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AN INDIRECT ADAPTIVE CONTROL SCHEME FOR DISTURBED MIMO SYSTEMS

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ABSTRACT :

An adaptive on line pole placement control scheme is presented which involves at each step identification of the unknown system matrices followed by a nonlinear computation of the adaptive feedback matrices. We show that a priori knowlege of an upperbound of the degree of the numerator, and of column degrees of a column proper denominator is sufficient to get local boundedness even when the system is corrupted by bounded disturbances.

I - INTRODUCTION

During last years crucial advances have been made in discrete time adaptive control from a theoretical point of view. First global stability has been established for single input single output (SISO) minimum phase deterministic and stochastic systems using direct or implicit schemes [1],[2]. For SISO non minimum phase systems, preference has been put on indirect or explicit schemes, in the general context defined by de Larminat in [3] and using on line pole placement [4] or quadratic control [5]. For these techniques, local stability has been established.

As far as multi input multi output (MIMO) systems are concerned, results about SISO minimum phase systems have been extended in [6],[7]. On the other hand, among the works we are aware of, no rigorous results exist for non minimum phase MIMO systems.

In [9], Prager et al propose an on line pole placement technique without proving stability but also with a strong assumption about the system structure.

Here we propose to extend the scheme of [4] to MIMO systems. And with weaker a priori knowledge required about the system structure than in [9], we state conditionned stability even when the system is corrupted by bounded disturbances. In [16] this result is enhanced when the norm of the residuals between the true plant and the assumed linear model is bounded from above by the norm of the signals (reduced order model, weak coupling ...).

Section II is devoted to the control law description for known systems. In Section III we describe our indirect adaptive control scheme, giving general hypotheses to get local stability. Section IV shows how to achieve these hypotheses. Section V draws conclusions.

All ideas introduced here are developped in [12]. The details of the proofs may be found in [12] or [17].

II - LINEAR TIME INVARIANT CONTROL SCHEME

II.1. MIMO SYSTEM REPRESENTATION :

Consider a MIMO system, for which, at each time n, we let u_n be the control input vector (in R^m) and y_n be the output vector (in R^ℓ) and we assume the following representation :

There exist relatively left prime polynomial matrices A(b), B(b) of appropriate dimensions such that :

$$\forall u_n \ , \ A(b)y_n - B(b)u_n = w_n \tag{1}$$

$$\forall n \quad \|w_n\| < W \tag{2}$$

where :
- w_n is the residue between the true plant and the assumed linear model and is required to be only bounded.
- b is the backward shift operator :
$$bu_n = u_{n-1} \tag{3}$$
- A(0) is the identity matrix :
$$A(0) = I \tag{4}$$

II.2. CONTROL LAW

From the primeness of A(b),bB(b), for any polynomial r(b), one can find polynomial matrices C(b), D(b) such that (see [12] or [17]) :

$$\det \begin{vmatrix} A(b) & -bB(b) \\ D(b) & C(b) \end{vmatrix} = r(b) \tag{5}$$

where det|.| denotes the determinant. Note that if r(0) is different from zero, C(0) is invertible.

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Then let us take the following control law:

$$p(b)C(b)u_{n+1}=-p(b)D(b)y_n+r(b)E(b)y_n^* - p(b)F(b)w_n \qquad (6)$$

Where $p(b)$, $E(b)$, $F(b)$ are respectively a polynomial and polynomial matrices and y_n^* is an output reference.

The closed loop behaviour may be characterized in the following way : Using the adjoint matrix, there exist polynomial matrices $X(b)$, $Y(b)$, $A_1(b)$, $B_1(b)$ such that :

$$\begin{pmatrix} X(b) & bB_1(b) \\ -Y(b) & A_1^-(b) \end{pmatrix} \begin{pmatrix} A(b) & -bB(b) \\ D(b) & C(b) \end{pmatrix} = r(b)\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \qquad (7)$$

Hence :

$$r(b)p(b)\begin{pmatrix} y_n \\ u_{n+1} \end{pmatrix} = \begin{pmatrix} X(b) & bB_1(b) \\ Y(b) & A_1^-(b) \end{pmatrix} \begin{pmatrix} p(b) & 0 \\ -p(b)F(b) & r(b)E(b) \end{pmatrix} \begin{pmatrix} w_n \\ y_n^* \end{pmatrix} \qquad (8)$$

and if $r(b)$ and $p(b)$ are stable polynomials, since w_n is bounded by assumption and if y_n^* is also bounded, y_n and u_n are bounded. Moreover let us factorize $B_1(b)$ in

$$B_1(b)=B_{1u}(b)B_{1s}(b) \qquad (9)$$

where $B_{1u}(b)$ is a left invertible polynomial matrix whose zeros are in the closed unit disk and $B_{1s}(b)$ is a right invertible polynomial matrix whose zeros are strictly outside the unit disk. Then the closed loop transfer functions are :

- for tracking

$$y_n = B_{1u}(b)\,\widetilde{M}_t(b)y_n^* \qquad (10)$$

with

$$\widetilde{M}_t(b) = b\,\frac{B_{1s}(b)E(b)}{p(b)} \qquad (11)$$

- for regulation

$$y_n = \left(\frac{X(b)}{r(b)} - B_{1u}(b)\widetilde{M}_r(b)\right)w_n \qquad (12)$$

with

$$\widetilde{M}_r(b) = b\,\frac{B_{1s}(b)F(b)}{r(b)} \qquad (13)$$

Then following [8], we verify that with appropriate choice of $p(b)$, $r(b)$, $E(b)$, $F(b)$ we can exhaustively describe the set of tracking and regulation transfer functions the process can assume by linear closed loop control while meeting constraint of robust internal stability. Hence the control law design may be reduced to finding stable polynomials $p(b)$, $r(b)$ and polynomial matrices $E(b)$, $F(b)$ such that these closed loop transfer functions are as "best" as possible.

Simply to get the "best" asymptotic behaviour, it is sufficient to take :

$$E(b)=E=(C(1)B^+(1)A(1)+D(1))\,\frac{p(1)}{r(1)} \qquad (14)$$

$$F(b)=F=C(1)B^+(1) \qquad (15)$$

where $B^+(1)$ is a pseudo inverse for $B(1)$.

<u>Remark</u> : This control law may also be used when the plant is observed through a stable polynomial filter $q(b)$, since if we let

$$q(b)\tilde{u}_n = u_n \qquad (16)$$

$$q(b)\tilde{y}_n = y_n \qquad (17)$$

$$q(b)\tilde{w}_n = w_n \qquad (18)$$

we may write

$$A(b)\tilde{y}_n = B(b)\tilde{u}_n + \tilde{w}_n + \delta_n \qquad (19)$$

where δ_n due to initial conditions tend to zero.

III - <u>AN INDIRECT ADAPTIVE CONTROL SCHEME</u>

III.1. <u>DESCRIPTION OF THE SCHEME</u> :

When $A(b)$, $B(b)$ are unknown, the control given by (6) cannot be computed as $C(b)$, $D(b)$ are unknown.

At each time n, we solve this problem in three steps :

1. Identification of the linear model (1). This gives time varying polynomial matrices $A_n(b)$, $B_n(b)$.
2. Computation of polynomial matrices $C_n(b)$, $D_n(b)$ such that :

$$\det \begin{vmatrix} A_n(b) & -bB_n(b) \\ D_n(b) & C_n(b) \end{vmatrix} = r(b) \qquad (20)$$

   From a practical point of view, one may just use an approximation method as it will be made more precise later on.
3. Computation of the control using (6).

Let us make these steps more explicit.

III.2. <u>IDENTIFICATION STEP</u> :

Let $\overline{y_n}$, $\overline{u_n}$ denote output and input respectively when filtered by the polynomial $p(b)$:

$$\overline{y_n} = p(b)y_n \qquad (21)$$

$$\overline{u_n} = p(b)u_n \qquad (22)$$

Let Θ_n be a block matrix defined as follows:

$$\Theta_n^t = \left[-A_n^1 \;\ldots - A_n^{N_a} \; B_n^0 \;\ldots\; B_n^{N_b}\right] \qquad (23)$$

where :

- A_n^i (resp B_n^i) are $\ell \times \ell$ (resp $\ell \times m$) scalar matrices,
- N_a (resp N_b) is the maximal assumed degree of A (resp B).

Let ϕ_n be the vector given by :

$$\phi_n^t = \left[\overline{y}_{n-1}^t \;\ldots\; \overline{y}_{n-N_a}^t \;\; \overline{u}_n^t \;\ldots\; \overline{u}_{n-N_b}^t\right] \qquad (24)$$

Then we can rewrite (1) in the following way :

$$\bar{y}_n = \Theta_n^t \, \Phi_n + \bar{\varepsilon}_n \tag{25}$$

where $\bar{\varepsilon}_n$, a vector in R^ℓ, is the filtered residue of our time varying linear model.

The identification step consists in defining a new matrix Θ_n given all past and present observations.

We shall assume that the adaptive identification scheme we use meets the following specifications (compare with lemma 3.1. in [4]) :

HI1 : $\|\Theta_n\| \leq M_1$

HI2 : $\|\bar{\varepsilon}_n\| \leq M_2 + a_n \|\Phi_n\|$

with a_n a sequence of positive real numbers such that $\lim_{n \to \infty} a_n = 0$

HI3 : $\forall \delta$, $\exists(\phi_1, N_1)$ <u>such that</u> :

$\forall k > N_1$, $\forall q$ such that

$\forall n \in [q+1, q+k]$ $\|\phi_n\| > \phi_1$

<u>then</u> $\frac{1}{k} \sum_{n=q+1}^{q+k} \|\Theta_n - \Theta_{n-1}\| < \delta$

HI1 requires the uniform boundedness of the model estimates Θ_n. HI2 ensures that the norm of the a posteriori error $\|\varepsilon_n\|$ stays very small compared with the norm of the signals $\|\phi_n\|$ when this norm becomes large. Finally HI3 means that for any time interval where $\|\phi_n\|$ is growing at a ligh level the mean difference between successive model estimates becomes smaller.

III.3. <u>COMPUTATION STEP</u>

In [17] we show that if $A_n(b)$, $bB_n(b)$ are relatively left prime, there exist $C_n(b)$, $D_n(b)$ such that, with N_a, N_b as defined in (23) :

$$D_n(b) = \sum_{k=0}^{N_a-1} b^k \, D_n^k \tag{26}$$

$$C_n(b) = I + \sum_{k=1}^{N_b} b^k \, C_n^k \tag{27}$$

and (20) is satisfied with degree of r(b) smaller than $(\deg(\det A_n(b)) + mN_b)$ and :

$$r(0) = 1 \tag{28}$$

Let ψ_n be the following matrix

$$\psi_n^t = (D_n^0 \ldots D_n^{N_a-1} \; I \; C_n^1 \ldots C_n^{N_b}) \tag{29}$$

To get ψ_n from the m-linear relation (20) we use an algorithm which gives an estimation $\hat{\psi}_n$ of ψ_n. Then let us note $\hat{C}_n(b)$, $\hat{D}_n(b)$ the corresponding polynomial matrices and $\hat{r}_n(b)$ the following polynomial :

$$\det \begin{vmatrix} A_n(b) - bB_n(b) \\ \hat{D}_n(b) \quad \hat{C}_n(b) \end{vmatrix} = \hat{r}_n(b) \tag{30}$$

Note that :

$$\hat{r}_n(0) = r(0) = 1 \tag{31}$$

Let $\hat{R}_n$ and R be vectors whose entries are the coefficients of $\hat{r}_n(b)$ and $r(b)$. We respectively assume that the following properties hold for the approximation algorithm we use :

HR1 : $\|\hat{\psi}_n\| < M_3$

HR2 : $\forall \gamma > 0$, $\exists(N_2, \mu)$ <u>such that</u> :

$\forall k > N_2$, $\forall q$ such that :

$\frac{1}{k} \sum_{n=q+1}^{q+k} \|\Theta_n - \Theta_{n-1}\| \leq \mu$

<u>then</u> :

$\frac{1}{k} \sum_{n=q+1}^{q=k} \|\hat{\psi}_n - \hat{\psi}_{n-1}\| \leq \gamma$

and

$\frac{1}{k} \sum_{n=q+1}^{q+k} \|\hat{R}_n - R\| \leq \gamma$

HR1 requires the uniform boundedness of the controller estimates $\hat{\psi}_n$ and HR2 ensures that when the mean difference between successive model estimates goes to zero so does the mean difference between successive controller estimates and the mean difference between the real and the desired closed loop poles.

III.4. <u>CONTROL STEP</u> :

Given $A_n(b)$, $B_n(b)$, $\hat{C}_n(b)$, $\hat{D}_n(b)$, first we compute $E_n(b)$, $F_n(b)$ (see discussion in section II) such that :

HCI : $E_n(b)$, $F_n(b)$ are polynomial matrices whose coefficients are locally bounded functions of Θ_n, $\hat{\psi}_n$ and whose degree is bounded from above.

Then the next step control u_{n+1} is given by

$$\hat{\psi}_n^t \, \Phi_{n+1} = r(b) E_n(b) y_n^* - F_n(b) \bar{\varepsilon}_n \tag{32}$$

Or, equivalently :

$$\bar{u}_{n+1} = \left[\sum_{i=0}^{N_e} E_n^i y^*_{n-i} - \sum_{h=0}^{N_f} F_n^h \bar{\varepsilon}_{n-h} - \sum_{k=0}^{N_b-1} \hat{C}_n^{k+1} \bar{u}_{n-k} - \sum_{j=0}^{N_a-1} \hat{D}_n^j \bar{y}_{n-j} \right] \tag{33}$$

$$u_{n+1} = -\sum_{i=0}^{N_p-1} p_{i+1}u_{n-i}+\overline{u}_{n+1} \quad (34)$$

where :

$$p(b) = 1 + \sum_{i=1}^{N_p} b^i p_i \quad (35)$$

$$E_n(b) = \sum_{i=0}^{N_e} b^i E_n^i \quad (36)$$

$$F_n(b) = \sum_{i=0}^{N_f} b^i F_n^i \quad (37)$$

III.5. BEHAVIOURAL THEOREM :

If HI1-HI2-HI3, HR1-HR2, HC1 hold and if $p(b)$,$r(b)$ are stable polynomials then :

(i) u_n, y_n are bounded

(ii) if Moreover we have :

$$\text{HI3'} : \lim_{n\to\infty} \|\Theta_n-\Theta_{n-1}\| = 0$$

$$\text{HR2'} : \lim_{n\to\infty} \|\Theta_n-\Theta_{n-1}\| = 0$$

$$\text{implies} \begin{cases} \lim_{n\to\infty} \|\hat{\psi}_n-\hat{\psi}_{n-1}\| = 0 \\ \lim_{n\to\infty} \|R_n-R\| = 0 \end{cases}$$

$$\text{HC2} \quad \lim_{n\to\infty} E_n(b) - E_{n-1}(b) = 0$$

$$\lim_{n\to\infty} F_n(b) - F_{n-1}(b) = 0$$

Then we asymptotically meet the first equation (8), namely :

$$\lim_{n\to\infty} \|r(b)(\overline{y}_n-B_{1n}(b)E_n(b)y^*_{n-1}) - (X_n(b)-bB_{1n}(b)F_n(b))\overline{\varepsilon}_n\| = 0 \quad (36)$$

IV - HOW TO ACHIEVE THE BEHAVIOURAL THEOREM HYPOTHESES

IV.1. INTRODUCTION :

In III.5., no specific hypothesis about the underlying functional link between the two sequences u_n and y_n was needed. Here, for a more concrete discussion, we shall assume that the representation in II.1. is valid.

In this case, as we will see later on, Identification hypotheses (HI) are satisfied as soon as "good" algorithms are chosen. In fact the main difficulties arise from (HR). Here is the common problem of all indirect schemes : is the adapted model stabilizable as supposed for the assumed model ? A positive answer implicitely yields putting constraint on the identification scheme.

For instance this constraint may be formulated in the following way :

Let Θ represent $(A(b),B(b))$ as in (23) and ρ be a positive constant, we substitute assumption HI1' for HI1 :

$$\text{HI1'} : \forall n, \ \|\Theta_n-\Theta\| < \rho$$

Compared with HI1, the assumed model Θ appears. The practical meaming of this fact is the requirement of an a priori knowledge of some structure characteristics (see IV.3.). An other limitation is given by the parameter ρ which is computed in such a way that (HR) could be met. Moreover as the initial condition Θ_o must also satisfy HI1', this confines our behavioural theorem to a local property.

Let us now give examples of Identification and Computation algorithms.

IV.2. IDENTIFICATION ALGORITHM :

The problem here is to meet conditions HI1' , HI2 , HI3 given the representation of section II.1. In fact all classical identification algorithms are sufficient for this purpose. For instance when the filtered disturbance bound $\overline{W}$ is known, we may use the following algorithm :

Let ν_n be the a priori identification error,

$$\nu_n = \overline{y}_n - \Theta^t_{n-1}\phi_n$$

Θ_n is computed as follows :

$$\Theta_n = \Theta_{n-1} + g_n P_{n-1}\phi_n \nu_n^t \quad (40)$$

$$\Pi_n = P_{n-1} - g_n P_{n-1}\phi_n\phi_n^t P_{n-1} \quad (41)$$

$$P_n \geq \Pi_n \quad (42)$$

$$g_n = \frac{\alpha_n}{\mu_n+\phi_n^t P_{n-1}\phi_n} \ \text{Max}\left\{1-\frac{\overline{W}}{\|\nu_n\|},0\right\} \quad (43)$$

where - (42) means that P_n is any matrix greater than Π_n in the sense of definite symmetric matrices and such that :

$$0 < \lambda \leq \lambda\min P_n \leq \lambda\min P_n \leq \Lambda \quad (44)$$

where $\lambda\min$ (resp. $\lambda\max$) denotes the minimum (resp maximum) eigen value.

- α_n , μ_n are sequences of positive real numbers such that :

$$0 < \alpha \leq \alpha_n \leq 1 \quad (45)$$

$$0 < \mu \leq \mu_n \leq M \quad (46)$$

This algorithm may be reduced to many commonly used algorithms such as stochastic approximation [5], modified least squares

[15], or constant trace algorithm [18]. It verifies the following property :

If $\overline{y}_n$ satisfies

$$\| \overline{y}_n - \Theta^t \phi_n \| \leqslant \overline{W} \tag{47}$$

Then HI1', HI2, HI3' hold.

Moreover if $\| \phi_n \|$ is bounded from above then

$$\limsup_{n \to \infty} \| \varepsilon_n \| \leqslant \overline{W} \tag{49}$$

This last inequality is very attractive when brought together with (38) and compared with (8).

When the bound $\overline{W}$ is unknown we may use a projection algorithm as in [19]

IV.3. CONTROL LAW COMPUTATION ALGORITHM

We have here to show existence of ρ and a priori knowledge about structure characteristics such that when Θ_n satisfy HI1' one can find a computation algorithm which realizes HR1 , HR2.

We may proceed on the following conceptual way.

Choice of ρ and structure characteristics : Let A(b), B(b) be the polynomial matrices of the assumed model where A(b) is column proper. With the following a priori knowledge :

N_i , i^{th} colomn degree of A(b) is known

N_b , upperbound of degree of B(b) is known

Then in [17], we show that if in (5) :

$$\deg r(b) \leqslant \sum_{i=1}^{\ell} N_i + m\, N_b \ , \tag{50}$$

We may impose i^{th} column of D(b) to have degree strictly less than N_i and degree of C(b) to be less than or equal to N_b. Moreover if

$$r(0) = 1 \tag{51}$$

we may choose

$$C(0) = I \tag{52}$$

Then let us write

$$(D(b)C(b)) = \begin{array}{cc} \begin{pmatrix} d(b) & c(b) \\ D^*(b) & C^*(b) \end{pmatrix} & \begin{array}{c} 1 \\ m-1 \end{array} \\ \begin{array}{cc} \ell \qquad & \qquad m \end{array} & \end{array} \tag{53}$$

where d(b), c(b) are polynomial row vectors. Using a Laplace's expansion of (5), we show in [17] that coefficients of the entries of d(b), c(b) are solutions of an invertible linear system we symbolically note :

$$\mathcal{A}\, X = \mathcal{B} \tag{54}$$

where $\mathcal{A}$ is made up from coefficients of minors of :

$$\begin{pmatrix} A(b) & -bB(b) \\ D^*(b) & C^*(b) \end{pmatrix}$$

Now using a continuity argument, with an appropriate topology defined on the polynomial coefficient space, there exists ρ such that for any $\hat{\Theta}(\hat{A}(b),\hat{B}(b))$ in the open sphere with centre $\Theta(A(b),B(b))$ and radius ρ, minors of :

$$\begin{pmatrix} \hat{A}(b) & -b\hat{B}(b) \\ D^*(b) & C^*(b) \end{pmatrix}$$

give a "strictly" nonsingular matrix $\hat{\mathcal{A}}$ (this "strictly" will be defined in the next section).

Choice of an algorithm : In the previous section we have shown that if HI1' is satisfied, by keeping constant the (m-1) last rows of $(D_n(b)C_n(b))$, the computation step may be reduced to solve an invertible linear system where unknowns are coefficients of the first row of $(D_n(b)C_n(b))$.

Let

$$\mathcal{A}_n\, X_n = \mathcal{B}_n \tag{55}$$

be this time varying linear system. To get a "good" estimation of X_n, only one step of any strictly contractive iterative algorithm is sufficient.

For instance, if $\mathcal{A}$ is said to be strictly nonsingular when there exists a square matrix C such that :

$$\| I - C\mathcal{A} \| \leqslant \eta < 1 \ , \ \| C \| < \Gamma \tag{56}$$

then following the previous section, HI1' implies $\mathcal{A}_n$ to be strictly nonsingular. Moreover the following sequence of vectors

$$\hat{X}_n = \hat{X}_{n-1} - C_n(\mathcal{A}_n \hat{X}_{n-1} - \mathcal{B}_n) \tag{57}$$

meets conditions (HR) (note that $\mathcal{A}_n$, $\mathcal{B}_n$ are polynomial functions of Θ_n's entries).

Remark : This algorithm is very restrictive and it seems reasonnable that sufficient a priori knowledge about structure characteristics is, when A(b) is column proper :

N_i , upperbound of i^{th} column degree of A(b)

N_b , upperbound of degree of B(b)

N , underbound of degree of det A(b)

Then we impose :

$\deg r(b) \leqslant N + m\, N_b$

i^{th} column degree of $D(b) = N_i - 1$

degree of $C(b) = N_b$

For further discussion see [12].

V - CONCLUSION

An adaptive on line pole placement control scheme is presented which involves at each step identification of the unknown system matrices followed by a nonlinear computation of the adaptive feedback matrices. We show that a priori knowledge of an upperbound of the degree of the numerator polynomial matrix, and of column degrees of a column proper denominator polynomial matrix (compare with [4]) is sufficient to get local boundedness of input and output signals. Moreover this scheme allows us to deal with systems such that the residue between the true plant and the assumed linear model is bounded whatever input is bounded or unbounded. In fact this result is extended in [16] to the following assumption :

$\exists \varepsilon$, $\exists(\phi,K)$ such that :

$\forall k \geqslant K$, $\forall q$ such that $\forall n \in [q+1,q+k]$ $\|\phi_n\| > \phi$

then $\frac{1}{k} \sum_{n=q+1}^{q+k} \frac{\|w_n\|}{\|\phi_n\|} < \varepsilon$

where w_n is this residue and ϕ_n is the identification information vector. In peculiar this allow us to deal with fast unmodeled modes or neglected weak coupling.

As for any indirect scheme, boundedeness is conditionned. Here left primeness of numerator and denominator polynomial matrices is required. To round this problem, the following algorithm is explored in [20]:

If "good" left primeness
  then use nominal scheme
  else if problem holds on
    then change degrees
    else use dual control.

The dual control mentionned here is obtained by :

$$\underset{u_{n+1}}{\mathrm{Min}} \ \underset{\|x\|=1}{\mathrm{Max}} \ \|(I-K_{n+1}\phi_{n+1}^{t})x\|$$

where K_n is the gain matrix of the identification algorithm. Note that this technique also allows to deal with ill modeled systems.

At last let us mention the technique used here to proove boundedness. It relies upon a state representation of the adaptive closed loop behaviour and uses a technical lemma established in [11]. It is in fact very general and can be used for many other schemes [21].

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