2019 18th European Control Conference (ECC)
Napoli, Italy, June 25-28, 2019

A gyroless adaptation of attitude complementary filtering

Aurélien Fiot[1] Christophe Combettes[2] Sébastien Changey[2] Nicolas Petit[3]

***Abstract*— This article proposes a methodology for estimating the attitude of a rigid body from vector measurements only, i.e. without gyrometers. Classically, two vector measurements are enough to determine the attitude of a rigid body, through an algebraic calculation which is sometimes considered as computationally costly or tedious to implement. Instead, simple observers are often implemented such as the attitude complementary filter, using the feedback of a (possibly biased) gyrometer. What we show in this article is that the angular velocity can be simply estimated with the vector measurements, thus making the gyro redundant. We propose an adaptation of the complementary filter that is simple to implement and gyroless. A proof of convergence is given, and simulation and experimental results are provided.**

INTRODUCTION

Attitude estimation of a rigid body is a central question in numerous fields of engineering and applied science, especially those including motion control. It has been vastly studied and there are many implementation for UAVs [1], [2], [3], UGVs [4], aerospace systems [5], [6], [7], [8], or so-called smart objects [9], [10] among others. Classically (see e.g. [11]), two vectors measurements, usually assumed to be obtained using accelerometers and magnetometers, are sufficient to algebraically reconstruct the attitude of a rigid body. This vastly documented method (see [12], [13]) has been improved in many applications with multi-sensor data fusion, adding gyrometers to the set of sensors, most frequently using Kalman filtering (see e.g. [14]) or, more recently, complementary filtering as in [15], [16]. This last solution is appealing because of its simplicity of implementation (relying on a few nonlinear equations that are readily implemented onboard any embedded system) and the simplicity of its straightforward tuning procedure (very few tuning gains being at stake). Gyrometers add robustness to vector measurements failures, and provide dynamic responsiveness to the estimation filter. Various experiments and works [17], [18], [19], [20], [21] [22], [23], [24], [25], [26], [27], [28] offer alternatives and comparisons of the various methods implementing the attitude estimation technique from gyrometers, accelerometers and magnetometers.

In some applications though, gyrometers are unreliable. This is the case for systems subjected to strong accelerations or high spinning rates, such as gun-launched ammunitions (see [29], [30], [31]). Moreover, their high cost and the improvement of low-cost sensors performance are multiplying the cases where a gyroless approach seems a viable and desirable alternative. Instead of directly measuring the angular velocity, some works have developped solutions for the problem of estimating it (see e.g. [32], [33]). In particular, [34], [35], [36] have offered a way of estimating the angular velocity from vector measurements, even when an unknown torque is applied.

As will appear in this paper, the angular velocity estimation technique plays a central role in the attitude estimation problem. This paper offers a gyroless adaptation of the complementary filter by [16]. This algorithm is simple to implement and only requires a minimal number of sensors (the method being gyroless, only direction sensors are required).

The paper is organized as follows. In Section I, the problem statement is formulated. In Section II, the proposed observer is presented. Its convergence is established. In Section III, simulation and experimental results are reported. Finally some perspectives are given in III-B.

I. NOTATIONS AND PROBLEM STATEMENT

A. Notations

Vectors in $\mathbb{R}^3$ are denoted with small letters, square matrices in $\mathbb{R}^{3\times 3}$ with capital letters. $|x|$ is the Euclidian norm of x in $\mathbb{R}^3$. For any two matrices A, B in $\mathbb{R}^{3\times 3}$ the inner product and Frobenius norm are defined by

$$\langle\langle A, B\rangle\rangle = \mathrm{Tr}(A^T B), \quad ||A|| = \sqrt{\langle\langle A, A\rangle\rangle}$$

For $z \in \mathbb{C}$, $\Re(z)$ is the real part of z ; for any matrix $M \in \mathbb{R}^{3\times 3}$, the anti-symmetric part of M is defined by

$$\mathbb{P}_a(M) = \frac{1}{2}(M - M^T)$$

For any two matrices A, B in $\mathbb{R}^{3\times 3}$, one notes $[A, B] = AB - BA$; finally for x in $\mathbb{R}^3$, $[x_\times]$ is the skew-symmetric cross-product matrix associated with x, i.e. $\forall y \in \mathbb{R}^3, [x_\times]y = x \times y$. Namely,

$$[x_\times] = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

where x_1, x_2, x_3 are the coordinates of x in the standard basis of $\mathbb{R}^3$.

B. Problem statement

Consider a rigid body whose inertia matrix J is known, which is subjected to a slowly-varying torque τ, and which is equipped with two embedded vector sensors producing measurements

$$v_i = R^T \mathring{v}_i, \quad i = 1, 2 \tag{1}$$

where R is the rotation matrix describing the rigid body attitude w.r.t. a set inertial frame, and $\mathring{v}_1, \mathring{v}_2$ are two constant vectors expressed in the inertial frame. Without loss of generality, the vectors $\mathring{v}_1, \mathring{v}_2$ are unit vectors [1]. They are

[1]Aurélien Fiot is a PhD candidate at CAS - Centre automatique et systèmes, MINES ParisTech, PSL Research University, and the Guidance, Navigation and Control department at ISL

[2]Sébastien Changey and Christophe Combettes are Dr. Eng. in the Guidance, Navigation and Control department at ISL

[3]Nicolas Petit is Professor, CAS - Centre automatique et systèmes, MINES ParisTech, PSL Research University

[1]in practical applications these vectors corresponds to fixed directions, e.g. direction to the Sun, or to the center of the Earth, local magnetic field, among others.

assumed to be non-colinear, i.e. $\mathring{v}_1^T \mathring{v}_2 \neq 0$. The dynamics of each v_i, the angular velocity in the body frame ω and the attitude matrix R are the following

$$\begin{cases} \dot{R} = R[\omega_\times] \\ \dot{v}_1 = v_1 \times \omega \\ \dot{v}_2 = v_2 \times \omega \\ \dot{\omega} = J^{-1}(J\omega \times \omega + \tau) \end{cases} \tag{2}$$

The equation governing ω is a forced Euler equation. For simplicity, we note

$$E(\omega) \triangleq J^{-1}(J\omega \times \omega), \; p \triangleq J^{-1}\tau$$

so that the Euler equation rewrites (using p as the (normalized) torque) as

$$\dot{\omega} = E(\omega) + p$$

The problem we adress in this paper is the estimation of the attitude matrix R from the measurements (1), and the model (2). The main assumption we formulate is the following.

Assumption 1: The torque p is slowly varying ; it is not known but it generates a rotation which remains bounded so that $|\omega| \leq \omega_{max}$ for all $t \geq 0$.

This assumption will be instrumental in the convergence analysis and the practical use of the observer we propose. The fact that the torque p is unknown is very useful for applications, as this variable is difficult to directly measure. In numerous cases, the observer we propose can be used with the purpose of estimating this quantity[2]. The "slowly-varying" assumption will be used to model p as (piece-wise w.r.t. time) constant vector, as is often done in linear observer design, following the ideas of [37]. Mathematically, we will consider that Assumption 1 yields

$$\dot{\tau} = 0, \quad \dot{p} = 0 \tag{3}$$

Here "slowly-varying" means with respect to "fast-varying" variables which are the other variables (attitude, directions, angular velocity), in the mathematical sense of singular perturbations of [38], e.g.

The boundedness of the angular velocity formulated in Assumption 1 is in fact a very mild assumption. Its practical necessity is easily imagined as direction measurements most likely do not produce any insight if the rigid body is rotating at infinite angular velocity.

II. Observer design and proof of convergence

Our observer aims to estimate the extended state $X = (R \quad v_1 \quad v_2 \quad \omega \quad p)$. Following a direct inspiration from [16], [36], we design the following observer

$$\begin{cases} \dot{\hat{R}} = \hat{R}\left([(\hat{\omega})_\times] + k_P[\sigma_\times]\right) \\ \quad \sigma = k_1 v_1 \times (\hat{R}^T \mathring{v}_1) + k_2 v_2 \times (\hat{R}^T \mathring{v}_2) \\ \dot{\hat{v}}_1 = v_1 \times \hat{\omega} + k(v_1 - \hat{v}_1) \\ \dot{\hat{v}}_2 = v_2 \times \hat{\omega} + k(v_2 - \hat{v}_2) \\ \dot{\hat{\omega}} = E(\hat{\omega}) + \hat{p} + k^2(v_1 \times \hat{v}_1 + v_2 \times \hat{v}_2) \\ \dot{\hat{\varpi}} = E(\hat{\varpi}) + \hat{p} + \gamma_1\sqrt{k}(\hat{\omega} - \hat{\varpi}) \\ \dot{\hat{p}} = \gamma_2 k(\hat{\omega} - \hat{\varpi}) \end{cases} \tag{4}$$

[2]in space applications, e.g., the study of anomalies of rotation boils down to estimating external torques and identifying them among a list of possible sources (impact of micrometeorites, solar wind, gazeous ejection, eddy currents, among others).

where k, k_P, γ_1 and γ_2 are constant positive tuning parameters.

By introducing the scaled errors

$$\tilde{R} = \hat{R}^T R \in \mathbb{R}^{3\times 3} \tag{5}$$

$$X = \begin{pmatrix} v_1 - \hat{v}_1 \\ v_2 - \hat{v}_2 \\ \frac{\omega - \hat{\omega}}{k} \end{pmatrix} \triangleq \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \in \mathbb{R}^9 \tag{6}$$

$$Y = \begin{pmatrix} \frac{\varpi - \hat{\varpi}}{k} \\ \frac{p - \hat{p}}{k\sqrt{k}} \end{pmatrix} \triangleq \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \in \mathbb{R}^6 \tag{7}$$

we obtain, using (3),

$$\begin{cases} \dot{\tilde{R}} = [\tilde{R}, [\omega_\times]] - k_P[\sigma_\times]\tilde{R} + [(\tilde{\omega})_\times]\tilde{R} \\ \dot{X}_1 = -kX_1 + kv_1 \times X_3 \\ \dot{X}_2 = -kX_2 + kv_2 \times X_3 \\ \dot{X}_3 = k(v_1 \times X_1 + v_2 \times X_2) + \frac{E(\omega) - E(\hat{\omega})}{k} + \sqrt{k}Y_2 \\ \dot{Y}_1 = -\gamma_1\sqrt{k}Y_1 + \sqrt{k}Y_2 + \gamma_1\sqrt{k}X_3 + \frac{E(\omega) - E(\hat{\omega})}{k} \\ \dot{Y}_2 = -\gamma_2\sqrt{k}Y_1 + \gamma_2\sqrt{k}X_3 \end{cases} \tag{8}$$

The proposed observer (4) is a cascade of two pre-existing observers, found in [16] and in [36] respectively. This will be instrumental in the convergence analysis. As will appear, the linearized interconnection of the three variables $\tilde{R}, X, Y$ is organized as pictured in Fig. 1. Interestingly, the (X, Y) dynamics (which are coupled together) are cascaded onto the $\tilde{R}$ dynamics without any feedback.

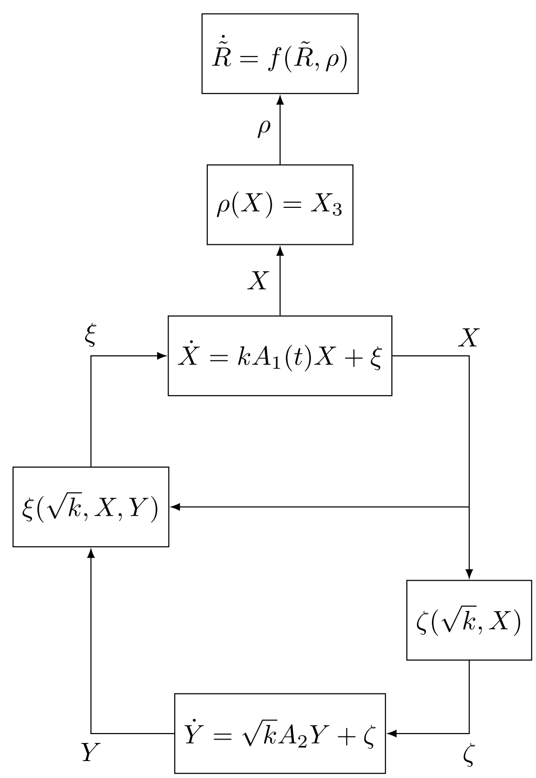

Fig. 1. Interconnection of systems

The main result of this paper is that the three subsystems $\tilde{R}, X, Y$ are exponentially stable, with X and Y converging to zero and $\tilde{R}$ converging to the identity matrix. This implies that the observer state $(\hat{R}, \hat{v}_1, \hat{v}_2, \hat{\omega}, \hat{\varpi}, \hat{p})$ converges to $(R, v_1, v_2, \omega, \omega, p)$. We prove this result (formulated in Theorem 3) in the following.

A. Recalls

The proposed result is actually a combination of two distinct convergence results, Theorem 1 and Theorem 2.

Theorem 1: [Angular velocity and torque observer [36]] Under Assumption 1, for any choice of $\gamma_1, \gamma_2 > 0$, the observer defined by

$$\begin{cases} \dot{\hat{v}}_1 = v_1 \times \hat{\omega} + k(v_1 - \hat{v}_1) \\ \dot{\hat{v}}_2 = v_2 \times \hat{\omega} + k(v_2 - \hat{v}_2) \\ \dot{\hat{\omega}} = E(\hat{\omega}) + \hat{p} + k^2(v_1 \times \hat{v}_1 + v_2 \times \hat{v}_2) \\ \dot{\hat{\varpi}} = E(\hat{\varpi}) + \hat{p} + \gamma_1 \sqrt{k}(\hat{\omega} - \hat{\varpi}) \\ \dot{\hat{p}} = \gamma_2 k(\hat{\omega} - \hat{\varpi}) \end{cases} \tag{9}$$

defines an error which, for sufficiently large $k > 0$, converges locally uniformly exponentially to zero.

Theorem 2: [Explicit complementary filter [16]] The filter defined by

$$\begin{cases} \dot{\hat{R}} = \hat{R}\left([(\omega^y - \hat{b})_\times] + k_P[\sigma_\times]\right) \\ \dot{\hat{b}} = -k_I \sigma \\ \quad \sigma = k_1 v_1 \times (\hat{R}^T \mathring{v}_1) + k_2 v_2 \times (\hat{R}^T \mathring{v}_2) \end{cases} \tag{10}$$

where $\omega^y = \omega + b$ is the measurement from an embedded gyro assumed to be corrupted with a constant bias b, and where k_I and k_P are constant positive tuning parameters, has three unstable equilibria characterized by

$$(\hat{R}_{\star i}, \hat{b}_{\star i}) \triangleq (U_0 D_i U_0^T R, b), i = 1, 2, 3$$

where $D_1 = \text{diag}(1, -1, -1)$, $D_2 = \text{diag}(-1, 1, -1)$ and $D_3 = \text{diag}(-1, -1, 1)$, and $U_0 \in SO(3)$ such that

$$M_0 \triangleq \sum_{i=1}^{2} k_i \mathring{v}_i \mathring{v}_i^T = U_0 \Lambda U_0^T$$

with Λ a diagonal matrix. Its error $(\tilde{R}(t), \tilde{b}(t))$ is locally exponentially stable to $(I, 0)$ and for almost all initial conditions $(\tilde{R}_0, \tilde{b}_0) \neq (\hat{R}_{\star i}^T R, b)$ the trajectory $(\hat{R}(t), \hat{b}(t))$ converges to the trajectory $(R(t), b)$.

We will now show that replacing the gyro in Mahony's explicit complementary filter by the estimation provided by (9), provides the desired convergence properties, yielding the following result.

Theorem 3 (Main Result): Under Assumption 1, given $k_P > 0$, for any $\gamma_1, \gamma_2 > 0$, the filter whose dynamics are defined by (4) defines an error (8), which, for k large enough, converges locally exponentially to $(I, 0, 0)$ for almost all initial conditions $\tilde{R}_0 \neq \hat{R}_{\star i}^T R$.

B. Asymptotic behavior of the (X, Y)-subsystem

We reproduce, in a summarized way, some calculus from [36], for self-containment purposes. Following (Khalil, 1996, Th. 3.13 [39]), we establish the exponential stability of the linearization about the origin $(X, Y) = (0, 0)$ and conclude on the nonlinear dynamics of the subsystem (X,Y). Linearization gives

$$\begin{cases} \dot{X} = kA_1(t)X + \xi \\ \dot{Y} = \sqrt{k}A_2 Y + \zeta \end{cases} \tag{11}$$

with

$$A_1(t) \triangleq \begin{pmatrix} -I & 0 & [a(t)_\times] \\ 0 & -I & [b(t)_\times] \\ [a(t)_\times] & [b(t)_\times] & 0 \end{pmatrix},$$

$$\xi \triangleq \begin{pmatrix} 0 \\ 0 \\ \nabla E(\omega) X_3 + \sqrt{k} Y_2 \end{pmatrix}$$

$$A_2 \triangleq \begin{pmatrix} -\gamma_1 & 1 \\ -\gamma_2 & 0 \end{pmatrix}, \quad \zeta \triangleq \begin{pmatrix} \gamma_1 \sqrt{k} X_3 + \nabla E(\omega) X_3 \\ \gamma_2 \sqrt{k} X_3 \end{pmatrix}.$$

Thus the linearized error dynamics (11) can be rewritten as the interconnection of the two systems

$$\dot{X} = kA_1(t)X \tag{12}$$

$$\dot{Y} = \sqrt{k}A_2 Y \tag{13}$$

which are actually disturbed by the input terms ξ and ζ defined above.

A detailed analysis of the time varying matrix $A_1(t)$ presented in [34] establishes the existence of a Lyapunov function $V_1(t, X)$ and two constants

$$0 < \alpha_1 \leq \beta_1 \tag{14}$$

depending only on the (constant) value of the scalar product $\mathring{v}_1^T \mathring{v}_2 \neq 0$, such that for all (t, X), along the trajectories of system (12), one has

$$\alpha_1 |X|^2 \leq V_1(t, X) \leq \beta_1 |X|^2, \quad |\nabla V_1(t, X)| \leq 2\beta_1 |X|$$

$$\dot{V}_1(t, X) = -k|X|^2$$

For any choice of $\gamma_1, \gamma_2 > 0$, the eigenvalues λ_1, λ_2 of A_2 have strictly negative real parts. Further, with $\gamma_1^2 \neq 4\gamma_2$ one has $\lambda_1 \neq \lambda_2$. Note e_1, e_2 two associated eigenvectors, $P = [e_1 \; e_2]$ the corresponding invertible matrix and $\lambda = -\max(\Re(\lambda_1), \Re(\lambda_2)) > 0$. We have, for all Y,

$$e^{\sqrt{k}A_2 t} Y = P \begin{pmatrix} e^{\sqrt{k}\lambda_1 t} & 0 \\ 0 & e^{\sqrt{k}\lambda_2 t} \end{pmatrix} P^{-1} Y$$

from which we deduce $|e^{\sqrt{k}A_2 t} Y| \leq ||P|| ||P^{-1}|| e^{-\sqrt{k}\lambda t} |Y|$. Defining V_2 as

$$\begin{aligned} V_2(Y) &\triangleq \sqrt{k} \; Y^T \int_0^\infty e^{\sqrt{k}A_2^T t} e^{\sqrt{k}A_2 t} dt \; Y \\ &= \sqrt{k} \int_0^\infty |e^{\sqrt{k}A_2^T t} Y|^2 dt \end{aligned}$$

we have, for all Y,

$$\alpha_2 |Y|^2 \leq V_2(Y) \leq \beta_2 |Y|^2 \tag{15}$$

with $\alpha_2 \triangleq \frac{1}{2||A_2||}, \frac{||P||^2 ||P^{-1}||^2}{2\lambda} \triangleq \beta_2$. Finally, by upper-bounding the gradient of V_2, we obtain $|\nabla V_2(Y)| \leq 2\beta_2 |Y|$. Moreover, using (13), one gets

$$\dot{V}_2(Y) = -\sqrt{k}|Y|^2.$$

We now investigate the convergence of the interconnection (X, Y). Consider the candidate Lyapunov function

$$V(t,(X,Y)) \triangleq V_1(t,X) + V_2(Y).$$

Note

$$\alpha \triangleq \min(\alpha_1,\alpha_2), \quad \beta \triangleq \max(\beta_1,\beta_2), \quad Z \triangleq \begin{pmatrix} X \\ Y \end{pmatrix}$$

We have

$$\alpha|Z|^2 \leq V(t,Z) \leq \beta|Z|^2$$

and the derivative of V along the trajectories of (8) satisfies

$$\dot{V}(t,Z) = -k|X|^2 + \nabla V_1(t,X)\xi - \sqrt{k}|Y|^2 + \nabla V_2(Y)\zeta.$$

A direct calculation gives

$$||\nabla E(\omega)|| \leq \sqrt{2}\omega_{max}.$$

Hence, the coupling terms can be bounded as

$$\begin{aligned} |\xi| &\leq \sqrt{2}\omega_{max}|X| + \sqrt{k}|Y| \\ |\zeta| &\leq \left(\sqrt{2}\omega_{max} + (\gamma_1+\gamma_2)\sqrt{k}\right)|X|. \end{aligned}$$

It directly follows that

$$\begin{aligned} \dot{V}(t,Z) \leq & -k|X|^2 + 2\beta_1\sqrt{2}\omega_{max}|X|^2 \qquad (16) \\ & + 2\sqrt{k}\beta_1|X||Y| - \sqrt{k}|Y|^2 \\ & + 2\beta_2\left(\sqrt{2}\omega_{max} + (\gamma_1+\gamma_2)\sqrt{k}\right)|X||Y| \\ = & \sqrt{k}Z^T \begin{pmatrix} -\sqrt{k} & \beta_1+\beta_2(\gamma_1+\gamma_2) \\ \beta_1+\beta_2(\gamma_1+\gamma_2) & -1 \end{pmatrix} Z \\ & + 2\beta_1\sqrt{2}\omega_{max}|X|^2 + 2\beta_2\sqrt{2}\omega_{max}|X||Y|. \qquad (17) \end{aligned}$$

In this last expression, the bilinear terms can be shown to be dominated by the quadratic terms. Indeed, for sufficiently large k, the symmetric matrix

$$M \triangleq \begin{pmatrix} -\sqrt{k} & \beta_1+\beta_2(\gamma_1+\gamma_2) \\ \beta_1+\beta_2(\gamma_1+\gamma_2) & -1 \end{pmatrix}$$

is definite negative. Therefore, by choosing a sufficiently large k, the first term in (17) is made dominant over the other terms that are not scaled by $\sqrt{k}$. Under these conditions, $\dot{V}$ is definite negative and the system (8) is uniformly exponentially stable.

As a result we deduce the following inequality (this will be instrumental in the connection of the full error system): there exists $C > 0$ and $\lambda > 0$ such that :

$$|\tilde{\omega}(t)| < Ce^{\lambda(t-t_0)} \qquad (18)$$

C. Asymptotic behavior of the $\tilde{R}$ subsystem

In this part we are considering the error dynamics (10).

Let us define M as

$$M \triangleq R^T M_0 R \text{ with } M_0 \triangleq \sum_{i=1}^{2} k_i \mathring{v}_i \mathring{v}_i^T$$

In [16], it is showed using $W \triangleq \sum_{k=1}^{2} k_i(1-\langle v_i, \hat{R}^T\mathring{v}_i\rangle)$ that the norm of the anti-symmetric part of $\tilde{R}M$ has to converge asymptotically to zero, and that a consequence of this fact implies either $\tilde{R} = I$ or $\mathrm{Tr}(\tilde{R}) = -1$.

Further, [16] showed this implies the filter has one exponentially stable equilibrium $\hat{R} = R$ and three unstable equilibria characterized by $\hat{R}_{*i} = U_0 D_i U_0^T R$, with D_i being diagonal matrices with a 1 on position i and -1 elsewhere, and $U_0 \in SO(3)$ such that $M_0 = U_0 \Lambda U_0^T$ with Λ a diagonal matrix (this is the result stated in Theorem 2).

D. Connecting the subsystems

Following the preceding recalls, let us consider the function

$$W \triangleq \sum_{k=1}^{2} k_i(1-\langle v_i, \hat{R}^T\mathring{v}_i\rangle) = k_1 + k_2 - \mathrm{Tr}(\tilde{R}M)$$

which is analogous to the candidate Lyapunov function in [16] for the subsystem $\tilde{R}$; it will not be a proper Lyapunov function since it is not always decreasing, as expected due to the external forcing term appearing in the interconnection pictured in Figure 1, but we can use it to show that $||\mathbb{P}_a(\tilde{R}M)||$ converges to zero.

Using (8), the derivative of W is given by

$$\begin{aligned} \dot{W} &= -\mathrm{Tr}\left(\dot{\tilde{R}}M + \tilde{R}\dot{M}\right) \\ &= -\mathrm{Tr}\left(-k_P[\sigma_\times]\tilde{R}M + [\tilde{R}M,[\omega_\times]] + [\tilde{\omega}_\times]\tilde{R}M\right) \\ &= k_P\,\mathrm{Tr}\left([\sigma_\times]\mathbb{P}_a(\tilde{R}M)\right) - \mathrm{Tr}\left([\tilde{\omega}_\times]\tilde{R}M\right) \end{aligned}$$

which yields

$$\dot{W} = -k_P||\mathbb{P}_a(\tilde{R}M)||^2 - \mathrm{Tr}([\tilde{\omega}_\times]\tilde{R}M)$$

Integrating this relation from 0 to t gives

$$\int_0^t -k_P||\mathbb{P}_a(\tilde{R}M)||^2 dt = W(t) - W(0) + \int_0^t \mathrm{Tr}([\tilde{\omega}_\times]\tilde{R}M) \qquad (19)$$

Cauchy-Schwarz inequality ensures that

$$\mathrm{Tr}\left([\tilde{\omega}_\times]\tilde{R}M\right) < ||[\tilde{\omega}_\times]|| \times ||\tilde{R}M||$$

Stemming from the fact that $||\tilde{R}M|| = ||\hat{R}^T M_0 R||$ is less than the product of the norms of those three matrixes, with the Frobenius norm of M_0 and any rotation matrix all being non-varying, and the exponential convergence of $\tilde{\omega}$ to zero (from (18)) is the existence of a strictly positive number C' such that

$$\mathrm{Tr}\left([\tilde{\omega}_\times]\tilde{R}M\right) < C'e^{-\lambda(t-t_0)} \qquad (20)$$

As a result $\mathrm{Tr}([\tilde{\omega}_\times]\tilde{R}M)$ is integrable. Consequently, the left-hand side integral of (19) is not only decreasing over time, but it has a lower bound (note that W is positive definite). Then, the function $t \mapsto \int_0^t -k_P||\mathbb{P}_a(\tilde{R}M)||^2 dt$ converges when t goes to $+\infty$. Once it is shown that the integrand is uniformly continuous, Barbalat lemma will ensure that the integrand has to converge to zero.

Indeed, the function $t \mapsto -k_P||\mathbb{P}_a(\tilde{R}M)||^2$ is uniformly continuous, because, thanks to Assumption 1, ω is bounded, and $\tilde{R}M = \hat{R}^T M_0 R$ with $\dot{R}$ and $\dot{\hat{R}}$ bounded.

As a result $\lim_{t\to+\infty} \mathbb{P}_a(\tilde{R}M)(t) = 0$. The conclusion follows along the lines of [16].

III. Validation of the gyroless attitude observer

A. Simulation data

Our proposed gyroless observer (4) has been tested on simulation data, with various inertia matrices and applied torques ; below are examples of estimation of angular velocities, torques and attitudes.

Figure 2 reports estimates of angular velocities during a simulation where a triangle signal torque is applied to the third axis of inertia. None of the values of the three torques are known to the observer which is able to estimate them, as reported in Figure 3. In this figure, it is worth noting that the assumption of slowly varying torque is reasonable, but prevents the observer to truly estimate the value of the triangle signal (some lag in the estimate is present, as expected). Interestingly, the coupling term $\tilde{\omega}$ is not completely vanishing, but it is kindly attenuated in the attitude estimate as reported in Figure 4.

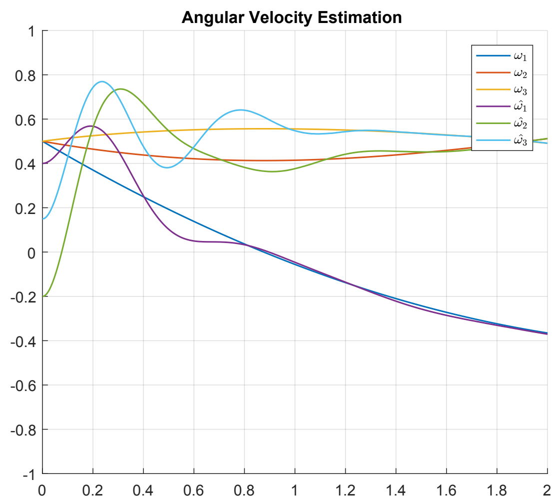

Fig. 2. Estimation of the angular velocity [simulation results] (zoom-in view)

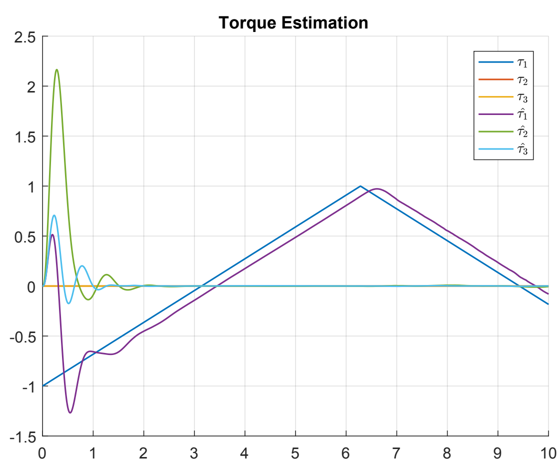

Fig. 3. Estimation of the torques [simulation results]

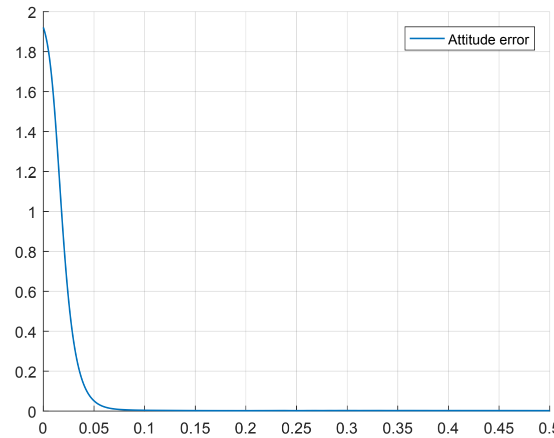

Fig. 4. Norm of the attitude error matrix [simulation results] (zoom-in view)

B. Experiments

The gyroless attitude observer (4) was also tested on experimental signals obtained using the low-cost sensors of smartphone. The direction sensors of the smartphone (magnetometer and accelerometer) are used. Only those measurements are used in the implementation of (4). The angular velocitys readings are used in an implementation of the filter (10) for comparisons.

The smartphone under consideration is a Blackberry KEY-one™Android smartphone which embeds a tri-axis gyro, a tri-axis accelerometer and a tri-axis magnetometer. The data are streamed, using a local Wifi connection, to a laptop PC implementing the observers in discrete-time under the form of Matlab scripts. The data are received at a rate of approx. 100Hz. The inertia matrix J is roughly estimated using the exterior dimensions of the device. Comparisons of the attitude, decomposed into three usual Euler-angles are reported in Figure 5. During the experiments, the smartphone is gently reoriented and the two observers (4) and (10) produce concurring estimates, as is reported in Figure 5. Although no absolute truth can be assessed in the abscence of a reference attitude, it is expected that (10) captures more directly the transients (which are directly measured by the gyros) than (4) which has to estimate them. The results reported in Figure 5 stress this fact. They also stress that the proposed observer is actually working, producing smooth estimated of the attitude without any gyro, but by filtering through several layers of model dynamics the measurement of the direction sensors.

Conclusion

This paper offers a simple alternative to the classic complementary filter of [16] for attitude filtering. The proposed alternative observer is gyroless, and features the appealing simplicity of the complementary filters (compared to state of the art Kalman filters, and directe resolution of Wahba's problem). Even though gyros arguably add robustness in case of one measurement failure, gyroless approaches remain of interest for low-cost applications or extreme conditions common gyros cannot handle (strong

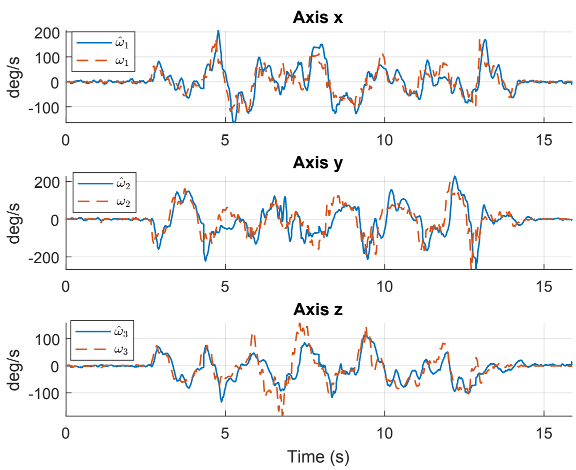

Fig. 5. Estimated angular velocitys compared against gyroscope readings [experimental results]

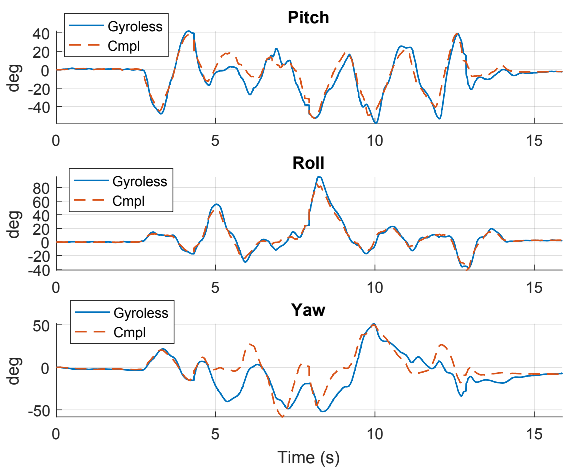

Fig. 6. Estimated Euler angles of the proposed gyroless observer compared against the complementary filter [experimental results]

acceleration, saturating spinning rates).

An interesting extension of this work would be a gyroless attitude observer with one direction sensor only (this may prove useful for example when magnetic disturbances are occurring, or in free-flight, when accelerometers do not provide a measurement of the gravity field). The classic complementary filter of [16] is known to partially converge when one single vector measurement is used (the rotation around the measured vector being unobservable). Concerning angular velocity, [40] has shown that it can still be estimated with one vector measurement, through a simple assumption of persistent excitation (being almost always satisfied, in particular in free-rotation). An analogous proof to the main result of this paper shows the (legitimate) angular velocity observer can still be used in the attitude matrix observer (since the former remains exponentially stable), thus providing a partial estimation of the attitude matrix. In the end, this adaptation of the gyroless observer works with a single direction sensor, and partially gives the attitude of a rigid body, which may prove useful in various application (ballistic free-flight or heavily disturbed magnetic environments).

REFERENCES

[1] N. Metni, J.-M. Pflimlin, T. Hamel, and P. Souères, "Attitude and gyro bias estimation for a VTOL UAV," *Control Engineering Practice*, vol. 14, no. 12, pp. 1511 – 1520, 2006.

[2] D. Jung and P. Tsiotras, "Inertial attitude and position reference system development for a small UAV," in *AIAA Infotech at Aerospace*, 2007.

[3] M. D. Hua, G. Ducard, T. Hamel, R. Mahony, and K. Rudin, "Implementation of a nonlinear attitude estimator for aerial robotic vehicles," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 1, pp. 201–213, 2014.

[4] I. Skog and P. Handel, "In-car positioning and navigation technologies – a survey," *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, no. 1, pp. 4–21, 2009.

[5] J. K. Bekkeng and M. Psiaki, "Attitude estimation for sounding rockets using microelectromechanical system gyros," *Journal of Guidance, Control, and Dynamics*, vol. 31, no. 3, pp. 533–542, 2008.

[6] B. O. Sunde, "Sensor modelling and attitude determination for micro-satellites," Master's thesis, NTNU, 2005.

[7] L. Magnis and N. Petit, "Rotation estimation for a satellite from sun sensors," in *Control Conference (ECC), 2013 European*. IEEE, 2013, pp. 852–859.

[8] J. C. Springmann, A. J. Sloboda, A. T. Klesh, M. W. Bennett, and J. W. Cutler, "The attitude determination system of the rax satellite," *Acta Astronautica*, vol. 75, pp. 120–135, 2012.

[9] K. Kitani, K. Horita, and H. Koike, "Ballcam!: dynamic view synthesis from spinning cameras," in *Adjunct proceedings of the 25th annual ACM symposium on User interface software and technology*. ACM, 2012, pp. 87–88.

[10] S. Changey, E. Pecheur, and T. Brunner, "Attitude estimation of a projectile using magnetometers and accelerometers, experimental validation," in *Position, Location and Navigation Symposium - PLANS 2014, 2014 IEEE/ION*, 2014, pp. 1168–1173.

[11] J. L. Crassidis, F. L. Markley, and Y. Cheng, "Survey of nonlinear attitude estimation methods," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 1, pp. 12–28, 2007.

[12] I. Y. Bar-Itzhack, "REQUEST - a new recursive algorithm for attitude determination," *Proceedings of the National Technical Meeting of The Institude of Navigation*, pp. 699–706, 1996.

[13] M. D. Shuster, "Kalman filtering of spacecraft attitude and the QUEST model," *The Journal of the Astronautical Sciences*, vol. 38, no. 3, pp. 377–393, 1990.

[14] D. Titterton and J. Weston, *Strapdown Inertial Navigation Technology*, 2nd ed. Reston, USA: The American Institute of Aeronautics and Aeronautics, 2004.

[15] R. Mahony, T. Hamel, and J.-M. Pflimlin, "Complementary filter design on the special orthogonal group SO(3)," in *Proc. of the 44th IEEE Conf. on Decision and Control, and the European Control Conference 2005*, 2005.

[16] ——, "Nonlinear complementary filters on the special orthogonal group," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1203–1218, June 2008.

[17] A. Cavallo, A. Cirillo, P. Cirillo, G. De Maria, P. Falco, C. Natale, and S. Pirozzi, "Experimental comparison of sensor fusion algorithms for attitude estimation," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 7585–7591, 2014.

[18] M.-D. Hua, "Attitude estimation for accelerated vehicles using gps/ins measurements," *Control Engineering Practice*, vol. 18, no. 7, pp. 723–732, 2010.

[19] F. Königseder, W. Kemmetmüller, and A. Kugi, "Attitude estimation using redundant inertial measurement units for the control of a camera stabilization platform," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 5, pp. 1837–1844, 2016.

[20] L. Benziane, A. El Hadri, A. Seba, A. Benallegue, and Y. Chitour, "Attitude estimation and control using linearlike complementary filters: theory and experiment," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 6, pp. 2133–2140, 2016.

[21] A. Tayebi, A. Roberts, and A. Benallegue, "Inertial vector measurements based velocity-free attitude stabilization," *IEEE Transactions on Automatic Control*, vol. 58, no. 11, pp. 2893–2898, 2013.

[22] A. Filippeschi, N. Schmitz, M. Miezal, G. Bleser, E. Ruffaldi, and D. Stricker, "Survey of motion tracking methods based on inertial sensors: a focus on upper limb human motion," *Sensors*, vol. 17, no. 6, p. 1257, 2017.

[23] R. Kottath, P. Narkhede, V. Kumar, V. Karar, and S. Poddar, "Multiple model adaptive complementary filter for attitude estimation," *Aerospace Science and Technology*, vol. 69, pp. 574–581, 2017.

[24] T. Michel, P. Genevès, H. Fourati, and N. Layaïda, "On attitude estimation with smartphones," in *Pervasive Computing and Communications (PerCom), 2017 IEEE International Conference on*. IEEE, 2017, pp. 267–275.
[25] R. G. Valenti, I. Dryanovski, and J. Xiao, "Keeping a good attitude: A quaternion-based orientation filter for imus and margs," *Sensors*, vol. 15, no. 8, pp. 19 302–19 330, 2015.
[26] K. Feng, J. Li, X. Zhang, C. Shen, Y. Bi, T. Zheng, and J. Liu, "A new quaternion-based kalman filter for real-time attitude estimation using the two-step geometrically-intuitive correction algorithm," *Sensors*, vol. 17, no. 9, p. 2146, 2017.
[27] E. Bergamini, G. Ligorio, A. Summa, G. Vannozzi, A. Cappozzo, and A. M. Sabatini, "Estimating orientation using magnetic and inertial sensors and different sensor fusion approaches: Accuracy assessment in manual and locomotion tasks," *Sensors*, vol. 14, no. 10, 2014.
[28] P. Martin and I. Sarras, "A global observer for attitude and gyro biases from vector measurements," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 15 409–15 415, 2017.
[29] R. L. McCoy, *Modern exterior ballistics*, 2nd ed. Schiffer, 1998.
[30] S. Changey, E. Pecheur, P. Wey, and E. Sommer, "Real-time estimation of projectile roll angle using magnetometers: in-lab experimental validation," *EUCASS Proceedings Series*, vol. 6, pp. 35–44, Dec. 2013.
[31] A. Fiot, S. Changey, C. Combettes, and N. Petit, "Estimation of air velocity for a high velocity spinning projectile using transverse accelerometers," in *Proc. of the AIAA Guidance, Navigation, and Control Conference, AIAA Science and Technology Forum and Exposition*, 2018.
[32] D. Thakur, F. Mazenc, and M. R. Akella, "Partial lyapunov strictification: smooth angular velocity observers for attitude tracking control," in *AIAA/AAS Astrodynamics Specialist Conference*, 2014, p. 4420.
[33] A. Berkane, S. Abdessameud and A. Tayebi, "Global exponential angular velocity observer for rigid body systems," in *Proc. of the 55th IEEE Conference on Decision and Control*, 2016.
[34] L. Magnis and N. Petit, "Angular velocity nonlinear observer from vector measurements," *Automatica*, vol. 75, pp. 46–53, 2017.
[35] ——, "Angular velocity nonlinear observer from single vector measurements," *Automatic Control, IEEE Transactions on*, vol. 61, no. 9, pp. 2473–2483, 2016.
[36] ——, "Angular velocity and torque observer from vector measurements," *Proceedings of the 20th World Congress of IFAC*, pp. 9330–9337, 2017.
[37] G. Hostetter and J. Meditch, "Observing systems with unmeasurable inputs," *IEEE Transactions on Automatic Control*, vol. 18, no. 3, pp. 307–308, 1973.
[38] P. V. Kokotovic and H. K. Khalil, *Singular Perturbations in Systems and Control*. IEEE Press, New York, 1986.
[39] H. Khalil, *Nonlinear Systems*. Prentice-Hall, second edition, 1996.
[40] L. Magnis and N. Petit, "Estimation of attitude from a single-direction sensor," in *Multisensor attitude estimation: fundamental concepts and applications*, H. Fourati and D. E. C. Belkiat, Eds. CRC Press, 2016, pp. 261–276.