TOWARDS A DIRECT ADAPTIVE CONTROL SCHEME FOR GENERAL DISTURBED MIMO SYSTEMS

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ABSTRACT :

The existence of direct adaptive control schemes for not necessarily minimum phase Multiinput-Multioutput systems is stated. This result is based on identification of a model with a bilinear structure when on line pole placement is to be achieved. We establish global boundedness even when the norm of the residuals between the true plant and the assumed linear model is bounded from above by the norm of the signals.

I - INTRODUCTION

Nowadays one of the most important problem of adaptive control theory is that of direct adaptive control of non minimum phase systems [4]. It may be opposed to indirect adaptive control for which conditionned boundedness has been established [1],[11]. This conditionning generally requires that the identified model be stabilizable and implies constrained identification [12], [13]. We will here see that, in direct adaptive control, non linear identification is substituted for constrained identification.

Since both input and output boundedness is required, two kinds of informations about the system are needed. In indirect adaptive control they are given by system identification on the one hand and feedback computation on the other hand [1],... In direct adaptive control of minimum phase systems, they are given by controller identification on the one hand and knowledge of both the minimum phase property and the system interactor on the other hand [2],... For direct adaptive control of non minimum phase systems, they will be given by both system and controller identification which are intricately connected [3]. As a first consequence, there is an increase of the parameter number as one can find as a common feature of the works we are aware of [4],[5],[6],[7]. The second consequence is a bilinear observation of the parameters in the output prediction error model as shown by Aström in [4]. To get around this problem Elliott in [5] uses a partial state prediction error model which is linear in the parameters. However this only yields one piece of information about the system and therefore we think that only local stability may be ascertained (hence staying in the vicinity of the true system is the second piece of information). In [4],[6], the bilinear parameter estimation problem is solved by relaxation consisting at the first step of a classical system identification which is then used at the second step to linearize the bilinear observation equation. This is in fact very close to an indirect scheme with the problem of stabilizability coming in. In [7], an on line criterion minimization scheme is proposed but since, in this algorithm, no attention is paid to input boundedness, it seems that only local boundedness can be ascertained. Nevertheless, in this case and with some restrictions, convergence to the global optimum is established.

We here propose to solve the bilinear estimation problem by considering both input and output prediction error models. However our result should be considered rather as a theoretical existence result than as a practical algorithm. Much more work remain to be done for converting our idea into an implementable scheme. Nevertheless our framework make it possible to establish global boundedness even when the norm of the residuals between the true plant and the assumed linear model is bounded from above by the norm of the signals. In this formulation we can imbed such problems as reduced order model, neglected weak coupling or time variation effects.

Section II is devoted to the control law description assuming the system is perfectly known. In section III we describe our direct adaptive control scheme and using [10] we give sufficient conditions about the estimation scheme in order to get global boundedness. Section IV states the existence of a modified stochastic gradient procedure which meets these conditions. Section V draws conclusions.

All proofs are omitted (see [14]).

II - LINEAR TIME INVARIANT CONTROL SCHEME

II.1. Choice of Multiinput-Multioutput system representation :

Consider a MIMO system, for which, at each stage n we let u_n be the control input vector (in R^m) and y_n be the output vector (in R^ℓ) and we assume that the following controllable representation holds for the system :
There exist relatively right prime polynomial matrices $A_1(b)$, $B_1(b)$ of appropriate dimensions such that :

$$\begin{aligned} A_1(b)z_n &= u_n \\ y_n &= B_1(b)z_n + w_n \end{aligned} \tag{1}$$

where - w_n is the residue between the true plant and the linear model.
- z_n (in R^m) is a partial state.
- b is the backward shift operator,

$$bu_n = u_{n-1} \tag{2}$$

- $A_1(0)$ is equal to the identity matrix

$$A_1(0) = I \tag{3}$$

By definition this representation is appropriate for control but unfortunately not for observation. Then let us introduce input and output prediction error representations : For any polynomial r(b), one can find polynomial matrices C(b), D(b) such that (see [8]) :

$$C(b)\ A_1(b) + b\ D(b)\ B_1(b) = r(b)\ I \tag{4}$$

Note that :

$$\deg r(b) \leqslant \mathrm{Max}\,\{\deg C(b)+\deg A_1(b),\ \deg D(b)+\deg B_1(b)+1\} \tag{5}$$

if deg denotes degree, and that if :

$$r(0) = 1 \tag{6}$$

Then C(0) is equal to the identity matrix.
Together with (1), (4) yields :

$$r(b)z_n = C(b)u_n + D(b)y_{n-1} - D(b)w_{n-1} \tag{7}$$

$$r(b)u_{n+1} = A_1(b)(C(b)u_{n+1} + D(b)y_n) - A_1(b)D(b)w_n \tag{8}$$

$$r(b)y_n = B_1(b)(C(b)u_n + D(b)y_{n-1}) + (r(b)I-bB_1(b)D(b))w_n \tag{9}$$

And if we let :

$$r(b) = 1 + br'(b) \tag{10}$$

$$A_1(b) = I + b\,A_1'(b) \tag{11}$$

$$C(b) = I + b\,C'(b) \tag{12}$$

this yields :

$$\begin{aligned} &r'(b)u_n=C'(b)u_n+D(b)y_n+A_1'(b)(u_n+C'(b)u_{n-1}+D(b)y_{n-1})-A_1(b)D(b)w_n \\ &y_n+r'(b)y_{n-1}=B_1(b)(u_n+C'(b)u_{n-1}+D(b)y_{n-1})+(r(b)I-bB_1(b)D(b))w_n \end{aligned} \tag{13}$$

Equations (13) may be considered as an observable form of the basic model as stated by the following theorem.

<u>Theorem</u> : For any sequence u_n , let y_n be given as follows :

$$y_n = B(b)u_n - A'(b)y_{n-1} \tag{14}$$

Then if the following relations also hold :

$$\begin{aligned} r'(b)u_n &= C'(b)u_n+D(b)y_n+A_1'(b)(u_n+C'(b)u_{n-1}+D(b)y_{n-1}) \\ y_n+r'(b)y_{n-1} &= B_1(b)(u_n+C'(b)u_{n-1}+D(b)y_{n-1}) \end{aligned} \tag{15}$$

we have :

$$(I+bC'(b))(I+bA_1'(b))+bD(b)B_1(b) = (1+br'(b))I \tag{16}$$

$$(I+bA'(b))B_1(b) = B(b)(I+bA_1'(b)) \tag{17}$$

Note that (17) establishes the connection one has between observable and controllable representations (see [9]), and that in this implicit model, the prediction errors are bilinear in the entries of $A_1(b)$, $B_1(b)$, $C(b)$ and $D(b)$.

II.2. Control law description :

Let us formally introduce the following control law which may be seen as a partial state model reference control :

$$p(b)C(b)u_{n+1} + p(b)D(b)y_n = r(b)E(b)y_n^* + p(b)F(b)e_n \tag{18}$$

where $p(b)$, $E(b)$, $F(b)$ are respectively a polynomial and polynomial matrices, y_n^* is an output reference signal, and e_n is the output prediction error as given by (9) :

$$e_n = r(b)y_n - B_1(b)(C(b)u_n + D(b)y_{n-1}) \tag{19}$$

$$= (r(b)I - bB_1(b)\ D(b))w_n \tag{20}$$

The closed loop behaviour is obtained from (8),(9) as follows :

$$p(b)r(b)u_n = \begin{cases} +A_1(b)E(b)r(b)y_{n-1}^* \\ +A_1(b)[F(b)(r(b)I-bB_1(b)D(b))-D(b)]w_n \end{cases} \tag{21}$$

$$p(b)r(b)y_n = \begin{cases} +B_1(b)E(b)r(b)y_{n-1}^* \\ +(I+bB_1(b)F(b))(r(b)I-bB_1(b)D(b))p(b)w_n \end{cases} \tag{22}$$

Hence if $r(b)$ and $p(b)$ are stable polynomials, the closed loop system will be stable and its boundedness will only depend on the characteristics of w_n and y_n^*.

Moreover following [3], we verify that with appropriate choice of $p(b)$, $r(b)$, $E(b)$, $F(b)$ we can exhaustively describe the set of tracking and regulation transfer functions the plant can assume by linear closed loop control while meeting constraint of robust internal stability. Hence the control design may be reduced to finding stable polynomials $p(b)$, $r(b)$ and polynomial matrices $E(b)$, $F(b)$ such that the closed loop transfer functions are as "best" as possible.

Simply to get the "best" asymptotic behaviour, it is sufficient to take :

$$E(b) = E = B_1(1)^+\ p(1) \tag{23}$$

$$F(b) = F = -B_1(1)^+ \tag{24}$$

where $B_1(1)^+$ is a pseudo inverse for $B_1(1)$.

III - A DIRECT ADAPTIVE CONTROL SCHEME

III.1. Description of the scheme :

In order to be able to use the partial state model reference design procedure described in the previous section, the matrices $A_1(b)$ and $B_1(b)$ must be known. The following algorithm will work even if the model parameters are not known :

At each sampling time n, we proceed in two steps :

1 - Identification of both model and controller matrices using prediction errors representation (13). This gives time varying polynomial matrices $A'_{1n}(b)$, $B_{1n}(b)$, $C_n(b)$, $D_n(b)$.

2 - Computation of the control signal using (18).

Let us make these steps more explicit.

III.2. Identification step :

Let $\bar{y}_n$, $\bar{u}_n$ denote the output and input signals respectively when filtered by the polynomial $p(b)$:

$$\bar{y}_n = p(b)y_n \tag{25}$$

$$\bar{u}_n = p(b)u_n \tag{26}$$

Let θ_n be a block matrix defined as follows :

$$\theta_n^t = (D_n^o \ldots D_n^{N_d} \quad C_n^1 \ldots C_n^{N_c}) \tag{27}$$

where - D_n^i (resp. C_n^i) are $m\times\ell$ (resp. $m\times m$) scalar matrices.

- N_d (resp. N_c) is the maximal assumed degree of $D(b)$ (resp. $C(b)$).

Let ϕ_n be the vector

$$\phi_n^t = (\bar{y}_{n-1}^t \ldots \bar{y}_{n-1-N_d}^t \quad \bar{u}_{n-1}^t \ldots \bar{u}_{n-N_c}^t) \tag{28}$$

Let A_n^i (resp. B_n^i) be $m\times m$ (resp. $\ell\times m$) scalar matrices and

N_a (resp. N_b) be the maximal assumed degree of $A_1'(b)$ (resp. $B_1(b)$).

Then we can rewrite observation equations (13) in the following way :

$$\bar{y}_n + r'(b)\bar{y}_{n-1} = \sum_{i=o}^{N_b} B_n^i(\theta_n^t\phi_{n-i}+\bar{u}_{n-i}) + \varepsilon_n^y \tag{29}$$

$$r'(b)\bar{u}_n = \theta_n^t\phi_{n+1} + \sum_{i=o}^{N_a} A_n^i(\theta_n^t\phi_{n-i}+\bar{u}_{n-i}) + \varepsilon_n^u \tag{30}$$

where ε_n^y (resp. ε_n^u), a vector in R^ℓ (resp. R^m), is the filtered residue of our time varying linear model.

This identification step consists in defining new matrices θ_n, A_n^i, B_n^i given all past and present observations. As mentionned earlier, this is a bilinear filtering problem.

Let us here just give a full list of sufficient conditions this filter would have to meet in order to get global boundedness (in section IV we describe such a filter).

Let ψ_n be the following block matrix :

$$\psi_n = (A_n^o \ldots A_n^{N_a} \; B_n^o \ldots B_n^{N_b}) \tag{31}$$

and let φ_n be the following vector :

$$\varphi_n^t = (\bar{y}_n^t \ldots \bar{y}_{n-1-N_d-N}^t \quad \bar{u}_n^t \ldots \bar{u}_{n-N_c-N}^t) \tag{32}$$

where :

$$N = \text{Max}\,\{N_a\,,\,N_b\} \tag{33}$$

Definition : a sequence $\{v_n\}$ of positive real numbers is said to have the property of mean η - smallness relatively to $\{s_n\}$ iff :

$$\begin{aligned} &\exists(\Sigma,K) \quad \text{such that :} \\ &\forall k,\ \forall q \quad \text{such that :} \quad \forall n \in [q+1,\ q+k]\text{-}\ s_n \geq \Sigma \\ &\text{then} \quad : \quad \frac{1}{k}\sum_{n=q+1}^{q+k} v_n \leq \eta \end{aligned} \tag{34}$$

Our set of assuptions is :

$$\text{HI1} : \ \|\theta_n\| \leq M_1 \ ,\ \|\psi_n\| \leq M_2 \tag{35}$$

HI2 : There exists a sequence s_n of positive real numbers such that :

$$\text{Max}\ \{s, \|\varphi_n\|\} \leq s_n \leq \lambda\ s_{n-1} + \text{Max}\ \{s, \|\varphi_n\|\} + S \tag{36}$$

where s,S are strictly positive constants, and :

$$0 \leq \lambda < 1 \tag{37}$$

and such that :

(i) $$\frac{\|\varepsilon_n^y\| + \|\varepsilon_n^u\|}{s_n} \leq \nu < 1 \tag{38}$$

(ii) $\left\{\frac{\|\varepsilon_n^y\| + \|\varepsilon_n^u\|}{s_n}\right\}$ has the property of mean η_1 - smallness relatively to $\{s_n\}$

(iii) $\left\{\|\theta_n - \theta_{n-1}\|\right\}$ has the property of mean η_2 - smallness relatively to $\{s_n\}$

<u>Remark</u> : Since the norm used in (38) has not been defined, using norm equivalence, this inequality has to be considered as follows :
For any norm, there exists an imposed constant such that (38) holds with this constant instead of 1.

III.3. <u>Control step</u> :

Given polynomials $p(b), r(b)$ and the time varying matrices θ_n, ψ_n, first we compute $E_n(b)$, $F_n(b)$ (see discussion in Section II) such that :

HC1 : $E_n(b)$, $F_n(b)$ are polynomial matrices whose coefficients are locally bounded functions of θ_n, ψ_n and whose degrees are bounded from above.

Then the next stage control u_{n+1} is given by

$$\bar{u}_{n+1} = E_n(b)\ r(b) y_n^* + F_n(b)\ \varepsilon_n^y - \theta_n^t\ \phi_{n+1} \tag{39}$$

$$u_{n+1} = - \sum_{i=o}^{N_p - 1} p_{i+1}\ u_{n-i} + \bar{u}_{n+1} \tag{40}$$

where

$$p(b) = 1 + \sum_{i=1}^{N_p} b^i \, p_i \qquad (41)$$

III.4. Behavioural Theorem :

if : HI1, HC1 hold

y_n^* is bounded

p(b) and r(b) are stable polynomials

$$\deg r(b) \leq \mathrm{Max}\,\{N_a\ ,\ N_b\} + \mathrm{Min}\,\{N_d\ ,\ N_c\} + 1 \qquad (42)$$

Then there exist strictly positive η_1 , η_2 such that if HI2 holds then :

(i) u_n , y_n are bounded

(ii) if moreover, we have

HI3 : $\lim_{n \to \infty} \| \Theta_n - \Theta_{n-1} \| = 0$

HC2 : $\lim_{n \to \infty} \| E_n(b) - E_{n-1}(b) \| = 0$

$\lim_{n \to \infty} \| F_n(b) - F_{n-1}(b) \| = 0$

Then we asymptotically meet (22), namely

$$\lim_{n \to \infty} \| r(b)(\overline{y}_n - B_{1n}(b)E_n(b)y^*_{n-1}) - (I + bB_{1n}(b)F_n(b))\varepsilon_n^y \| = 0 \qquad (43)$$

IV - A DEAD ZONE STOCHASTIC GRADIENT ESTIMATION ALGORITHM

IV.1. Estimation algorithm :

The aim of this section is essentially to show that algorithms meeting assumptions HI1, HI2 do exist. For that, let us assume that representation (13) hold, with w_n characterized as follows :
Let $\varepsilon_n(\Theta_n\ ,\ \psi_n)$ be the identification error vector as defined by (29), (30) we assume the following :

HP : There exists unknown matrices θ^*, ψ^* such that

$$\forall n, \left\| \frac{\varepsilon_n(\theta^*,\psi^*)}{s_n} \right\| \leqslant v_n \leqslant V < 1 \qquad (44)$$

where - $\{s_n\}$,$\{v_n\}$ are sequences of known strictly positive real numbers such that $\{v_n\}$ has the property of mean η - smallness relatively to $\{s_n\}$ and $\{s_n\}$ meets (36).

- an upperbound X^2 of $\|\theta^*\|^2 + \|\psi^*\|^2$ is known.
- upper bounds of N_a , N_b , N_c , N_d are known.

Note that the existence of θ^*, ψ^* implicitely yields a relation such (4) and therefore implies a consistent choice of degrees of $A_1(b)$, $B_1(b)$, $C(b)$, $D(b)$, $r(b)$. In peculiar since $A_1(b)$ may be improper and $B_1(b)$, $D(b)$ singular, a confident choice which also meets (42) is :

$$\deg r(b) \leqslant N_c \qquad (45)$$

The assumption here is very mild and allows to deal with such problems as reduced order model, neglected weak coupling, non linearities or time variation effects (see [11]).

Let $J_n(\theta,\psi)$ be a least square criterion with forgetting factor defined as follows :

$$J_n(\theta,\psi) = \sum_{i=1}^{n} \mu^{n-i} \left\| \frac{\varepsilon_i(\theta,\psi)}{s_i} \right\|^2 \qquad (46)$$

A classical estimation procedure lies in minimizing $J_n(\theta,\psi)$. Unfortunately, this criterion is not convex and HI2 (iii) does not hold. Therefore we propose the following procedure :

Let θ_n,ψ_n be recursively defined as follows :

$$(\theta_n,\psi_n) \in \operatorname{Arg} \min_{\theta,\psi \in X_n} \|\theta-\theta_{n-1}\|^2$$

where X_n is the set of matrices θ,ψ such that :

C1 : $\|\theta\|^2 + \|\psi\|^2 \leqslant X^2$

C2 : $J_n(\theta,\psi) \leqslant j_n$

C3 : $\|\theta-\theta_i\|^2 \leqslant \|\theta_{i+1}-\theta_i\|^2 \quad \forall i \in [n-I,\ n-2]$.

where - I is a fixed integer

- $\mu < \frac{1-V}{1+V}$ (48)

- $j_n = \mu\, j_{n-1} + v_n \quad , \quad j_o = 0$ (49)

The introduction of C2 makes the algorithm look like a stochastic gradient algorithm, while C1, C3 force the new estimation to lie outside a time varying dead zone :

We have the following property :

IV.2. Identification theorem :

θ_n, ψ_n are well defined and for any n, we have

i) $\|\theta_n\|^2+\|\psi_n\|^2 \leqslant X^2$

ii) $\left\| \frac{\varepsilon_n(\theta_n,\psi_n)}{s_n} \right\| \leqslant \frac{1+V}{2} < 1$

and $\left\{\left\| \frac{\varepsilon_n(\theta_n,\psi_n)}{s_n} \right\|\right\}$ has the property of mean η_1 - smallness relatively to $\{s_n\}$, where η_1 is any number such that :

$$\eta_1 > \frac{\eta}{1-\mu} \tag{50}$$

and η has been introduced in HP.

iii) For any η_2 , there exists an integer I such that $\{\|\theta_n-\theta_{n-1}\|\}$ has the property of mean η_2 - smallness relatively to $\{s_n\}$

IV.3. Discussion :

We have stated the existence of identification algorithms which meet conditions HI with very mild assumption about the plant.
Nevertheless a direct implementation of those algorithms may be very difficult. On the contrary, indirect schemes work out decomposed implementation. This decomposition consists at the first step in a linear model estimation and at the second step in the controller computation (see [1]). However this computation may be untractable

without any explicit coordination between these two steps, coordination which is implicit in direct schemes.

V - CONCLUSION

An adaptive direct control scheme is presented in this paper. It is obtained with a pole placement as underlying design method. The characteristics of this technique are :

- estimation of more parameters than those required for control,
- an estimated model which is bilinear in the parameters and may be seen as both input and output prediction errors model.

Looking at the underlying time varying control scheme we give a full list of sufficient conditions the estimation algorithm must meet in order to get global boundedness. Then we show how a modified stochastic gradient procedure can meet these conditions even when the norm of the residuals between the true plant and the assumed linear model is bounded from above by the norm of the signals.

Nevertheless our statement should be considered rather as a theoretical existence result than as a practical algorithm. In peculiar we have considered only the boundedness problem. Much more work remain to be done for converting our idea into an implementable scheme with both boundedness and "good" behaviour properties. An interesting opening may be found in the methods described in [7] but modified so as to include sufficient properties we have exhibited here for global boundedness.

Aknowledgment :

The author wishes to thank Dr. J. LEVINE for many stimulating discussions. Dr. G. COHEN for valuable comments on the original manuscript and constant support throughout the research behind this paper.

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