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ROBUST MODEL REFERENCE ADAPTIVE CONTROLLERS. PART II: RELAXING THE STABILITY CONDITIONS

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ABSTRACT

Several modifications to the adaptive schemes aimed at enhancing its robustness properties are studied from the point of view of classical compensation and sensitivity theories. Bounded output disturbance cancellation and reduced order modelling are considered. The effect of outer-loop compensation, usually carried in practice, is theoretically formalized. Some open questions and further research topics are presented.

1. INTRODUCTION

In this paper we shall be concerned with the problem of verifying the stability conditions that arise in the theory of robust discrete adaptive control [1,2]. We will consider two classes of uncertainty: bounded output disturbances (BOD) and reduced order modelling.

The following assumptions will be made regarding the process: linear time invariancy, finite dimension, state invertibility and known delay. For this class of systems the robust stability conditions reduce to the *existence* of a regulator parameter vector verifying: i) the resulting closed loop transfer function is inside a conic sector (contained in the passivity sector), ii) the regulator incorporates the internal models of the reference and BOD signals. Sidestepping of both conditions will be studied in the paper from the point of view of classical compensator design. Although the results presented in [3] relax the requirement of ii) preserving L_∞-stability, we believe it is important to properly understand its implications in view of its effects on performance deterioration.

The effect on the conicity condition of two types of filters is established. It is shown that *phase lead filtering of the adaptation error* and *low pass filtering of the regressor and reference sequences* affects in an identical way the conicity condition. Constructive procedures to design these filters are given when either one of the following a priori informations is available: an upper bound on the profile of the phase shift induced by the neglected dynamics or a "sufficiently tight" conic bound for the closed-loop process transfer function. Model reference and minimum variance controllers are also compared from this perspective.

Incorporation of a *regulation loop around the tracking error*, which has proven useful in practice, is also evaluated. Both adaptive and fixed gain compensation are considered and its effect on the conicity condition formally established. Using the recent results on optimal sensitivity [4] a design process consisting of two separate stages is sketched: 1) Approaching the plant to the model reference with the adaptive controller. That is shrinking the radius of the cone centered at the model reference which contains the adaptively controlled system. 2) Filtering of the resulting uncertainty with the outer-loop compensator. A possible alternative for applying the results in [4,section VII] to give bounds on the conicity condition verification (that is the achievable sensitivity reduction) in terms of the singularity measure of the model reference is outlined.

The condition of *BOD cancellation* is shown to be easily verified by a modified scheme incorporating the BOD internal model.

A brief presentation of the adaptive controller and the main stability theorem is done in Section 2. The conicity condition relaxation by filtering and outer-loop compensation are presented in Sections 3 and 4 respectively. Section 5 is devoted to the BOD cancellation problem.

2. ADAPTIVE CONTROLLER AND STABILITY CONDITIONS[1]

2.1. Adaptive scheme

Let the *known delay* process described by

$$A(q^{-1})Y_t = q^{-d}B(q^{-1})U_t + V_t \tag{2.1}$$

be adaptively controlled with

$$W_{t+d} = \hat{S}_t(q^{-1})U_t + \hat{R}_t(q^{-1})Y_t \triangleq \hat{\theta}_t^T \phi_t \tag{2.2.a}$$

$$\phi_t \triangleq [U_t, U_{t-1}, \ldots, U_{t-n_S}, Y_t, Y_{t-1}, \ldots Y_{t-n_R}]^T \tag{2.2.b}$$

$$\hat{\theta}_t = \hat{\theta}_{t-d} + \lambda''_t F_t \phi_{t-d} e_t \tag{2.2.c}$$

$$F_t^{-1} = \lambda'_t F_{t-d}^{-1} + \lambda''_t \phi_{t-d} \phi_{t-d}^T \quad ; \lambda'_t > 0 \ , \ \lambda''_t = \rho_t^{-1} \tag{2.2.d}$$

$$\rho_t = \mu\rho_{t-1} + \max(|\phi_{t-d}|^2, \underline{\rho}) \ ; \ \underline{\rho} > 0 \ , \ \mu \in (0,1) \tag{2.2.e}$$

being our desired objective

$$e_t \triangleq Y_t - \frac{1}{C_R} w_t \to 0 \tag{2.3}$$

that is the model reference is q^{-d}/C_R.

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[1] See [2,3] for a detailed description of this section.

A.1 (μ-Stabilizability assumption). Let n_S, n_R and μ be given scalars as in (2.2.b) and (2.2.e) respectively. For the closed-loop polynomial

$$C \triangleq S_* A + q^{-d} R_* B \tag{2.4}$$

the following condition of stabilizability holds:

$$\Theta_s \triangleq \{ \theta_* \varepsilon R^n : C(q) \neq 0 , \forall q \varepsilon C, |q| > \mu^{1/2} \} \neq \emptyset \tag{2.5}$$

where $n \triangleq n_S + n_R + 2$ and θ_* contains the coefficients of the polynomials[1] S_*, R_*.

2.2. Error model stability

Combining (2.1), (2.2.a), (2.3) and (2.4) we get the error model

$$e_t = -H_2 \psi_t + e_t \; ; \; e_t \triangleq (H_2 - C_R^{-1}) \omega_t + C^{-1} S_* V_t \; ; \; H_2 \triangleq C^{-1} B \tag{2.6.a}$$

$$\phi_{t-d} = W_1 (\omega_t - \psi_t) + W_2 V_t \tag{2.6.b}$$

where

$$\psi_t \triangleq (\hat{\theta}_{t-d} - \theta)^T \phi_{t-d} \tag{2.7}$$

$$W_1 \triangleq C^{-1} [A, q^{-1}A, \ldots, q^{-n_R} A \; ; \; q^{-d}B, q^{-d-1}B, \ldots q^{-d-n_S} B] \tag{2.8.a}$$

$$W_2 \triangleq C^{-1} [-q^{-d}R_*, -q^{-d-1} R_*, \ldots -q^{-d-n_R} R_* \; ; \; -q^{-d}S_*, q^{-d-1} S_*, \ldots q^{-d-n_S} S_*]^T \tag{2.8.b}$$

Notice that H_2 *is the transfer function* Y_t^*/w_t, i.e. the process in closed-loop with a μ-stabilizing regulator.

Global L_2-stability of the adaptively controlled process is established in the following theorem [2].

Theorem 2.1. (L_2- Stability conditions). Consider the known delay LTI system (2.1) in closed-loop with the adaptive controller (2.2), (2.3). If assumption A.1 holds and

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[1] Refered in the sequel as μ-stabilizing regulator.

(conicity) 1) $|\bar{\sigma} H_2 [(\mu^{1/2} e^{j\theta})^{-1}] - 1| < \sqrt{1-\bar{\sigma}}$, $\forall \theta \in [0,\pi]$,

$$\bar{\sigma} > 1/(1 + \| \lambda_t' \|_\infty) \quad (2.9.a)$$

(BOD cancelation)[1] 2) $e_t^* \in L_2$ (2.9.b)

then ψ_t, $e_t \in L_2$ (hence $\to 0$) and $\phi_t \in L_\infty$.

2.3. Discussion

We will separately discuss the various implications of the assumptions above.

1. The *process delay* is needed to be known to get a Reparametrization in terms of causal operators. For the unknown delay case see [3].This is a fundamental assumption for robustness studies. It allows to insure that H_2 is proper hence its global phase shift (when taking $\theta \in [0,\pi]$) is zero for all stably invertible processes. This in its turn implies that the phase shift required to verify the conicity condition can always be provided with a stable filter.

2. The *condition* $e_t^* \in L_2$ implies that $[(C_R - q^{-d} R_*)B - S_* A]$ and S_* contain the internal models of ω_t and V_t respectively. While the former may be interpreted as a model-following requirement, i.e. H_2 approaches $1/C_R$ (see 2.6.a) the latter is of a completely different nature. Although not requiring the a priori knowledge of the internal models (2.9.b) is a restrictive assumption. It constitutes a second restriction on the set of allowable regulator parameters, the first being μ-stabilizability. Furthermore, simulated evidence [5] has proven that convergence is usually extremely slow in the presence of BOD. In Section 5 an alternative scheme that explicitly incorporates the BOD internal model (if this is available) and preserves closed-loop stability is proposed.

3. Understanding the nature of the two parameters ($\bar{\sigma}$ and μ) appearing in the *conicity condition* (2.9.a) furnishes considerable insight into the design problem. $\bar{\sigma}$ ranges in [0,1] and is a designer choosen parameter. When the parameter adaptation algorithm (PAA) does not include *normalization* i.e. $\lambda_t'' \neq \rho_t^{-1}$, it will depend on the level of excitation (ϕ_t) and the speed of convergence ($\lambda^{max}(F_t)$) [2]. Better robustness properties, that is a larger allowable cone for H_2, is obtained with smaller $\bar{\sigma}$. At the limit (2.9.a) becomes a passivity condition of the form[2] Re $\{H_2\} > 1/2$. The coefficient μ establishes an *alertness-robustness trade-off*,

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[1] This condition has been relaxed in [3] in a L_∞ context.

[2] By suitable selection of λ_t'' we may relax to Re $\{H_2\} > 0$.

appearing its robustness aspect at both the μ-stabilizability (2.5) and conicity (2.9.b) assumptions. PAA alertness is directly affected since μ defines the normalization filter time constant (2.2.e). $\bar{\sigma}$ can also be proved to be related to the contraction factor of the PAA, hence affecting the speed of convergence.

4. In [2,3] the adaptation error was defined as the *tracking error* (2.3) *filtered by* C_R. The conicity condition is transformed to:

$$C_R H_2 \in A \triangleq \text{CONE}\ (1/\bar{\sigma},\ \sqrt{1-\bar{\sigma}}/\bar{\sigma}) \tag{2.10}$$

and it has the nice interpretation of requiring $H_2 \approx C_R^{-1}$. We will find convenient to use this filtered adaptation error in the sequel, it will be denoted e_t^c

$$e_t^c \triangleq C_R\, e_t \tag{2.11}$$

5. Notice that we have been able to recapture in the stability conditions (2.9) the usual performance (in the sense of pole-placement) and disturbance rejection design objectives.

3. CONICITY CONDITION RELAXATION BY FILTERING

Two different filters will be considered in this section: phase lead filtering of the adaptation error and low pass filtering of the regressor and reference sequence. Its effect on the conicity condition will be proved to be equivalent. Also it will be shown that from this perspective, the robustness of minimum variance and model reference controllers is identical.

3.1. Adaptation error phase lead filtering

In most practical situations the closed-loop transfer function H_2 will have a phase lag larger than[1] 90°. However, this may only happen at a frequency range strictly contained in $(0,\pi)$, since $\Im\{H_2(e^{j0})\} = \Im\{H_2(e^{j\pi})\}$. This motivates the inclusion of a phase lead filtered adaptation error

$$e_t^L \triangleq L\, e_t \quad ; \quad L \in \mathcal{R}(q^{-1}) \tag{3.1}$$

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[1] Specifically, any multiplicative modelling error causing more than 30° phase lag violates (2.9.a).

in the PAA (eq. 2.2.c) to insure $LH_2 \in A$. It is clear that if an upper bound on the profile of the phase shift of H_2 is known it is always possible to construct L verifying the passivity condition. Stable invertibility of H_2 further insures the stability of L.

It is customary in robust linear design to use conic bounded transfers functions to characterize coarsely defined systems [4]. Since the conicity condition is given *for any* μ-stabilizing regulator, in our adaptive context we only require the knowledge of a conic bound for a closed-loop transfer function attainable with the choosen regulator. In particular for stable systems we may simply take the open-loop bound. It is reasonable to assume that the bounds on H_2 will be tighter than the open-loop bounds. Furthermore a natural choice for the cone center is simply the model reference. With this idea in mind a procedure to choose L is given below.

Lemma 3.1. If $|H_2-C_2| < |R_2|$, $C_2 R_2 \in R\,(q^{-1})$ with C_2, R_2 proper stable and stably invertible

and L is chosen as

$$L = (\bar{\sigma}\ C_2)^{-1} \tag{3.2}$$

then $LH_2 \in A$ for all (sufficiently tight) cones verifying

$$\| R_2 C_2^{-1} \|_2 < \sqrt{1 - \bar{\sigma}} \tag{3.3}$$

Proof

$$|H_2-C_2| < |R_2| \Rightarrow |C_2^{-1} H_2 - 1| < |C_2^{-1} R_2| < \sqrt{1-\bar{\sigma}} \qquad \text{(from eq.3}$$

$$\Rightarrow |L\ \bar{\sigma}\ H_2 - 1| < \sqrt{1-\bar{\sigma}} \qquad \text{(from eq.3}$$

$$\Rightarrow L\ H_2 \in A$$

Notice that though filtering e_t by L or C_R affects H_2 in a similar way, L is significant only in the adaptive design and it *does not change the desired closed loop performance*. Prefiltering of the reference sequence by C_R in (2.2.a) would add the interpretation of observer polynomial to C_R.

3.2. Regressor and reference sequence low-pass filtering

We will prove in this section that the well established practice of low-pass filtering of the regressor and reference signals improves the robustness of the adaptive scheme in the sense that phase advance is added to H_2. The following lemma defines the error model for the modified scheme.

Lemma 3.2. For a given $M \in \mathcal{R}(q^{-1})$, proper, stable and stably invertible define

$$\phi_t^M \triangleq M^{-1} \phi_t \;;\; \omega_t^M \triangleq M^{-1} \omega_t \;;\; \psi_t^M \triangleq (\hat{\theta}_{t-d}-\theta_*)^T \phi_{t-d}^M \tag{3.4}$$

and use them to replace ϕ_t and ω_t in (2.2). Under these conditions the tracking error verifies

$$e_t = - H_2 M \psi_t^M + e_t^* \tag{3.5}$$

Proof. The modified regulator equation (2.2.a) may be written as

$$\omega_{t+d} = P_M(\hat{\theta}_t)^T \phi_t \tag{3.6}$$

where $P_M(.)$ is an operator defined as

$$P_M(X_t) \triangleq M X_t M^{-1}$$

From the linearity and invariancy to constant X_t of P_M, i.e. $P_M(X) = X$

$$\omega_{t+d} = \theta_*^T \phi_t + M(\hat{\theta}_t-\theta_*)^T \phi_t^M \tag{3.7}$$

Since

$$Y_t = H_2 \theta_*^T \phi_{t-d} + C^{-1} S_* V_t \tag{3.8}$$

we can, substituting (3.7) and (3.8) in (2.3) get (3.5).

A stability theorem analogous to Th.2.1 can be easily derived, defining an operator $H_1^M : e_t \to \psi_t^M$ which verifies the same conicity conditions as $H_1 : e_t \to \psi_t$ [2,5]. Notice that in contrast to L and C_R, M does not affect e_t^*. However its effect on H_2 is identical. Remark also that a *low pass* M^{-1} *gives phase lead to* H_2, hence robustness improvement.

3.3. Regressor extension

Minimum variance (MV) controllers [6] require, to insure convergence, to augment the regressor vector in the PAA by

$$\phi_t^{MV} \triangleq [\, \phi_t \;\vdots\; \omega_{t+d-1}, \omega_{t+d-2}, \dots \omega_{t+d-n_\gamma}]^T$$

in order to estimate a prefilter $\hat{\gamma}_t(q^{-1})$ of the reference **sequence.** Consequently the MV control law is given as

$$\omega_{t+d} = \hat{\theta}_t^{MV} \phi_t^{MV} \tag{3.10}$$

Define the following polynomial

$$C_{MV} \triangleq AS_* + q^{-d}BR_* + q^{-1}\Gamma_* B = C + q^{-1}\Gamma_* B \tag{3.11}$$

with Γ_* of degree $n_\gamma - 1$. Multiplying (3.11) by Y_t

$$C_{MV}Y_t = B\theta_*^T \phi_{t-d} + B\Gamma_* Y_{t-1} + S_* V_t \triangleq B(\theta_*^{MV})^T \phi'_{t-d} + S_* V_t \tag{3.12}$$

Notice that for $C_R = 1$

$$(\theta_*^{MV})^T(\phi'_{t-d} - \phi_{t-d}^{MV}) = q^{-1}\Gamma_* e_t \tag{3.13}$$

substituting in (3.12) and using (2.3)

$$C_{MV}e_t = B[(\theta_*^{MV})^T \phi_{t-d}^{MV} + q^{-1}\Gamma_* e_t] + S_* V_t - C_{MV}\omega_t$$

Using (3.11) and adding and substracting $B\omega_t$ we get

$$C\, e_t = B[(\theta_*^{MV} - \hat{\theta}_{t-d}^{MV})^T \phi_{t-d}^{MV}] + (B - C_{MV})\omega_t + S_* V_t$$

consequently using (3.11) and definning a corresponding ψ_t^{MV}

$$e_t = -H_2\psi_t^{MV} + [H_2(1-q^{-1}\Gamma_*) - 1]\,\omega_t + C^{-1} S_* V_t \tag{3.14}$$

Compare (3.14) with (2.6). We remark that from the point of view of the conicity and the BOD cancelation condition no improvement is obtained by adapting the reference signal precompensator. However, we remark that the component of e_t^* due to the reference sequence is modified.

4 - OUTER-LOOP COMPENSATION

In this section we will study a modification to the adaptive scheme consisting of an outer-loop compensator taken around the error signal (see Fig.1). Both fixed and adaptive compensators will be considered. Such a modification has been proved useful in practice [7], its theoretical implications are formalized here.

Fig. 1

4.1. Fixed and adaptive compensation

Adding to the control law (2.2.a) the regulation-loop gives

$$\omega_{t+d} = \hat{\theta}_t^T \phi_t + H_{OL}(q^{-1})e_t \quad ; \quad H_{OL} \in \mathcal{R}(q^{-1}) \tag{4.1}$$

and the error model

$$e_t = (1 + q^{-d}H_2H_{OL})^{-1}\,[-H_2\psi_t + e_t^*] \tag{4.2}$$

Compare (4.2) with (2.6.a). Notice that the outer-loop compensation is equivalent to a feedback interconnection of H_2 and $q^{-d}H_{OL}$. This can be easily seen from Fig.2 where we have equated the dotted section transfer function to $q^{-d}H_2$ and recalling that due to the definition of ψ_t (eq.2.7) the process delay is reflected in ω_t (hence in the feedback branch).

An interesting linear control problem arises immediately. How to characterize (in the frequency domain) the class of transfer functions we can obtain from the feedback interconnection of a proper, stable and stably invertible system (H_2) and a stable compensator with fixed relative degree ($q^{-d}H_{OL}$)?. A first preliminary answer arises from the restriction of causality of H_{OL}. This simply implies that the first d-1 Markov parameters of the closed-loop transfer function and those of H_2 must be equal. In continuous time this means that the high frequency behaviour of the closed-loop system approach that of H_2 as $\omega \to \infty$. We will further elaborate this point in section 4.2.

Fig. 2

Since the choice of H_{OL} strongly depends on the knowledge of H_2, adaptive theoreticians would simply suggest to let the PAA search the "best" H_{OL}, e.g. to adapt also H_{OL}. In fact this can easily be done augmenting the regressor (assuming for simplicity $H_{OL} \in R[q^{-1}]$)

$$\phi_t^{OL} \triangleq [\phi_t \,\vdots\, e_t, e_{t-1}, \dots e_{t-n_H}] \quad ; \quad n_H \triangleq \deg\{H_{OL}\} \tag{4.3}$$

The new (μ-stabilized) characteristic polynomial would be

$$C_{OL} \triangleq AS_* + q^{-d}BR_* + q^{-d}BH_3^* = C + q^{-d}BH_3^* \tag{4.4}$$

and the error model

$$e_t = -C_{OL}^{-1}B\psi_t^{OL} + C_{OL}^{-1}(B - C\,C_R^{-1})\omega_t + C_{OL}^{-1}S_*v_t \tag{4.5}$$

where ψ_t^{OL} is defined as in (2.7) with ϕ_t^{OL} and the augmented parameter vectors. Remark that $C_{OL}^{-1}B$ is the closed-loop transfer function Y_t^*/ω_t. It is clear from (4.4) that from the point of view of the conicity condition (now imposed to $C_{OL}^{-1}B$) adding $H_{OL}(t,q^{-1})$ is equivalent to augmenting the order of $\hat{R}_t(q^{-1})$. Hence no (structurally new) robustness improvement is obtained.

4.2. Interpretation as a plant uncertainty filter

In this section we will give an interpretation to the outer-loop compensated scheme in terms of the following design problem: given a model reference C_R^{-1} and an uncertainty bound on H_2 (the closed-loop transfer function attainable placing a μ-stabilizing regulator around an uncertain plant $q^{-d}B/A$), find the filters H_{OL} and L (or M) which shrink the ball of uncertainty (the new allowable cone equivalent to condition (2.9.a)), define bounds on the optimal shrinkage and look at its dependence on C_R^{-1} and the uncertainty over H_2. Compare with Problem 2 of [4]. The motivations to pose in that way the problem are twofold. Firstly, we believe that this formulation helps us to *approach classical sensitivity theory to adaptive control theory*, which is our final objective. Secondly, in our framework, the two *stages of control-law synthesis*: design of a control law for a nominal plant and filtering of plant uncertainty are clearly identified. The former, being restricted to the well structured part of the process, is realized with a model reference adaptive controller. The outer-loop and adaptation error (or regressor and reference) compensation are then viewed as filters for the plant uncertainty. In contrast with the linear case, the design stages in adaptive control are not independent and they are related by the conicity condition (2.9.a).

Depending on the error (e_t, e_t^c, e_t^L) introduced to the PAA and the outer-loop compensator, the transfer function required to satisfy the conicity condition may take different forms. In all cases, it is possible to represent it as the transfer between nodes 1 and 2 of the model reference scheme of Fig.3.

Fig. 3

Typically P represents a plant with a controller attached, $\hat{P}$ is an estimate of P and Q provides the additional filtering required to the extent that P differs from $\hat{P}$.

Assume P is known to the extent that it lies inside a cone of the form

$$P \in \mathcal{P} \overset{\Delta}{=} \mathrm{CONE}(\hat{P},\ \delta W)\ , \quad \delta > 0 \tag{4.6}$$

where W is a unitary norm filter. The following propositions regarding the scheme of Fig.3 are proved in [4].

Proposition 4.1. The feedback system is stable if

$$\| W\ Q \|_2 < \delta^{-1} \tag{4.7}$$

Proposition 4.2. Let $K_{12}(P)$ be the operator mapping P into the (closed-loop) transfer function between nodes 1 and 2. It is easy to see that

$$K_{12}(\hat{P}) = \hat{P}\,T \tag{4.8}$$

$$K_{12}(P) - K_{12}(\hat{P}) = (1-\hat{P}Q)(P-\hat{P})\,[\,1+Q(P-\hat{P})]^{-1}T \tag{4.9}$$

Propositions 4.3. Define

$$v(P,\hat{P},\delta) \triangleq \sup_{P\varepsilon P}\{\| W[K_{12}(P) - K_{12}(\hat{P})]\|_2 \delta^{-1}\}$$ (sensitivity to plant perturbations)

$$\mu(P) \triangleq \inf_Q\{\| W(1-PQ)\|_2\}$$ (singularity measure of P)

$$\underline{v}(\hat{P},\delta) \triangleq \inf_Q \{ v(P,\hat{P},\delta)\}$$ (optimal sensitivity)

The following relations hold:

$$v(P,\hat{P},\delta) \geq \mu(\hat{P}) \tag{4.10}$$

$$\lim_{\delta\to o} \underline{v}(\hat{P},\delta) = \mu(\hat{P}) \tag{4.11}$$

$$\underline{v}(\hat{P},\delta) \leq \| W(1-\hat{P}Q)\|_2 (1-\delta\|Q\|_2)^{-1} , \forall \delta \varepsilon [0,\|Q\|_2^{-1}] \tag{4.12}$$

For our application P and $\hat{P}$ represent H_2 and C_R^{-1} respectively and T,Q contain all the additional filters introduced to enhance the robustness. Current research is under way to establish the relationship between v and the conicity condition. To date only an indirect interpretation is available.

5 - BOUNDED OUTPUT DISTURBANCE CANCELLATION

When the internal model of the BOD is *known a priori*, it is possible to incorporate it into the adaptive controller to cancel their effect. The only modification required is in the regressor, which should be taken as

$$\phi_t^D \triangleq [DU_t, DU_{t-1}, \ldots DU_{t-n_S} ; Y_t, Y_{t-1}, \ldots Y_{t-n_R}]^T$$

where

$$DV_t = 0 \quad , \quad D \varepsilon R [q^{-1}]$$

It is easy to show that the new error model is

$$e_t = - C_D^{-1} B \psi_t^D + (C_D^{-1} B - C_R^{-1})\omega_t$$

$$C_D \triangleq S_* D A + q^{-d} R_* B$$

Notice that to insure $e_t^* \varepsilon L_2$ we still require $(BC_R - C_D)\omega_t = 0$. This is however a requirement that is satisfied whenever the pole-placement objective is fulfilled. Since constant disturbances are always present, an integrating term in D should usually be added. It is clear that simply augmenting the order of $\hat{S}_t$ would allow the PAA to accommodate the BOD internal model, however simulated evidence [5] has shown that convergence is extremely slow. Remark also that when explicitly adding D, the order of $\hat{R}_t$ should be incremented.

The condition $e_t^* \varepsilon L_2$, which is equivalent to say that robust servo behaviour is possible for the choosen regulator orders, is not only a technical requirement. The existing adaptive schemes are highly sensitive to its violation. The flexibility available for the PAA to search to satisfy the pole-placement requirement is considerably reduced since it is constrained to the parameter subspace containing the BOD internal model. In order to reduce the BOD deleterious effects the design objective should take into account, besides the usual pole-zero placement some frequency selective mechanism.

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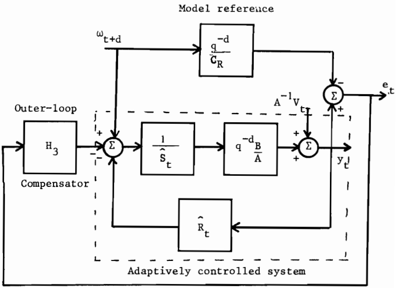

Fig. 1.

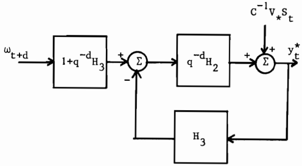

Fig. 2.

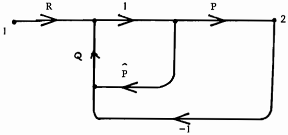

Fig. 3.