TOWARDS A GLOBALLY STABLE DIRECT ADAPTIVE CONTROL SCHEME FOR NOT NECESSARILY MINIMUM PHASE SYSTEMS

L. PRALY

CAI - ECOLE NATIONALE SUPERIEURE DES MINES DE PARIS
35, Rue Saint-Honoré 77305 Fontainebleau Cedex France
Tel : (6) 422.48.21

ABSTRACT

A direct adaptive control scheme for not necessarily minimum phase systems is presented. The algorithm is based on simultaneous identification of input and output prediction models. This leads to a bilinear parameters estimation problem for which a least squares criterion minimization is proposed. This framework makes it possible to establish global convergence without any extra condition. We give examples to illustrate practical features.

I - INTRODUCTION

Nowadays one of the most important problem of adaptive control theory is that of direct adaptive control of non minimum phase systems (sampled continuous time systems frequently are not minimum phase [9]). It may be opposed to indirect adaptive control for which conditionned boundedness has been established [2]. This conditionning generally requires that the identified model be stabilizable and implies constrained identification [3]. We will here see that, in direct adaptive control, non linear identification is substituted for constrained identification.

The primary problem of adaptive control algorithm is to guarentee boundedness of both input and output signals. To get this double property, two kinds of informations about the closed loop system are needed. In indirect adaptive control they are given by model parameter estimation on the one hand and controller parameter computation on the other hand [2], [10]. In direct adaptive control of minimum phase systems they are given by controller estimation on the one hand and knowledge of the minimum phase property on the other hand [4]. For direct adaptive control of non minimum phase systems, they will be given by controller plus extra parameters estimation.

In [1] or [8], these extra parameters are those of the model. However this leads to parameter estimation based on a bilinear observation equation. A relaxation method is used to solve this problem. It consists at the first step in a classical linear model parameter estimation whose result is then used at the second step to linearize the bilinear observation equation. This is in fact very close to an indirect scheme with the problem of stabilizability coming in.

In [5] the extra parameters are those of a partial state predictor. The observation of the parameters is linear. But it only yields one piece of information about the system. Therefore we think that without any persistency of excitation assumption only local stability may be ascertained (hence staying in the vicinity of the true system is the second piece of information).

Here we introduce an input prediction model which together with an output prediction model define an implicit prediction model which is bilinear in the parameters. To solve this estimation problem a least squares criterion minimization is proposed. This framework makes it possible to establish global stability. Moreover in [6] we have shown that our formulation can be extended to imbed such problems as MIMO systems with reduced order model, neglected weak coupling or some time variation effects and non linearities.

However, to day we do not know any computationnally efficient algorithm to solve the minimization problem involved in our approach. Nevertheless to get some insight into practical features, we offer a more implementable algorithm and we present simulation results.

II - DIRECT ADAPTIVE CONTROL SCHEME

<u>Assumptions</u> : Let a system with $y(t)$, $u(t)$ as scalar output and input respectively. The following assumptions will be used :

A0. there exists (unknown) scalar polynomials in the unit delay operator q^{-1} such that the system can be represented by :

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t) \tag{1}$$

with

$$A(q^{-1}) = 1+a_1 q^{-1}+\dots+a_{n_A} q^{-n_A} \tag{2}$$

$$B(q^{-1}) = b_0+b_1 q^{-1}+\dots+b_{n_B} q^{-n_B} \tag{3}$$

A1. $n \geqslant \max\{n_A, n_B\}$ is known

A2. An upperbound (which does not need to be small) of the coefficients of $A(q^{-1}), B(q^{-1})$ is known.

A3. Given an (known) exponentially stable polynomial $R(q^{-1})$

0191-2216/83/0000-0199 $1.00 © 1983 IEEE

$$R(q^{-1})=1+r_1 q^{-1}+\ldots+r_{n_R} q^{-n_R}\ ,\ n_R \leq n \quad (4)$$

There exists (unknown) polynomials $C(q^{-1})$, $D(q^{-1})$ of degree $n,(n-1)$ respectively with $C(0) = 1$, such that :

$$A(q^{-1})\ C(q^{-1}) + q^{-1}\ B(q^{-1})\ D(q^{-1}) = R(q^{-1}) \quad (5)$$

Moreover an upperbound of the coefficients of $C(q^{-1})$, $D(q^{-1})$ is known.

<u>System reparametrization</u> : Using assumption A3, let

$$\begin{aligned} A(q^{-1}) &= 1 + q^{-1}\ \bar{A}(q^{-1}) \\ C(q^{-1}) &= 1 + q^{-1}\ \bar{C}(q^{-1}) \\ R(q^{-1}) &= 1 + q^{-1}\ \bar{R}(q^{-1}) \end{aligned} \quad (6)$$

From (1), (5) we get:

$$\bar{R}(q^{-1})u(t) = \begin{cases} +\bar{C}(q^{-1})u(t)+D(q^{-1})y(t)+\bar{A}(q^{-1})u(t) \\ \qquad (7.a) \\ +\bar{A}(q^{-1})[\bar{C}(q^{-1})u(t-1)+D(q^{-1})y(t-1)] \end{cases}$$

$$R(q^{-1})y(t)=B(q^{-1})[u(t)+\bar{C}(q^{-1})u(t-1)+D(q^{-1})y(t-1)] \quad (7.b)$$

Equations (7) may be considered as equivalent to equations ((1),(5)). Given $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$, $D(q^{-1})$, $R(q^{-1})$ this is an implicit input-output prediction model. Given $R(q^{-1})$, $\{u(t)\},\{y(t)\}$ this is a bilinear observation equations in the coefficients of $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$, $D(q^{-1})$.

<u>Feedback control law</u> : If the input $u(t)$ is generated by the causal feedback control law :

$$u(t+1)=-\bar{C}(q^{-1})u(t)-D(q^{-1})y(t)+E\ \ R(q^{-1})y^M(t) \quad (8)$$

where $\{y^M(t)\}$ is an arbitrary bounded set point sequence and E is a scalar, then the resulting closed loop is :

$$R(q^{-1})(u(t)-A(q^{-1})E\ y^M(t-1))=0 \quad (9)$$

$$R(q^{-1})(y(t)-B(q^{-1})E\ y^M(t-1))=0 \quad (10)$$

It is exponentially stable since $R(q^{-1})$ is exponentially stable. It achieves a traking objective if :

$$1 = B(1)E \quad (11)$$

Since no assumption is made about the stability of the inverse system, it may not be possible that $y(t)$ tends to $y^M(t)$. Therefore, at least special consideration has to be given to the question of steady-state errors in output sequence.

<u>Adaptive control scheme</u> : In this paper we shall be concerned with the situation when $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$, $D(q^{-1})$ are unknown. Only $R(q^{-1})$ is given and the input $u(t)$ and the output $y(t)$ are measured. Our problem is to determine an algorithm to achieve the following objectives :

- $\{u(t)\}$ and $\{y(t)\}$ remain bounded.
- the closed loop polynomial approaches $R(q^{-1})$ for the given set point sequence $\{y^M(t)\}$

In section III we investigate the properties of the following algorithm : at each sampling time t, we proceed in two steps :

1 - Identification of both system and controller polynomials using prediction model (7). This gives time varying polynomials $\bar{A}(t,q^{-1})$, $B(t,q^{-1})$, $\bar{C}(t,q^{-1})$, $D(t,q^{-1})$.

2 - Computation of the control as

$$u(t+1)=-\bar{C}(t,q^{-1})u(t)-D(t,q^{-1})y(t)+E(t)R(q^{-1})y^M(t) \quad (12)$$

with :

$$E(t)= \begin{cases} \dfrac{1}{B(t,1)} & \text{if } |B(t,1)| > \varepsilon \\ 1 & \text{if not} \end{cases} \quad (13)$$

III - <u>BILINEAR ESTIMATION AND STABILITY</u>

<u>Bilinear estimation</u> : Let us derive a prediction error formulation [7] :

Let ψ^* , θ^* be the true system and controller parameter vector respectively :

$$\theta^* = (d_0 \ldots d_{n-1} c_1 \ldots c_n)' \quad (14)$$

$$\psi^* = (a_1 \ldots a_n b_0 \ldots b_n)' \quad (15)$$

For other particular value of these vectors say ψ,θ we may define from (7) the following prediction error vector

$$\varepsilon(t,\theta,\psi)=z(t)-(H_u(t)H_y(t))F(\theta,\psi) \quad (16)$$

where

$$z(t)=(\bar{R}(q^{-1})u(t)\quad R(q^{-1})y(t))' \quad (17)$$

$$H_u(t)= \begin{pmatrix} y(t)\ldots y(t-2n+1)u(t)\ldots u(t-2n+1) \\ 0 \qquad \ldots\ldots \qquad 0 \end{pmatrix}$$
$$H_y(t)= \begin{pmatrix} 0 \qquad \ldots\ldots \qquad 0 \\ y(t-1)\ldots y(t-2n)u(t)\ldots u(t-2n) \end{pmatrix} \quad (18)$$

$F(\theta,\psi)$ is a vector whose coordinates are the coefficients of $A(q^{-1})D(q^{-1})$, $A(q^{-1})C(q^{-1})$, $B(q^{-1})D(q^{-1})$, $B(q^{-1})C(q^{-1})$. The function $F(.,.)$ has the following properties :

<u>Lemma 1</u> :

i) F is a continuous function from $R^{(4n+1)}$ to $R^{(8n+1)}$

ii) if $\|F(\theta,\psi)\|$ is finite then so is $\|(\theta,\psi)\|$

iii) $F(.,.)$ is generally not monic. But if (θ,ψ) is an antecedent such that both corresponding $(C(q^{-1}),D(q^{-1}))$, $(A(q^{-1}),B(q^{-1}))$ are relatively prime, then it is unique.

Proof : due to space limitation, the proof is omitted.

Let $J(t,\theta,\psi)$ be the least squares criterion with forgetting factor recursively defined as follows :

$$J(t,\theta,\psi)=\mu\ \ J(t-1,\theta,\psi)+\varepsilon(t,\theta,\psi)'Q\varepsilon(t,\theta,\psi) \quad (19)$$

$$0 < \mu < 1 \quad (20)$$

$$J(0,\theta,\psi)= (F(\theta,\psi)-F(\theta(0),\psi(0)))' \times P_0(F(\theta,\psi)-F(\theta(0),\psi(0))) \quad (21)$$

where Q, P_0 are positive definite matrices, $(\theta(0),\psi(0))$ is a priori estimates.

A classical estimation procedure lies in minimizing $J(t,\theta,\psi)$. To introduce a less stringent algorithm, let J_0 be a positive scalar given by boundedness assumption in A2, A3, such that :

$$J(0,\theta^*,\psi^*) < J_0 \quad (22)$$

Let M_t be the set defined as follows :

$$M_t = \{(\theta,\psi)/J(t,\theta,\psi) \leq \mu^t J_o\} \quad (23)$$

Then at time t we take the estimates $(\theta(t),\psi(t))$ as an element of M_t. We have the following property :

Lemma 2 : Subject to assumptions A0, A1, A2, A3, we have :

I1 : $\|\theta(t)\|$, $\|\psi(t)\|$ remain bounded

I2 : $\lim_{t\to\infty} \|\varepsilon(t),\ \theta(t),\ \psi(t)\| = 0$

I3 : M_t is a decreasing sequence of compact subsets.

Proof : see appendix A.

However the limit set $\bigcap_{t=0}^{\infty} M_t$ does not generally consist of only one point. As a consequence, parameter time variations may be introduced without improving estimation. To round this difficulty, we have to define a rule to choose $(\theta(t),\psi(t))$ in M_t. For example let $(\theta(t),\psi(t))$ be the element of M_t that minimizes $\|\theta-\theta(t-1)\|^2$, i.e. $(\theta(t),\psi(t))$ is given by solving :

$$(\theta(t),\psi(t)) \in \text{Arg} \min_{(\theta,\psi)\in M_t} \|\theta-\theta(t-1)\|^2 \quad (24)$$

with

$$M_t = \{(\theta,\psi)/J(t,\theta,\psi) \leq \mu^t J_o\} \quad (25)$$

then we have :

Lemma 3 : Subject to assumptions A0, A1, A2, A3, if $(\theta(t),\psi(t))$ are as defined by (24), (25) then we have properties I1, I2, I3 and :

I3' : $\lim_{t\to\infty} \|\theta(t) - \theta(t-1)\| = 0$

Proof : see appendix A.

Now with properties I1, I2, I3' we are in position to state that our objectives are achieved :

Global stability :

Theorem : Let $(\theta(t),\psi(t))$ be such that properties I1, I2, I3' are verified, if u(t+1) is given by (12) , $\{u(t)\},\{y(t)\}$ remain bounded and :

$$\lim_{t\to\infty} (R(q^{-1})y(t) - B(t,q^{-1})y^*(t-1)) = 0 \quad (26)$$

$$y^*(t) = E(t)\ R(q^{-1})\ y^M(t) \quad (27)$$

Proof : see appendix A.

IV - TOWARDS AN IMPLEMENTABLE ALGORITHM

The algorithm presented in the previous section is conceptual. At each time t, it requires the computation of a minimizer of a quadratic criterion over an implicitely defined non convex set. However it should be noted that given θ (resp ψ) $\min_{\psi} J(t,\theta,\psi)$ (resp. $\min_{\theta} J(t,\theta,\psi)$) is a classical quadratic minimization problem. Consequently an alternate minimization in θ,ψ is a candidate for an efficient descent algorithm which converges to any stationnary point of $J(t,\theta,\psi)$. Since $J(t,\theta^*,\psi^*)$ is strictly smaller than $\mu^t J_o$ if $J(t,\theta,\psi)$ has only one stationnary point, one could expect that using this alternate minimization procedure, $\theta(t),\psi(t)$ can be computed in a finite number of operation. This leads to the following estimation algorithm :

Algorithm : at each time t :

$i=0$, $\theta_o=\theta(t-1)$, $\psi_o=\psi(t-1)$

1.1. $\theta_{i+1} = \text{Arg} \min_{\theta} J(t,\theta,\psi_i)$

1.2. $\psi_{i+1} = \text{Arg} \min_{\psi} J(t,\theta_{i+1},\psi)$

2. if $i \geq i$ max then $\theta(t)=\theta_{i+1}$, $\psi(t)=\psi_{i+1}$ end

3. $i=i+1$ return to 1.

Due to one-line consideration, ($i \leq i$ max) is substituted for $(J(t,\theta,\psi)\leq\mu^t J_o)$ to stop the iterations.

The computational complexity of this algorithm consists in 1.1, 1.2.: A positive symetric linear system has to be computed (approximatively n^4 operations) and solved.

Simulations using this algorithm are presented in appendix B.

Remark : In fact $J(t,\theta,\psi)$ may have several stationnary points. One of the reasons is the non monicity of the F(.,.) function (since J depends explicitely on F and not on (θ,ψ)). To round this difficulty, we can limit the set of admissible (θ,ψ) for the minimization such that F(.,.) is monic. Using lemma 1, this is met if (θ,ψ) are such that the corresponding polynomials verify :

$$A(q^{-1})C(q^{-1}) + q^{-1}\ B(q^{-1})\ D(q^{-1}) = 1 \quad (28)$$

Note that this leads to choose $R(q^{-1}) = 1$ as desired closed loop poles.

This constraint could be introduced in the estimation algorithm using penalty function : let $G(\theta,\psi)$ be the quadratic sum of the relations in the coefficients given by (28) (Note that again $G(\theta,\psi)$ is separately quadratic in (θ,ψ)), in the algorithm above we could replace $J(t,\theta,\psi)$ by :

$$I(t,\theta,\psi) = J(t,\theta,\psi) + \nu\ G(\theta,\psi) \quad (29)$$

where ν is a positive Lagrange multiplier.

V - CONCLUSION

An adaptive direct control scheme is presented in this paper. It is obtained with a pole placement as underlying design method. The characteristics of this technique are :

- estimation of both model and controller parameters
- an estimation procedure which is bilinear in these parameters and which is obtained from both input and output prediction error model.

We show that a conceptual least squares criterion minimization is sufficient to get global convergence with very weak assumptions:

. stabilizability of the system

. knowledge of an upper bound of the parameters and of the system order.

In particular no assumption is required about the estimated parameters.

However our statement should be considered rather as a theoretical existence result than as a practical scheme. Nevertheless, we have introduced a more implementable algorithm which leads to encouraging simulation results.

Much more work remain to be done for converting these ideas into an efficient algorithm. Our preliminary results suggest an interesting and surely fruitful subject for future study.

REFERENCES

[1] K.J. ASTROM
"Direct methods for non minimum phase systems" IEEE 19th CDC. December 1980.

[2] G.C. GOODWIN, K.S. SIN
"Adaptive control of non minimum phase system". IEEE Trans AC, April 1981.

[3] Ph. de LARMINAT
"On the stabilizability condition in indirect adaptive control". 6th IFAC Symp. On Identification and System Parameter Estimation. June 1982.

[4] G.C. GOODWIN, C.R. JOHNSON, K.S. SIN
"Global convergence for adaptive one-step ahead optimal controllers based on input matching". IEEE Trans AC, Dec.1981.

[5] H. ELLIOTT
"Direct adaptive pole placement with application to non minimum phase systems" IEEE Trans AC, June 1982.

[6] L. PRALY
"Towards a direct adaptive control scheme for general disturbed MIMO systems".
Analysis and Optimization of Systems. Lecture notes in Control and information sciences. Vol.44. Springer Verlag 1982.

[7] L. LJUNG
"Convergence analysis of parametric identification methods". IEEE Trans AC, October 1978.

[8] L. LJUNG, E. TRULSSON
"Direct minimization methods for adaptive control of non-minimum phase systems". 6th IFAC Symp. On Identification and system parameter estimation. June 1982.

[9] K.J. ASTROM
"Zeros of sampled systems". IEEE 19th CDC December 1980

[10] C. SAMSON
"An adaptive L.Q. controller for non minimum phase systems". Int. J. Control, Vol.35, N°1, 1982.

APPENDIX A

Proof of lemma 1 : First note that (θ^*,ψ^*) belongs to M_t which is consequently not empty. From the definition of $J(t,\theta,\psi)$ we have :

$$J(t-1,\theta,\psi) \leq \frac{1}{\mu} J(t,\theta,\psi) \qquad \text{(AP.2)}$$

This implies that if (θ,ψ) belongs to M_t , it also belongs to M_{t-1}. Moreover using the properties of F(.,.) (see lemma 1), we can state that M_o is a compact subset. This yields properties I1, I3.

Ultimately, property I2 can be obtained by noting that :

$$\varepsilon(t,\theta,\psi)'Q\varepsilon(t,\theta,\psi) \leq J(t,\theta,\psi) \qquad \text{(AP.2)}$$

Proof of lemma 2 : Since $\theta(t)$ minimizes $\|\theta-\theta(t-1)\|^2$ on M_t, and, for any integer s, $\theta(t+s)$ belongs to M_t (use property I3), we have :

$$\|\theta(t+s)-\theta(t-1)\| \geq \|\theta(t)-\theta(t-1)\| \qquad \text{(AP.3)}$$

Now by contradiction, suppose there exists $\varepsilon > 0$ and an infinite set of indices T such that :

$$\forall t \in T, \|\theta(t)-\theta(t-1)\| > \varepsilon \qquad \text{(AP.4)}$$

with (AP.3), we also have :

$$\forall t \neq s \in T, \|\theta(s-1)-\theta(t-1)\| > \varepsilon \qquad \text{(AP.5)}$$

Let $B(\theta(t-1),\frac{\varepsilon}{2})$ be the closed sphere around $\theta(t-1)$ with radius $\frac{\varepsilon}{2}$

The family $B(\theta(t-1),\frac{\varepsilon}{2})_{t\in T}$ covers a subset of $\overline{M}_o$, the compact subset of $\mathbb{R}^{2n}$ defined as

$$\overline{M}_o = \{\theta \mid \exists(\theta_o,\psi_o) \in M_o : \|\theta-\theta_o\| \leq \frac{\varepsilon}{2}\} \qquad \text{(AP.6)}$$

Moreover we have :

$$B(\theta(s-1),\frac{\varepsilon}{2}) \cap B(\theta(t-1),\frac{\varepsilon}{2}) = \phi \quad \forall t\neq s \in T \qquad \text{(AP.7)}$$

Then let v (resp. V) be the volume of $B(\theta,\frac{\varepsilon}{2})$ (resp. $\overline{M}_o$), and let card T denote the number of elements of T. It follows that :

$$v.\ \text{card}\ T \leq V \qquad \text{(AP.8)}$$

This implies that card T is finite which contradicts the definition of T.

Proof of theorem :
Let $\Phi(t)$ be the following vector :

$$\Phi(t)=(y(t)\ldots y(t-n+1)u(t)\ldots u(t-n+1))' \qquad \text{(AP.9)}$$

(12) may be rewritten in :

$$u(t+1)=-\theta(t)'\Phi(t)+y^*(t) \qquad \text{(AP.10)}$$

Let $\varepsilon_u(t),\varepsilon_y(t)$ be the coordinates of $\varepsilon(t,\theta(t),\psi(t))$ we have :

$$\varepsilon_u(t)=\begin{cases}\overline{R}(q^{-1})u(t)-\theta(t)'\phi(t)\\ -\sum_{i=1}^{n} a_i(t)[\theta(t)'\phi(t-i)+u(t+1-i)]\end{cases} \qquad \text{(AP.11)}$$

$$\varepsilon_y(t)=\begin{cases}R(q^{-1})y(t)\\ -\sum_{i=1}^{n} b_i(t)[\theta(t)'\phi(t-i-1)+u(t-i)]\end{cases}$$

It follows that :

$$R(q^{-1})u(t+1)=\begin{cases}A(t,q^{-1})y^*(t)+\varepsilon_u(t)\\ +\sum_{i=1}^{n} a_i(t)[\theta(t)-\theta(t-i)]'\phi(t-i)\end{cases} \qquad \text{(AP.12)}$$

$$R(q^{-1})y(t)=\begin{cases}B(t,q^{-1})y^*(t-i)+\varepsilon_y(t)\\ +\sum_{i=1}^{n} b_i(t)[\theta(t)-\theta(t-i-1)]'\phi(t-i-1)\end{cases}$$

Let us rewrite these equations in a more classical state space representation : let X(t) be the following vector :

$$X(t)=(y(t-1)\ldots y(t-2n)u(t)\ldots u(t-2n))' \qquad \text{(AP.13)}$$

We rewrite (AP.12) in :

$$X(t+1)=(F+\Delta F_t)X(t)+G\varepsilon(t,\theta(t),\psi(t))+H(t)Y^*(t) \quad \text{(AP.14)}$$

Where F is a companion matrix of $(R(q^{-1}))^2$

ΔF_t incorporates the following differences :

$$a_i(t)(\theta(t)-\theta(t-i)),b_i(t)(\theta(t)-\theta(t-i-1)$$

G is a constant matrix

$H(t)$ includes the coefficients of $A(t,q^{-1})$, $B(t,q^{-1})$

$Y^*(t)$ is the following vector :

$$Y^*(t)=(y^*(t)\ldots y^*(t-n-1))' \quad \text{(AP.15)}$$

Now F is exponentially stable. There exists a norm such that :

$$\| F \| \leqslant \sigma < 1 \quad \text{(AP.16)}$$

And with I1, we know that $a_i(t),b_i(t)$ in $\psi(t)$ are bounded, then with I3', there exists a time T such that :

$$\forall t \geqslant T \qquad \| \Delta F_t \| \leqslant \frac{1-\sigma}{2} \quad \text{(AP.17)}$$

With I2, I1 we know that $\varepsilon(t,\theta(t),\psi(t)),H(t)$ are bounded. It follows that $X(t)$ remain bounded.
Then using I2, I3' in (AP.12) leads to (27).

Remark : if (25), (26) are used to get $\psi(t)$, $\theta(t)$; before stating uniform boundedness of $X(t)$ we need to show that $J(t,\theta,\psi)$ makes sense for any time t, i.e. $(H_u(\tau)H_y(\tau))$ in (18) are finte for $\tau \leqslant t$.

Let us state this property recursively :
Assume that, for $\tau \leqslant t$, $(H_u(\tau)H_y(\tau))$ are finite. Then $X(t)$ and $\phi(t)$ are finite. But also M_t is well defined and therefore $\theta(t),\psi(t)$ are finite. Then using (AP.11) $\varepsilon_u(t)$ is finite. It follows from (AP.12) that $u(t+1)$ is finite and from (1) that $y(t+1)$ is finite. This yields $(H_u(t+1)H_y(t+1))$ finite.

APPENDIX B

To illustrate some practical features of the procedure proposed in this paper, we offer simulations for a very difficult example presented in [2].

The following system is considered :

$$A(q^{-1}) = 1.0 - 1.2q^{-1}$$
$$B(q^{-1}) = 1.0 - 3.1q^{-1} + 2.2q^{-2}$$

The objectives are :

$$R(q^{-1}) = 1.0$$

$$y^M(t) = \begin{cases} 1.0 & 0\leqslant t\leqslant 60 \\ -1.0 & 61\leqslant t\leqslant 80 \\ 1.0 & 81\leqslant t\leqslant 100 \end{cases}$$

It follows that the controller is defined by :

$$C(q^{-1}) = 1.0 + 22.8q^{-1} - 39.6q^{-2}$$
$$D(q^{-1}) = -21.6$$

Note very high coefficients. This is due to the proximity of roots of $A(q^{-1})$, $B(q^{-1})$ (1.2. for $A(q^{-1})$ and 1.1. for $B(q^{-1})$). This will also explain a very high sensitivity of the controller parameters with respect to variations of the model parameters when they are in the vicinity of the system parameters.

We use the estimation algorithm of section IV with the following conditions :

- $\mu = .8$
- imax = 28
- initial parameters :
  $A(q^{-1}) = 1.0 - 1.0q^{-1}$
  $B(q^{-1}) = -2.0q^{-1} + 3.0q^{-2}$
  $C(q^{-1}) = 1.0 + 1.0q^{-1} + 3.0q^{-2}$
  $D(q^{-1}) = 1.0$
- initial signals $\equiv 0$
- P_o is obtained by simulating the closed loop plant given by initial parameters with $y^M(t)$ a white noise qnd by taking :

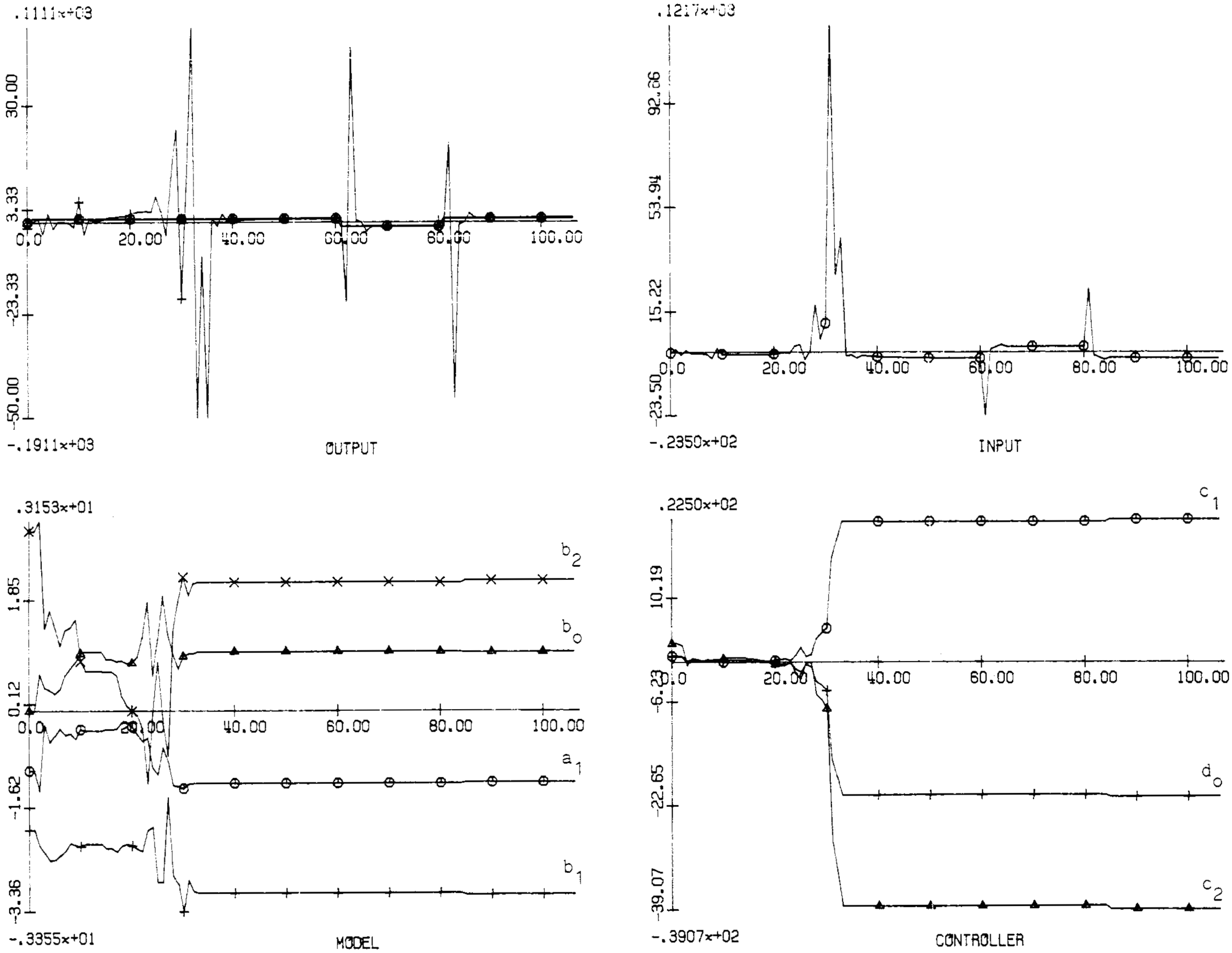
$$P_o = \sum_{t=o}^{T} \mu^{T-t}(H_u(t)H_y(t))(H_u(t)H_y(t))'$$

The figure shows the output and input of the system and the estimated parameters for our algorithm. We can see that :

i) the output converges to the reference output. However we have a bad step response due to the unstable zeros of the system, its small gain and the choice of closed loop poles (see (10)).

ii) the parameters converge to their true values. Note the jump of the controller parameters when the model parameters have converged.

With this very crude algorithm, we nearly have the same behaviours than those presented in [2] for an indirect scheme. By now the matter is not to compare these two schemes since our algorithm is too much time consuming. However the weakness of our assumptions and these first simulation results suggest an interesting subject for future study.


.1111x+03
30.00
3.33
-23.33
-50.00
-.1911x+03
0.0
20.00
40.00
60.00
80.00
100.00
OUTPUT
.1217x+03
92.66
53.94
15.22
-23.50
-.2350x+02
INPUT
.3153x+01
1.85
0.12
-1.62
-3.36
-.3355x+01
b_2
b_o
a_1
b_1
MODEL
.2250x+02
10.19
-6.23
-22.65
-39.07
-.3907x+02
c_1
d_o
c_2
CONTROLLER