho_2^{-1})\circ(\mathrm{Id}+\rho_3)^2\circ[\gamma_2^u\\ &\quad +\gamma_2^y\circ(\mathrm{Id}+\rho_1^{-1})\circ(\mathrm{Id}+\rho_3)^2\circ\gamma_1^u](s) \quad (10)\end{aligned}$$

Remark 2.3 Condition (8) has been introduced by Mareels and Hill in [10] to state the boundedness part of this small-gain theorem for nonlinear feedback systems. This condition is a nonlinear version of the classical small-gain condition (see, for instance, [2]). Sufficient conditions to guarantee the condition (8) are given in [10]. In particular one inequality implies the other. Our task here was to complete the result of [10] in order to take into account the effects of the initial conditions and to express the gain function γ of the closed loop system in terms of the gains of the two subsystems. Our result can be used to conclude asymptotic stability for the internal variables under the conditions of the next corollaries 2.1 and 2.2.

Remark 2.4 Theorem 2.1 deals with global practical stability. Its complement, local asymptotic stability, holds (as in [22]) when $d_i = D_i^o = 0$ and $\forall s \geq s_\ell$ is replaced by $\forall s \leq s_\ell$ in (8).

Remark 2.5 The IOpS properties (7) and the small gain condition (8) imply that the topological separation condition of [15, Theorem 2.1] holds. Indeed, to each t in $\mathbb{R}_+$ and each output pair (y_1, y_2), we can associate the real number :

$$d_t(y_1, y_2) = \|y_{2t}\| - \gamma_2^y(\|y_{1t}\|) \, . \tag{11}$$

Then (7) implies readily that [15, eq. (2.3.2)] holds with the symbol v representing $x_2(0)$, d_2 and u_2. Also (7) and (8) imply that [15, eq. (2.3.1)] holds for some function ϕ_1 of class K_∞ and with the symbol u representing s_ℓ, $x_1(0)$, d_1 and u_1.

Corollary 2.1 *Under the conditions of Theorem 2.1, if $s_\ell = d_i = D_i^o = 0\,(i = 1, 2)$ and the systems (5) and (6) are weakly zero-state detectable, the system (5)-(6) is GAS when $u = 0$.*

Remark 2.6 When establishing GAS results using Corollary 2.1 we will, in certain instances, assume that each subsystem is ISS since this is sufficient to guarantee that each subsystem has the UO property and is weakly zero-state detectable. See Proposition 3.1 and Corollary 3.1 for another motivation of the ISS assumption.

As stated in Remark 2.1, IOpS (resp. IOS) is ISpS (resp. ISS) when the state is seen as an output. In this case, the UO property with $D^o = 0$ is obviously satisfied. The following corollary is a particular case of Theorem 2.1.

Corollary 2.2 *Consider the special case of system (5)-(6) with $y_1 = x_1$ and $y_2 = x_2$, i.e. :*

$$\dot{x}_1 = f_1(x_1, y_2, u_1), \quad y_1 = x_1 \tag{12}$$

$$\dot{x}_2 = f_2(x_2, y_1, u_2), \quad y_2 = x_2 \tag{13}$$

Assume that both the x_1 and x_2 subsystems are ISpS (resp. ISS) with (y_2, u_1) and (y_1, u_2) considered as inputs, i.e. (7) holds. If, in addition, the small-gain condition (8) is satisfied, then the complete system (12)-(13) is ISpS (resp. ISS when s_ℓ in (8) is equal to zero) with (u_1, u_2) as input.

Another interesting fact relying upon the notion of IOpS is that it is possible to assign any gain to certain classes of systems including n integrators. Precisely, we have :

Theorem 2.2 (Gain assignment) : *Consider the control system*

$$\begin{aligned} \dot{\zeta} &= A\zeta + B(H\xi + \omega_0) \\ \dot{\xi} &= F\xi + Gu + \omega \end{aligned} \tag{14}$$

with $u \in \mathbb{R}$ as input, $\zeta \in \mathbb{R}^l$, $\xi \in \mathbb{R}^n$ as components of the state, $(\omega_0, \omega) \in \mathbb{R} \times \mathbb{R}^n$ as perturbations and ζ as output. Assume (A, B) is stabilizable, (F, G) is controllable, (F, H) is observable and (H, F, G) has maximal relative degree. Under these conditions, for any function γ of class K_∞, there exists a smooth function $u_n(\zeta, \xi)$, with $u_n(0) = 0$, such that the system (14) in closed-loop with $u = u_n(\zeta, \xi) + v$ is :

1. *ISS with (ω_0, ω, v) as input,*
2. *IOpS with (ω_0, ω, v) as input, ζ as output and the function γ as gain.*

Moreover, if the inverse function γ^{-1} of γ is linearly bounded on a neighborhood of 0, the closed-loop system (14) can be rendered IOS.

Remark 2.7 There is no contradiction between the ISS and IOpS properties. The "practical" in the latter means only that, in general, γ is actually assigned only outside a neighborhood of 0.

Remark 2.8 Following [24], a sufficient condition under which an ISS system $\dot{x} = f(x, u)$ has a linearly bounded gain near zero is that its zero-input system $\dot{x} = f(x, 0)$ is exponentially stable at $x = 0$.

To illustrate the interest of these two theorems, let us consider the following single-input system :

$$\begin{aligned} \dot{x} &= f(x, z) + u \\ \dot{z} &= h(x, z) \end{aligned} \tag{15}$$

where (u, x, z) is in $\mathbb{R} \times \mathbb{R} \times \mathbb{R}^p$, f and h are smooth functions. When $\dot{z} = h(x, z)$ is ISS with x as input, following the arguments in [17], it is easy to make the whole system (15) ISS with a full-state feedback law such as $u := -x - f(x, z) + v$. However, to the authors' knowledge, making the system (15) ISS with a *partial-state* feedback law like $u = \vartheta(x) + v$ is still an open issue. Here, we present a positive answer.

Corollary 2.3 *(1) If in (15) the z-subsystem is ISpS with x as input, then we can find a smooth partial-state feedback $\vartheta(x)$ which is zero at zero and such that, with $u = \vartheta(x) + v$, the system (15) is ISpS with v as input.*
(2) If the z-subsystem, with x as input, is ISS and, with $f(x,z)$ as output, is IOS with a gain function linearly bounded on a neighborhood of 0, then the system (15) is made ISS with v as input.
(3) If the z-subsystem, with x as input, is ISS and the matrix $\frac{\partial h}{\partial z}(0,0)$ is asymptotically stable, then $\vartheta(x)$ in closed loop with (15) gives GAS and LES.

Remark 2.9 This result extends to the partial-state feedback case or dynamic uncertain case the "adding one integrator technique" (compare with [23, Theorem 4]).

3 Further facts about the IOpS property

The main purpose of this section is to establish some properties of IOpS systems.

We first point out that the notions of IOpS (resp. IOS) and ISpS (resp. ISS) are strongly related. Indeed, for the control system with outputs :

$$\begin{array}{rcll} \dot{x} &=& f(x,u)\ , & x \in \mathbb{R}^n\ ,\ u \in \mathbb{R}^m \\ y &=& h(x,u)\ , & y \in \mathbb{R}^p \end{array} \tag{16}$$

where f and h are smooth functions, we have, similar to [16, Prop. 3.2 & Prop. 7.1],

Proposition 3.1 *If the x-system is ISpS (resp. ISS), then the system (16) with y as output is IOpS (resp. IOS if, in addition, $h(0,0) = 0$). Conversely, if the system (16) is IOpS (resp. IOS) and has the property SUO with a d^o in (3), then the x-system is ISpS (resp. ISS if, in addition, $d^o = 0$).*

Next result follows readily from Theorem 2.1 and Proposition 3.1.

Corollary 3.1 *Under the conditions of Theorem 2.1, if the systems (5) and (6) have the SUO property, the system (5)-(6) is ISpS (resp. ISS if $s_\ell = d_i^o = d_i = 0$ $(i = 1,2)$).*

In Theorem 2.1, we gave a nonlinear small-gain condition under which the interconnected system made of two IOpS systems is again IOpS. In some cases, this condition is trivially checked. More precisely, when the system (5)-(6) takes the following form :

$$\begin{array}{rcl} \dot{x} &=& f(x,z,u) \\ \dot{z} &=& g(z,u)\ , \end{array} \tag{17}$$

we have, similar to [16, Prop. 7.2],

Proposition 3.2 *If the x-subsystem of (17) is ISpS (resp. ISS) with (z,u) as input and the z-subsystem of (17) is ISpS (resp. ISS) with u as input, then the system (17) is ISpS (resp. ISS) with u as input.*

4 Applications

4.1 A detour from the center manifold reduction theorem

Consider the following system :

$$\begin{array}{rcl} \dot{z} &=& q(z,\zeta) \\ \dot{\zeta} &=& f(\zeta) + \omega(z,\zeta) \end{array} \tag{18}$$

with $(z,\zeta) \in \mathbb{R}^p \times \mathbb{R}^n$ as state, $\omega \in \mathbb{R}^n$ as coupling nonlinearity. Assume :

1. The vector field f is homogeneous with degree r and $\zeta = 0$ is an asymptotically stable equilibrium point of $\dot{\zeta} = f(\zeta)$.
2. The z-subsystem with ζ as input and $\omega(z,\zeta)$ as output has the SUO property with $d_z^o = 0$ in (3) and is IOS with gain $\gamma_z(s) \le \mu |s|^r$ for some nonnegative real number μ.

Then, as an immediate application of Corollary 3.1, we have :

Proposition 4.1 *Under these conditions and if μ is sufficiently small, the zero solution of (18) is GAS.*

This result generalizes [4, Lemma p.442] or [1, Lemma 4.3] where the local counterpart of this result is proved by applying the center manifold reduction theorem which imposes $f(\zeta) = F\zeta$ with F an asymptotically stable matrix. System (18) has been treated in a different way in [5, Section 4].

4.2 Linear systems with nonlinear, stable dynamic perturbations

Consider the following system :

$$\left\{ \begin{array}{rcl} \dot{z} &=& q(z,\zeta) \\ \dot{\zeta} &=& A\zeta + B(H\xi + \omega_0(z,\zeta)) \\ \dot{\xi} &=& F\xi + Gu + \omega(z,\zeta) \end{array} \right. \tag{19}$$

with $u \in \mathbb{R}$ as input, $(z,\zeta,\xi) \in \mathbb{R}^p \times \mathbb{R}^l \times \mathbb{R}^n$ as state, $(\omega_0,\omega) \in \mathbb{R} \times \mathbb{R}^n$ as coupling nonlinearities. Assume :

1. (A, B) is stabilizable, (F, G) is controllable, (F, H) is observable and (H, F, G) has maximal relative degree.
2. The z-subsystem with ζ as input and (ω_0, ω) as output has the SUO property with a $d_z^o = 0$ in (3) and is IOS with a gain function γ_z linearly bounded on a neighborhood of 0.

Using Theorem 2.2 and Corollary 3.1 gives :

Proposition 4.2 *Under these conditions, we can design a smooth partial-state global asymptotic stabilizer $u(\zeta, \xi)$ for the system (19) such that the system (19) with $u = u(\zeta, \xi) + v$ is ISS with respect to v.*

This proposition belongs to the class of results known for these so-called partially linear composite systems studied for example in [20, 14, 19, 21, 9]. As proved in [19], when $l > 1$, extra assumptions must be imposed on the z subsystem to guarantee controllability to the origin even when (A, B) is controllable and the coupling terms (ω_0, ω) are not present (see also [14, Theorem 3]). These extra assumptions are in place to guarantee that the state z remains bounded while the state ζ converges to zero, as in [18]. For example, growth conditions on q may be imposed ([14, Proposition 5], [20, Theorems 6.2 & 6.4]). Here, to address the coupling terms, we impose the SUO and IOS properties on the z subsystem with (ω_0, ω) as outputs. According to Corollary 2.1, this could be relaxed to UO + IOS + weakly zero-state detectable if only GAS is desired.

4.3 Perturbed pure-feedback systems

Consider the single-input system:

$$\begin{array}{rcl} \dot{x}_i & = & x_{i+1} + f_i(X_i, Z_i) \\ \dot{Z}_i & = & h_i(X_i, Z_i), \quad 1 \leq i \leq n-1 \\ \dot{x}_n & = & u + f_n(X_n, Z_n) \\ \dot{Z}_n & = & h_n(X_n, Z_n) \end{array} \tag{20}$$

where, for each i in $\{1, \ldots, n\}$, $X_i = (x_1, \ldots, x_i) \in \mathbb{R}^i$ is part of the measured system state components, $Z_i \in \mathbb{R}^{m_i}$ is unmeasured, $u \in \mathbb{R}$ is the input and the functions f_i's and h_i's are smooth.

This type of systems has been extensively studied by many researchers with different viewpoints including state feedback stabilization, or (dynamic) output feedback stabilization (see [8, 11, 13] and the references therein). In the absence of the dynamic uncertainties characterized here by the Z_i's, results on the global stabilization of (20) are available in [7, 8, 12, 3].

A recursive use of Corollary 2.3 and Proposition 3.2 leads to the following:

Proposition 4.3 *Suppose that for each $1 \leq i \leq n$, $\dot{Z}_i = h_i(X_i, Z_i)$ is ISpS with X_i as input. Then we can design a smooth partial-state feedback $u(x_1, ..., x_n)$ such that for any initial conditions all the trajectories of the system (20) in closed loop with $u = u(x_1, ..., x_n)$ are bounded. Moreover, if for each $1 \leq i \leq n$, $f_i(0) = 0$, $\frac{\partial h_i}{\partial Z_i}(0)$ is an asymptotically stable matrix and the Z_i-subsystem is ISS, we can design a global asymptotic partial-state stabilizer $u(x_1, ..., x_n)$ for the system (20).*

5 Conclusions

The notion of input-to-output practical stability (IOpS) introduced in this paper is a natural generalization of Sontag's input-to-state stability property. We have shown that the notion IOpS allows us to establish a generalized small-gain theorem (see Theorem 2.1 and Corollary 3.1) and a gain assignment theorem (see Theorem 2.2). The first one extends the small monotone gain theorem proved by Mareels and Hill in [10] by including a stability result of Lyapunov type. With these results, we have been able to prove a result in the spirit of the center manifold reduction theorem (see Proposition 4.1), to give conditions under which a linear system with nonlinear, stable dynamic perturbations is globally asymptotically stabilizable (see Proposition 4.2) and finally to show that the ISS property can be propagated through integrators by choosing an appropriate partial-state feedback (see Corollary 2.3). The latter provides an interesting tool for control design (see Proposition 4.3).

Acknowledgments :

Most of this work was done while the first author was with CAS, Ecole des Mines de Paris. The authors are indebted to Jean-Michel Coron and Eduardo Sontag for helpful remarks.

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