Prediction-based control for nonlinear systems with stochastic input delay

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Abstract This chapter focuses on nonlinear systems with a stochastic input delay, which can arise from communication lags for instance. For input-delay systems, the use of prediction-based control methods is quite standard. This method was first used for constant delays before being extended to time-varying delays. Yet, the design of a prediction-based controller to handle stochastic delays is still an open problem.

In this chapter, we propose to describe the stochastic input delay as a continuous-time Markov process with a finite number of values. This random delay is then robustly compensated through a constant-horizon state prediction. Applying the method of backstepping, we obtain that exponential stabilization of the closed-loop system is achieved, under a sufficient condition. This condition bears on the range of the possible values of the time lag, which should be sufficiently small and close enough to the prediction horizon.

1 Introduction

Time delays have always represented an important research topic, due to their frequent appearance and the performance limitation they often induce for control systems [29]. In the case where the delay affects the input of a dynamical system, active control techniques generically referred to as Smith Predictor [28, 32] have been developed to counteract these limitations. Grounding on a prediction of the system state, this so-called Finite Spectrum Assignment [21, 24] aims at compensating the input delay in the closed-loop dynamics and recovering the nominal delay-free performances. This technique, originally designed for constant delays, has gradually been extended to time-varying ones [3, 27] or more intricate cases such as state-

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dependent delays [1, 2], multiple delays [5] or additive disturbances [22]. In these cases, the prediction time horizon has to be carefully computed to account for future delay variations or implicit dependencies.

Recently, the rise of communication technologies has intensified the need to cope with transmission lags [11, 30, 31]. They result for instance from congestion in the transmission path(s) or changes in the routing algorithm and are usually modeled as stochastic variables to account for their sudden and random variations [6, 10, 26, 34].

However, while in recent years many monographs have studied Stochastic Delay Equations (SDEs), such as [12, 15, 25], the case where the delay is itself a random variable has been seldomly studied. Up to the authors' knowledge, one of the few examples to do so are [16, 17, 23, 33]. While [33] studies a piecewise constant process and [23] analyzes a deterministic delay term multiplied by a random variable, [16, 17] consider stochastic state delays modeled as a Markov process with a finite number of states. The authors then consider each delay value separately, following the so-called technique of probabilistic delay averaging. This constant delay reasoning inspired the core of the analysis methodology proposed in this chapter.

In this paper, we investigate the problem of prediction-based control design for nonlinear systems subject to stochastic input delay and extend the scope of our previous work [18] focusing on linear systems. We model the random input delay as a Markov process with a finite number of values. In lieu of computing a state prediction exactly compensating for the input delay (which cannot be achieved as future delay values are random and thus unknown), we propose to use a constant prediction horizon. Similarly to [18], we prove that such a controller achieves exponential stabilization of the origin, provided that the range of possible delay values is sufficiently narrow and that the prediction horizon is chosen close enough to these values.

This chapter is organized as follows. In Section 2, the problem statement and the main stabilization result are given. We then propose a PDE representation of the stochastic delay and a backstepping reformulation of the closed-loop system in Section 3, in view of Lyapunov stability analysis in Section 4.

Notations. In the following sections, for a signal $v : (x,t) \in [0,1] \times \mathbb{R} \mapsto v(x,t) \in \mathbb{R}$, we denote $\|v(t)\|$ its spatial $\mathcal{L}_2$-norm

$$\|v(t)\| = \sqrt{\int_0^1 v(x,t)^2 dx} \tag{1}$$

and $\|v\|_\infty$ its $\mathcal{L}_\infty$-norm for a measurable essentially bounded signal as

$$\|v(t)\|_\infty = \inf \{C \geq 0 \mid |v(x,t)| \leq C \text{ f.a.a. } x \in [0,1]\} = \lim_{p\to\infty} \|v(t)\|_p \tag{2}$$

Additionally, for a real matrix A, $|A|$ denotes its Euclidean norm

$$|A| = \sqrt{max(\lambda(A^T A))} \quad (3)$$

in which λ represents the eigenvalues of the matrix, and A^T denotes the transpose of A.

$\mathbb{E}(x)$ denotes the expectation of a random variable x. For a random signal $x(t)$ ($t \in \mathcal{T} \subset \mathbb{R}$), the conditional expectation of $x(t)$ at the instant t knowing that $x(s) = x_0$ at the instant $s < t$ is denoted $\mathbb{E}_{[s,x_0]}(x(t))$.

Finally, $e_i \in \mathbb{R}^r$ ($r \in \mathbb{N}_+$ and $i \in \{1, ..., r\}$) denotes the unit vector, that is, $e_1 = \begin{pmatrix}1 \ 0 \cdots 0\end{pmatrix}^T$, $e_2 = \begin{pmatrix}0 \ 1 \cdots 0\end{pmatrix}^T$, ..., $e_r = \begin{pmatrix}0 \ 0 \cdots 1\end{pmatrix}$. Besides, for $(x_1, x_2) \in \mathbb{R}^r \times \mathbb{R}^r$, we denote the Hadamard product of x_1 and x_2 as $x_1 \bullet x_2 = (x_{1i} x_{2i})_{1 \leq i \leq r} \in \mathbb{R}^r$.

2 Problem statement and control design

We consider the following controllable nonlinear dynamics

$$\dot{X}(t) = f(X(t), U(t - D)) \quad (4)$$

in which $X \in \mathbb{R}^n$ and $U \in \mathbb{R}$ are the state and control input, respectively. f is a nonlinear function of class C^1 such that $f(0, 0) = 0$. The stochastic delay D is a Markov process with the following properties:

(1) $D(t) \in \{D_i, i \in \{1, ..., r\}\}$, $r \in \mathbb{N}$ with $0 < \underline{D} \leq D_1 < D_2 < ... < D_r \leq \overline{D}$.

(2) The transition probabilities $P_{ij}(t_1, t_2)$, which quantify the probability to switch from D_i at time t_1 to D_j at time t_2 ($(i, j) \in \{1, ..., r\}^2$, $t_2 \geq t_1 \geq 0$), are differentiable functions $P_{ij} : \mathbb{R}^2 \to [0, 1]$ satisfying

$$\sum_{j=1}^{r} P_{ij}(t_1, t_2) = 1, \quad (0 \leq t_1 \leq t_2) \quad (5)$$

(3)The realizations of D are continuous from the right.

The control objective is to stabilize plant (4) following a prediction-based approach, despite the random nature of the input delay. This will be achieved by a robust delay compensation approach. Before detailing our control design, we first further characterize the delay-free characteristics of the plant under consideration.

Assumption 1 *There exists a feedback law κ of class C^1 such that, the plant*

$$\dot{X} = f(X, \kappa(X)) \quad (6)$$

is exponentially stable with basin of attraction $\mathcal{A} \subset \mathbb{R}^n$, i.e. there exists a positive definite function $W : \mathbb{R}^n \to \mathbb{R}_+$ such that, for all compact $C \subset \mathcal{A}$, there exist positive constants λ, C_1, C_2 and C_3 such that, for all $x \in C$,

$$\frac{dW}{dX}(X) f(X, \kappa(X)) \leq -\lambda |X|^2 \quad (7)$$

$$C_1|X|^2 \leq W(X) \leq C_2|X|^2 \tag{8}$$

$$\left|\frac{dW}{dX}(X)\right| \leq C_3|X| \tag{9}$$

Assuming exponential stability of the delay-free closed-loop dynamics in lieu of asymptotic stability is not restrictive, in the sense that some dynamical systems for which the origin is only asymptotically stable can have a zero-delay robustness property. This prohibits us from using a robust compensation controller as the one we aim at designing.

We now detail two additional technical assumptions.

Assumption 2 *The function $f(X,U)$ is globally Lipschitz, i.e. there exists $C_L > 0$ such that for $\forall (u_1, u_2) \in \mathbb{R}^2$ and $\forall (X_1, X_2) \in (\mathbb{R}^n)^2$*

$$|f(X_1,u_1) - f(X_2,u_2)| \leq C_L|(X_1,u_1) - (X_2,u_2)| \tag{10}$$

Assumption 3 *The feedback law κ defined in Assumption 1 is such that $\kappa(0) = 0$ and there exists a positive constant C_4 such that*

$$\left|\frac{d\kappa}{dX}\right| \leq C_4 \tag{11}$$

Contrary to Assumption 1, these last two assumptions are more restrictive and more technical. They aim at simplifying the stability analysis detailed in the following, but should be relaxed in future works.

Note that Assumption 2 jointly with the fact that $f(0,0) = 0$ guarantee that the plant does not escape in finite time, which is a necessary condition for stabilization to be achieved for any delay values. Therefore, we do not need to assume strong forward completeness here, as usually done for prediction-based control for nonlinear dynamics [3], but, in all likelihood, future works will have to replace Assumption 2 by this property.

The control law is chosen here as

$$U(t) = \kappa(\hat{P}(t)) \tag{12}$$

in which the predictor $\hat{P}$ is defined for $\theta \in [t - D_0, t]$ as

$$\hat{P}(\theta,t) = X(t) + \int_{t-D_0}^{\theta} f(\hat{P}(s,t), U(s))ds, \quad t - D_0 \leq \theta \leq t \tag{13}$$

To exactly compensate for the input delay, one would need the function $\phi : t \to t - D(t)$ to be invertible in order to define the predictor. However, the stochastic nature of the delay makes the computation of the inverse function $\phi^{-1}(t)$ (if it exists) impossible as it depends on future delay variations and is therefore not causal.

For this reason, we hence propose to use a constant time horizon prediction in the controller (12)–(13). Note that, if the time lag D were constant and equal to D_0, the control law would then involve the exact prediction of the state X over a time window of D_0 units of time. Consistently, we now formulate the main result of the paper which states that robust compensation is achieved if the time delay remains sufficiently close to D_0.

Theorem 1 *Consider the closed-loop system consisting of the system (4) satisfying Assumptions 1–3 and the control law (12). Let C be a compact set of $\mathcal{A}$ containing the origin. Then, there exist positive constants $\epsilon^\star$ and $\rho^\star$ such that, if*

$$|D_0 - D_j| \leq \epsilon^\star, \quad j \in \{1, ..., r\} \tag{14}$$

and if

$$\Upsilon(0) \leq \rho^\star \tag{15}$$

there exist positive constants R and γ (independent of the initial condition) such that

$$\mathbb{E}_{[0,(\Upsilon(0),D(0))]}(\Upsilon(t)) \leq R\Upsilon(0)e^{-\gamma t} \tag{16}$$

with

$$\Upsilon(t) = |X(t)|^2 + \int_{t-\overline{D}-D_0}^{t} U(s)^2 ds \tag{17}$$

Condition (14) guarantees that the prediction performed in (13) remains sufficiently accurate in the case of a stochastic delay. In details, (14) requires the sequence of the random delay $\mathbf{D} = \begin{pmatrix} D_1 & \cdots & D_i & \cdots & D_r \end{pmatrix}^T$ to be limited in a vicinity $\epsilon^\star$ of the constant D_0.

Note that this result is consistent with the delay-robustness results obtained in the deterministic delay case. Indeed, [4, 19] provide a similar robust compensation result for a time-differentiable delay function, under the assumptions that both the range of variation of the delay and its variation rate are sufficiently limited. A similar result was obtained in [13] but through a small-gain approach enabling to avoid restricting the delay rate. Hence, Theorem 1 falls within this framework and extends it to the stochastic context.

We now provide the proof of this theorem in the following sections.

3 PDE Representation of the Delay and Backstepping Transformation

First, to represent the control input which is subject to a stochastic delay, we define a distributed actuator vector as, for $x \in [0, 1]$, $\mathbf{v}(x, t) = \begin{pmatrix} v_1(x, t) & \cdots & v_k(x, t) & \cdots & v_r(x, t) \end{pmatrix}^T$ with $v_k(x, t) = U(t + D_k(x - 1))$. This enables to rewrite (4) as

$$\begin{cases} \dot{X}(t) = f(X(t), \delta(t)^T \mathbf{v}(0, t)) \\ \Lambda_D \mathbf{v}_t(x, t) = \mathbf{v}_x(x, t) \\ \mathbf{v}(1, t) = U(t)\mathbf{1} \end{cases} \tag{18}$$

in which $\Lambda_D = diag(D_1, ..., D_r)$, $\mathbf{1}$ is a r-by-1 all-ones vector and $\delta \in \mathbb{R}^r$ is such that, if $D(t) = D_j$, then $\delta(t) = e_j$, that is,

$$\delta_i(t) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Hence, δ is a random process with the same transition probabilities as the stochastic process D, but with the finite number of states (e_i) instead of (D_i). In the sequel, δ and D will thus be equivalently used.

Now, we introduce $\hat{v}(x,t)$ to represent the control input $U(t)$ within the interval $[t - D_0, t]$, and the corresponding input estimation error $\tilde{\mathbf{v}}(x,t)$ defined as

$$\begin{cases} \hat{v}(x,t) = U(t + D_0(x-1)) \\ \tilde{\mathbf{v}}(x,t) = \mathbf{v}(x,t) - \hat{v}(x,t)\mathbf{1} \end{cases} \tag{20}$$

Then, we define $\mu(x,t) = U(t - D_0 + \overline{D}(x-1))$ to represent the controller within the interval $[t - D_0 - \overline{D}, t - D_0]$. The extended state $(X(t), \hat{v}(x,t), \tilde{\mathbf{v}}(x,t), \mu(x,t))$ satisfies

$$\begin{cases} \dot{X}(t) = f(X(t), \hat{v}(0,t) + \delta(t)^T \tilde{\mathbf{v}}(0,t)) \\ D_0 \hat{v}_t(x,t) = \hat{v}_x(x,t) \\ \hat{v}(1,t) = U(t) \\ \Lambda_D \tilde{\mathbf{v}}_t(x,t) = \tilde{\mathbf{v}}_x - \Sigma_D \hat{v}_x \\ \tilde{\mathbf{v}}(1,t) = \mathbf{0} \\ \overline{D}\mu_t(x,t) = \mu_x(x,t) \\ \mu(1,t) = \hat{v}(0,t) \end{cases} \tag{21}$$

in which $\Sigma_D = \begin{pmatrix} \frac{D_1 - D_0}{D_0} & \cdots & \frac{D_r - D_0}{D_0} \end{pmatrix}^T$ and $\mathbf{0}$ is a r-by-1 all-zeros vector.

In view of stability analysis, we introduce the backstepping transformation (see [20])

$$w(x,t) = \hat{v}(x,t) - \kappa(\hat{p}(x,t)), \ 0 \le x \le 1 \tag{22}$$

in which the distributed predictor $\hat{p}$ is defined as

$$\hat{p}(x,t) = X(t) + D_0 \int_0^x f(\hat{p}(y,t), \hat{v}(y,t))dy, \ 0 \le x \le 1 \tag{23}$$

Notice that this backstepping transformation is similar to the one used in the deterministic context (see [20]) but applied here to our estimated distributed input $\hat{v}$.

Lemma 1 *The backstepping transformation (22), jointly with the control law (12)–*(13)*, transform the plant (21) into the target system* $(X, w, \tilde{\mathbf{v}}, \mu)$

$$\begin{cases} \dot{X}(t) = f(X(t), \kappa(X(t)) + w(0,t) + \delta(t)^T \tilde{\mathbf{v}}(0,t)) \\ D_0 w_t(x,t) = w_x(x,t) + r_w(x,t) \\ w(1,t) = 0 \\ \Lambda_D \tilde{\mathbf{v}}_t(x,t) = \tilde{\mathbf{v}}_x - \Sigma_D h(t + D_0(x-1)) \\ \tilde{\mathbf{v}}(1,t) = \mathbf{0} \\ \overline{D}\mu_t(x,t) = \mu_x(x,t) \\ \mu(1,t) = \kappa(X(t)) + w(0,t) \end{cases} \tag{24}$$

in which r_w is defined for $x \in [0,1]$ as

$$r_w(x,t) = -D_0 \frac{d\kappa}{dX}(\hat{p}(x,t))\Phi(x,0,t)\tilde{f}(t) \tag{25}$$

with

$$\tilde{f}(t) = f(X(t), \kappa(X(t)) + w(0,t) + \delta(t)^T \tilde{\mathbf{v}}(0,t)) - f(X(t), \kappa(X(t)) + w(0,t)) \tag{26}$$

Φ is the state-transition matrix associated with the equation

$$r_x(x,t) = D_0 \frac{\partial f}{\partial X}\big(\hat{p}(x,t), w(x,t) + \kappa(\hat{p}(x,t))\big) r(x,t) \tag{27}$$

and h is defined for $t \geq 0$ as

$$h(t) = D_0 \frac{d\kappa}{dX}(\hat{p}(1,t)) f(\hat{p}(1,t), w(0,t) + \kappa(X(t)) + \delta(t)^T \tilde{\mathbf{v}}(0,t)) \tag{28}$$

with $\hat{p}(x,t)$ defined in (23) a function of $X(t)$ and $w(y,t)$ for $y \in [0,x]$ and $x \in [0,1]$.

Proof Define $q(x,t) = D_0 \hat{p}_t(x,t) - \hat{p}_x(x,t)$. Taking time- and space-derivatives of $\hat{p}$ defined in (23), one obtains

$$\begin{aligned} & q(x,t) \\ =& D_0 \Big[f(X(t), \hat{v}(0,t) + \delta(t)^T \tilde{\mathbf{v}}(0,t)) + \int_0^x \frac{\partial f}{\partial X}(\hat{p}(y,t), \hat{v}(y,t)) D_0 \hat{p}_t dy \\ & + \int_0^x \frac{\partial f}{\partial U}(\hat{p}(y,t), \hat{v}(y,t)) \hat{v}_x dy - f(X(t), \hat{v}(0,t)) \\ & - \int_0^x \frac{\partial f}{\partial X}(\hat{p}(y,t), \hat{v}(y,t)) \hat{p}_x dy - \int_0^x \frac{\partial f}{\partial U}(\hat{p}(y,t), \hat{v}(y,t)) \hat{v}_x dy \Big] \\ =& D_0 \int_0^x \frac{\partial f}{\partial X}(\hat{p}(y,t), \hat{v}(y,t)) q(y,t) dy + D_0 \tilde{f}(t) \end{aligned} \tag{29}$$

with $\tilde{f}(t)$ defined in (26). This integral equation can be rewritten under the differential form

$$\begin{cases} q_x(x,t) = D_0 \dfrac{\partial f}{\partial X}(\hat{p}(x,t), \hat{v}(x,t)) q(x,t) \\ q(0,t) = D_0 \tilde{f}(t) \end{cases} \tag{30}$$

Thus, introducing the state-transition matrix Φ associated with (30), it follows that

$$q(x,t) = D_0 \Phi(x,0,t) \tilde{f}(t) \tag{31}$$

As the time-derivative of the backstepping transformation (22) is given as

$$w_t(x,t) = \hat{v}_t(x,t) - \frac{d\kappa}{dX}(\hat{p}(x,t)) \hat{p}_t(x,t) \tag{32}$$

and its space-derivative as

$$w_x(x,t) = \hat{v}_x(x,t) - \frac{d\kappa}{dX}(\hat{p}(x,t)) \hat{p}_x(x,t) \tag{33}$$

the dynamics of the backstepping variable w then follows.

In addition, the expression of $h(t)$ for $t \geq 0$ is obtained by taking a time derivative of (12)

$$h(t + D_0(x-1)) = \hat{v}_x(x,t) = D_0 \dot{U}(t + D_0(x-1)) = D_0 \frac{d\kappa}{dX}(\hat{p}(1,t)) \dot{\hat{P}}(t) \tag{34}$$

We are now ready to analyze the closed-loop system stablization.

4 Stability analysis of the closed-loop system

Let us define the state of the target system (24) as $\Psi = (X, w, \tilde{\mathbf{v}}, \mu) \in \mathbb{R}^n \times \mathcal{L}_\infty([0,1],\mathbb{R}) \times \mathcal{L}_\infty([0,1],\mathbb{R}^r \times \mathcal{L}_\infty([0,1],\mathbb{R}) \triangleq \mathcal{D}_\Psi$. Note that (24) was reformulated as a dynamical system involving a random parameter, as studied in [14] or [7]. We now formulate a well-posedness result for this system.

Following [14], by a weak solution to the closed-loop system (4) and (12)–(13), we refer to a $\mathbb{R}^n \times \mathcal{L}_\infty([-\overline{D}, 0],\mathbb{R}) \times \mathbb{R}$-valued random variable $(X(X_0,t), U_t(U_0,\cdot), D(t))$, the realizations of which satisfy an integral form of (4) and (12)–(13).

Similarly, by a weak solution to (24), we refer to a $\mathcal{D}_\Psi \times \mathbb{R}$-valued random variable $(X(X_0,t), w(w_0,\cdot,t), \tilde{\mathbf{v}}(\tilde{\mathbf{v}}_0,\cdot,t), \mu(\mu_0,\cdot,t), D(t))$, the realizations of which are a weak solution of (24), that is, in the standard weak solution sense of [8] for the transport PDEs and under an integral form for the ODE.

Lemma 2 *For every initial condition* $(X_0, U_0) \in \mathbb{R}^n \times \mathcal{L}_\infty([-\overline{D}, 0],\mathbb{R})$*, the closed-loop system consisting of the plant* (4) *satisfying Assumptions 1–2 and the control law defined in* (12)–(13) *has a unique weak solution.*

Consequently, for each initial condition in $\mathcal{D}_\Psi$*, the target system* (24) *also has a unique weak solution.*

Proof The proof is omitted due to space limitation. □

From Lemma 2, (Ψ, δ) thus defines a continuous-time Markov process and we can therefore introduce the following elements for stability analysis.

In the sequel, we consider the following Lyapunov functional candidate

$$\begin{aligned} V(\Psi) =& W(X) + bD_0 \int_0^1 (1+x) w(x)^2 dx + c \sum_{l=1}^{r} \int_0^1 (1+x)(e_l \bullet \mathbf{D})^T \tilde{\mathbf{v}}(x)^2 dx \\ &+ d\overline{D} \int_0^1 (1+x)\mu(x)^2 dx \end{aligned} \tag{35}$$

with $b, c, d > 0$, and $\mathbf{D} = \begin{pmatrix} D_1 & \cdots & D_i & \cdots & D_r \end{pmatrix}^T$ and where $\bullet$ denotes the Hadamard multiplication and the square in $\tilde{\mathbf{v}}(x)^2$ should be understood component-wise.

As the functional V is not differentiable with respect to time t when evaluated at $\Psi(t)$, we introduce the infinitesimal generator L (see [15] and [17]) as

$$LV(\Psi) = \limsup_{\Delta t \to 0^+} \frac{1}{\Delta t} \Big(\mathbb{E}_{[t,(\Psi,\delta)]} (V(\Psi(t+\Delta t))) - V(\Psi) \Big) \tag{36}$$

We also define L_j, the infinitesimal generator of the Markov process Ψ obtained by fixing the value $\delta(t) = e_j$, as

$$L_j V(\Psi) = \frac{dV}{d\Psi}(\Psi) F_j(\Psi) \tag{37}$$

in which F_j denotes the operator corresponding to the dynamics of the target system (24) with the fixed value $\delta(t) = e_j$, that is, for $\Psi = (X, w, \tilde{\mathbf{v}}, \mu)$,

$$F_j(\Psi)(x) = \begin{pmatrix} f(X(t), \kappa(X(t)) + e_j^T \tilde{\mathbf{v}}(0) + w(0)) \\ \frac{1}{D_0}\left[w_x(x) + r_w(x) \right] \\ \Lambda_D^{-1} \left[\tilde{\mathbf{v}}_x(x) - \Sigma_D h(\bullet + D_0(x-1)) \right] \\ \frac{1}{\overline{D}} \mu_x(x) \end{pmatrix} \tag{38}$$

For the sake of conciseness, in the sequel, we denote $V(t)$, $LV(t)$ and $L_j V(t)$, for short, instead of $V(\Psi(t), \delta(t))$, $LV(\Psi(t), \delta(t))$ and $L_j V(\Psi(t), t)$ respectively.

It is worth noticing that, due to the fact that V does not depend explicitly on δ, the infinitesimal generators are related as follows

$$\begin{aligned} LV(t) &= \sum_{j=1}^{r} P_{ij}(0,t) \frac{dV}{d\Psi}(\Psi(t)) F_j(\Psi(t)) + \sum_{j=1}^{r} \frac{\partial P_{ij}}{\partial t}(0,t) V(\Psi(t)) \\ &= \sum_{j=1}^{r} P_{ij}(0,t) L_j V(t) \end{aligned} \tag{39}$$

Therefore, in view of stability analysis, as a first step, one can focus on the derivative of the Lyapunov functional evaluated for a dynamic with a fixed delay, that is, $L_j V$. This is the approach we follow in the sequel.

4.1 Lyapunov analysis

Before detailing the Lyapunov analysis, we provide a technical lemma, useful in the following computations.

Lemma 3 *Consider the function h defined in* (28)*. There exists* $M > 0$ *such that*

$$\|h(t + D_0(\cdot - 1))\|^2 \leq MV(t), \quad t \geq D_0 \tag{40}$$

Proof From the definition of the control law (12)–(13), (34) and Assumptions 2–3, then

$$\begin{aligned}|h(t)|^2 =& D_0^2 \left(\frac{d\kappa}{dX}(\hat{p}(1,t))\hat{p}_t(1,t) \right)^2 \\ \leq& C_4^2 |f(\hat{p}(1,t), \hat{v}(1,t)) + \Phi(1,0,t)\tilde{f}(t)|^2 \\ \leq& C_4^2 \Big(|f(\hat{p}(1,t), \hat{v}(1,t))| + e^{D_0 C_L} |\tilde{f}(t)| \Big)^2\end{aligned} \tag{41}$$

Hence, using (26) and Assumption 2, one obtains

$$\begin{aligned}&\|h(t + D_0(\cdot - 1))\|^2 = \int_0^1 h(t + D_0(x-1))^2 dx \\ \leq& 2C_4^2 \Big(\int_0^1 \left| f\left(\hat{p}(1, t + D_0(x-1)), \hat{v}(1, t + D_0(x-1))\right) \right|^2 dx \\ &+ e^{2D_0 C_L} \int_0^1 |\tilde{f}(t + D_0(x-1))|^2 dx \Big) \\ \leq& 2C_4^2 \Big(C_L^2 \int_0^1 \left(|\hat{P}(t + D_0(x-1))| + |U(t + D_0(x-1))| \right)^2 dx \\ &+ 2C_L^2 e^{2D_0 C_L} \int_0^1 U(t + D_0(x-2))^2 dx \\ &+ 2C_L^2 e^{2D_0 C_L} \int_0^1 U(t + D_0(x-1) - D(t + D_0(x-1)))^2 dx \Big)\end{aligned} \tag{42}$$

Besides, from (13) and Assumption 2, for $\theta \in [t - D_0, t]$,

$$\begin{aligned} |\hat{P}(\theta)| \leq & |X(t)| + C_L \int_{t-D_0}^{\theta} (|\hat{P}(s)| + |U(s)|) ds \\ \leq & \Big(|X(t)| + C_L \int_{t-D_0}^{\theta} |U(s)| ds \Big) + C_L \int_{t-D_0}^{\theta} |\hat{P}(s)| ds \end{aligned} \tag{43}$$

Using Grönwall's inequality, it follows that

$$\begin{aligned} |\hat{P}(\theta)| \leq & |X(t)| + C_L \int_{t-D_0}^{\theta} |U(s)| ds \\ & + C_L \int_{t-D_0}^{\theta} \big(|X(t)| + \int_{t-D_0}^{s} |U(\xi)| d\xi \big) e^{C_L(\theta - s)} ds \\ = & e^{C_L(\theta - t + D_0)} |X(t)| + C_L \int_{t-D_0}^{\theta} e^{C_L(\theta - s)} |U(s)| ds \end{aligned} \tag{44}$$

Replacing (44) into (42), one has

$$\begin{aligned} \|h(t + D_0(\cdot - 1))\|^2 \leq & 2C_L^2 D_0^2 C_4^2 \int_0^1 \Big(e^{C_L D_0 x} |X(t)| + |U(t + D_0(x-1))| \\ & + C_L \int_{t-D_0}^{t+D_0(x-1)} e^{C_L(t + D_0(x-1) - s)} |U(s)| ds \Big)^2 dx \\ & + 4C_L^2 D_0^2 C_4^2 e^{2D_0 C_L} \Bigg(\int_0^1 U(t + D_0(x-2))^2 dx \\ & + \int_0^1 U(t + D_0(x-1) - D(t + D_0(x-1)))^2 dx \Bigg) \end{aligned} \tag{45}$$

Now, define the inverse of the backstepping transformation of (22), which is given as

$$\hat{v}(x,t) = w(x,t) + \kappa(\hat{\rho}(x,t)) \tag{46}$$

where $\hat{\rho}$ satisfies

$$\hat{\rho}(x,t) = X(t) + D_0 \int_0^x f(\hat{\rho}(y,t), w(y,t) + \kappa(\hat{\rho}(y,t))) dy \tag{47}$$

Similarly, using Assumption 2 and Grönwall's inequality, one can also bound $\hat{\rho}(x,t)$ as

$$\hat{\rho}(x,t) \leq e^{D_0 C_L (1+C_4) x} |X(t)| + D_0 C_L \int_0^x e^{D_0 C_L (1+C_4)(x-y)} |w(y,t)| dy \tag{48}$$

Then, from (46), one obtains

$$\|\hat{v}(t)\|^2 \leq 3C_4^2 e^{2D_0 C_L (1+C_4)} |X(t)|^2 + 3\big(C_4^2 D_0^2 C_L^2 e^{2D_0 C_L (1+C_4)} + 1 \big) \|w(t)\|^2 \tag{49}$$

Hence, observing that

$$\int_0^1 U(t+D_0(x-1)-D(t+D_0(x-1)))^2dx \le \sum_{j=1}^{r}\int_0^1 U(t+D_0(x-1)-D_j)^2dx$$
$$\le 2r\left(\|\hat{v}(t)\|^2+\|\mu(t)\|^2\right) \tag{50}$$

and using (49) in (45), it follows that

$$\|h(t+D_0(\cdot-1))\|^2 \le M_X|X(t)|^2+M_w\|w(t)\|^2+M_\mu\|\mu(t)\|^2 \tag{51}$$

in which (M_X, M_w, M_μ) are positive constants defined as follows

$$\begin{cases} M_X = 6C_L^2\overline{D}^2C_4^2\big(e^{2C_L\overline{D}}+C_4^2e^{2C_L\overline{D}(1+C_4)}(3+3C_L^2e^{2C_L\overline{D}}+2(1+2r)e^{2C_L\overline{D}})\big) \\ M_w = 6C_L^2\overline{D}^2C_4^2\big(C_4^2\overline{D}^2C_L^2e^{2\overline{D}C_L(1+C_4)}+1\big)(3+3C_L^2e^{2C_L\overline{D}}+2(1+2r)e^{2C_L\overline{D}}) \\ M_\mu = 4(1+2r)C_L^2\overline{D}^2C_4^2e^{2C_L\overline{D}} \end{cases} \tag{52}$$

Consequently, from the definition of the Lyapunov functional V in (35), one finally gets the desired result with $M=\overline{D}^2\max\left\{\frac{M_X}{\min(\lambda(P))},\frac{M_w}{b\underline{D}},\frac{M_\mu}{d\overline{D}}\right\}$. □

With this last ingredient, we are now ready to develop the Lyapunov analysis.

Lemma 4 *Consider the closed-loop system consisting of* (4) *and the control law* (12)–(13) *satisfying Assumptions 1–3. Let C be a compact set of $\mathcal{A}$ containing the origin. Assume there exists a positive constant ϵ such that*

$$|D_0-D_j|\le\epsilon, \quad j\in\{1,...,r\} \tag{53}$$

and that $X(t)\in C$ for a certain $t\ge\overline{D}$, then, there exist $b,c,d\in\mathbb{R}_+^{*3}$ which are independent of ϵ such that the Lyapunov functional V defined in (35) satisfies

$$LV(t)\le-(\eta-g(\epsilon))V(t) \tag{54}$$

with $\eta>0$ independent of ϵ and the function $g:\mathbb{R}_+\to\mathbb{R}_+$ satisfying $\lim_{\epsilon\to 0}g(\epsilon)=0$.

Proof Applying integrations by parts and Young's inequality, one obtains

$$\begin{aligned}\frac{dV}{d\Psi}(\Psi)F_j(\Psi) \leq &\frac{dW}{dX}f(X(t),\kappa(X(t))+w(0,t)+\tilde{v}_j(0,t))\\&-b\big(1-2\gamma_1\big)\|w(t)\|^2-bw(0,t)^2+2b\frac{1}{\gamma_1}\|r_w(x,t)\|^2\\&-c\sum_{l=1}^{r}\left(1-2\left|1-\frac{D_l}{D_0}\right|\gamma_2\right)\|\tilde{v}_l(t)\|^2-c\sum_{l=1}^{r}\tilde{v}_l(0,t)^2\\&+2c\sum_{l=1}^{r}\left|1-\frac{D_j}{D_0}\right|\frac{1}{\gamma_2}\|h(t+D_0(\bullet-1))\|^2\\&-d\mu(0,t)^2-d\|\mu(t)\|^2+2d\mu(1,t)^2\end{aligned}\tag{55}$$

From the definition of $\tilde{f}$ in (26) with $\delta(t)=e_j$ and Assumptions 2–3, it holds

$$\|r_w(x,t)\|^2=D_0^2C_4^2e^{2D_0C_L}C_L^2\tilde{v}_j(0,t)^2\tag{56}$$

Then, with Assumption 2, we obtain the following inequality

$$\begin{aligned}\frac{dV}{d\Psi}(\Psi)F_j(\Psi) \leq &\frac{dW}{dX}\Big(f(X(t),\kappa(X(t))+C_L|w(0,t)+\tilde{v}_j(0,t)|\Big)\\&-b\big(1-2\gamma_1\big)\|w(t)\|^2-bw(0,t)^2\\&+2b\frac{1}{\gamma_1}D_0^2C_L^2C_4^2e^{2D_0C_L}\tilde{v}_j(0,t)^2\\&-c\sum_{l=1}^{r}\left(1-2\left|1-\frac{D_l}{D_0}\right|\gamma_2\right)\|\tilde{v}_l(t)\|^2-c\sum_{l=1}^{r}\tilde{v}_l(0,t)^2\\&+2c\sum_{l=1}^{r}\left|1-\frac{D_l}{D_0}\right|\frac{1}{\gamma_2}\|h(t+D_0(\bullet-1))\|^2\\&-d\mu(0,t)^2-d\|\mu(t)\|^2+2d\mu(1,t)^2\end{aligned}\tag{57}$$

Therefore, applying Assumptions 2–3, Lemma 3 and Young's inequality, $LV(t)$ satisfies

$$\begin{aligned}LV(t)=&\sum_{j=1}^{r}P_{ij}(0,t)\frac{dV}{d\Psi}(\Psi)F_j(\Psi)\\\leq&-\left(\frac{\lambda}{2}-4dC_4^2\right)|X(t)|^2-b\big(1-2\gamma_1\big)\|w(t)\|^2\\&-c\sum_{l=1}^{r}\left(1-2\left|1-\frac{D_l}{D_0}\right|\gamma_2\right)\|\tilde{v}_l(t)\|^2-d\|\mu(t)\|^2-\left(b-4d-\frac{C_3^2C_L^2}{\lambda}\right)w(0,t)^2\\&-\left(c-2b\frac{1}{\gamma_1}D_0^2e^{2D_0C_L}C_4^2C_L^2-\frac{C_3^2C_L^2}{\lambda}\right)\sum_{j=1}^{r}P_{ij}(0,t)\tilde{v}_j(0,t)^2\end{aligned}$$

$$- c\sum_{j=1}^{r} P_{ij}(0,t)\sum_{l\neq j}\tilde{v}_l(0,t)^2 - d\mu(0,t)^2 + \frac{2cr\epsilon}{D_0}\frac{1}{\gamma_2}MV(t) \tag{58}$$

in which M is a positive constant which does not depend on ϵ and is defined in Lemma 3.

Observing that $D_0 \in [\underline{D}, \overline{D}]$, let us choose $(b, c, d, \gamma_1, \gamma_2) \in (\mathbb{R}_+^*)^5$ as follows

(a) $d < \frac{\lambda}{8C_4^2}$

(b) $b \geq 4d + \frac{C_3^2 C_L^2}{\lambda}$,

(c) $\gamma_1 \leq \frac{1}{4}$,

(d) $\gamma_2 \leq \frac{1}{4}\min\left\{(1 - \frac{D_1}{\overline{D}})^{-1}, (\frac{D_r}{\underline{D}} - 1)^{-1}\right\}$,

(e) $c \geq 2b\frac{1}{\gamma_1}\overline{D}^2 e^{2\overline{D}C_L} C_4^2 C_L^2 + \frac{C_3^2 C_L^2}{\lambda}$. □

From (58), one then obtains (54) with the well-defined positive constant $\eta = \min\left\{\frac{\lambda - 8dC_4^2}{2C_2}, \frac{1}{4\overline{D}}\right\}$, and the function

$$g(\epsilon) = \frac{2cr\epsilon}{D_0}\frac{1}{\gamma_2}M \tag{59}$$

which satisfies $\lim_{\epsilon\to 0} g(\epsilon) = 0$.

The proof of Theorem 1 now follows.

4.2 Proof of Theorem 1

Firstly, as $\lim_{\epsilon\to 0} g(\epsilon) = 0$, there exists $\epsilon^\star > 0$ such that $\eta - g(\epsilon) = \gamma > 0$ for $\epsilon < \epsilon^\star$. Therefore, according to Dynkin's formula [9, Theorem 5.1, p. 133], assuming temporally that $X(t) \in C$ for all $t \geq \overline{D}$, from (54), one obtains for $\epsilon < \epsilon^\star$

$$\begin{aligned} &\mathbb{E}_{[\overline{D},(\Psi,D)(\overline{D})]}(e^{\gamma t}V(t)) - e^{\gamma\overline{D}}V(\overline{D}) \\ &\qquad \leq \mathbb{E}_{[\overline{D},(\Psi,D)(\overline{D})]}\left(\int_{\overline{D}}^{t}\left[\gamma e^{\gamma s}V(s) + e^{\gamma s}LV(s)\right]ds\right) = 0 \end{aligned} \tag{60}$$

from which, using standard conditional expectation properties, one deduces

$$\mathbb{E}_{[0,(\Psi,D)(0)]}(e^{\gamma t}V(t)) \leq \mathbb{E}_{[0,(\Psi,D)(0)]}(e^{\gamma\overline{D}}V(\overline{D})) \tag{61}$$

Consequently, applying Gronwall's inequality to (60) (see [16])

$$\mathbb{E}_{[0,(\Psi,D)(0)]}(V(t)) \leq \mathbb{E}_{[0,(\Psi,D)(0)]}(V(\overline{D}))e^{-\gamma(t-\overline{D})} \tag{62}$$

Similarly to [19, Lemma 4], by applying Young's and Gronwall's inequalities to the backstepping transformation (22) and its inverse (46), there exist positive constants q_1 and q_2 such that for $\forall t \geq 0$, $q_1 V(t) \leq \Upsilon(t) \leq q_2 V(t)$. It thus follows that $\mathbb{E}_{[0,(\Upsilon(0),D(0))]}(\Upsilon(t)) \leq \frac{q_2}{q_1}\mathbb{E}_{[0,(\Upsilon(0),D(0))]}(\Upsilon(\overline{D}))e^{-\gamma(t-\overline{D})}$. To relate $\Upsilon(\overline{D})$ to $\Upsilon(0)$, we formulate the following lemma.

Lemma 5 *Consider the dynamics* (4) *satisfying Assumptions 1–3. There exists a constant R_0 such that the function Υ defined in* (17) *satisfies*

$$\Upsilon(t) \leq R_0 \Upsilon(0), \quad t \in [0, \overline{D}] \tag{63}$$

Proof Firstly, with Assumption 2, notice that

$$\begin{aligned} |X(t)| \leq & e^{C_L t}|X(0)| + C_L \int_0^t e^{C_L(t-s)}|U(s-D(s))|ds \\ \leq & e^{C_L t}|X(0)| + C_L \sum_{j=1}^{r} \int_0^t e^{C_L(t-s)}|U(s-D_j)|ds \end{aligned} \tag{64}$$

For $t \in [0, \overline{D}]$, with $U(t) = U_0(t)$ for $t \leq 0$, (64) then gives

$$|X(t)|^2 \leq N_1|X(0)|^2 + N_2 \int_{-\overline{D}}^{\min\{t-\underline{D},0\}} U_0(s)^2 ds + N_2 \int_{\min\{t-\underline{D},0\}}^{t-\underline{D}} U(s)^2 ds \tag{65}$$

with $N_1 = 2e^{2C_L\overline{D}}$ and $N_2 = rC_L^2 N_1$. Besides, using Assumption 3 and (44), the prediction-based control law satisfies the inequality

$$|U(t)| = |\kappa(\hat{P}(t))| \leq C_4 \left(e^{C_L D_0}|X(t)| + C_L \int_{t-D_0}^{t} e^{C_L(t-s)}|U(s)|ds \right) \tag{66}$$

from which one obtains

$$U(t)^2 \leq N_3|X(t)|^2 + N_4 \int_{t-D_0}^{t} U(s)^2 ds \tag{67}$$

with $N_3 = 2C_4^2 e^{2C_L D_0}$ and $N_4 = C_L^2 N_3$.

Replacing (67) into (65) and using again (65), one obtains

$$\begin{aligned} |X(t)|^2 \leq & N_1|X(0)|^2 + N_2 \int_{-\overline{D}}^{\min\{t-\underline{D},0\}} U_0(s)^2 ds + N_2 N_3 \int_{\min\{t-\underline{D},0\}}^{t-\underline{D}} X(s)^2 \\ & + N_2 N_4 D_0 \int_{\min\{t-\underline{D},0\}-D_0}^{t-\underline{D}} U(s)^2 ds \end{aligned} \tag{68}$$

$$\leq N_5|X(0)|^2 + N_6 \int_{-\overline{D}}^{t-\underline{D}} U(s)^2 ds \tag{69}$$

with

$$N_5 = N_1 \left(1 + N_2 N_3 (\overline{D} - \underline{D})\right) \tag{70}$$

$$N_6 = N_2 \left(1 + N_2 N_3 (\overline{D} - \underline{D}) + N_4 D_0\right) \tag{71}$$

Hence, with (67), it follows that

$$U(t)^2 \leq N_3 N_5 |X(0)|^2 + (N_3 N_6 + N_4) \int_{-\overline{D}}^{t} U(s)^2 ds \tag{72}$$

and, applying Gronwall's lemma, that

$$U(t)^2 \leq N_7 |X(0)|^2 + N_8 \int_{-\overline{D}}^{t} U_0(s)^2 ds \tag{73}$$

with $N_7 = N_3 N_5 e^{(N_3 N_6 + N_4)\overline{D}}$ and $N_8 = (N_3 N_6 + N_4) e^{(N_3 N_6 + N_4)\overline{D}}$. Consequently, it also holds

$$X(t)^2 \leq N_9 |X(0)|^2 + N_{10} \int_{-\overline{D}}^{t} U_0(s)^2 ds \tag{74}$$

with $N_9 = N_5 + N_6(\overline{D} - \underline{D}) N_7$ and $N_{10} = N_6(1 + (\overline{D} - \underline{D}) N_8)$. Therefore, as from (17), it holds

$$\Upsilon(t) = |X(t)|^2 + \int_{t-\overline{D}-D_0}^{0} U_0(s)^2 ds + \int_0^t U(s)^2 ds \tag{75}$$

the conclusion follows from (73) and (68). □

With Lemma 5, we can now conclude the proof of Theorem 1. Indeed, the function Υ thus satisfies $\mathbb{E}_{[0,(\Psi,D)(0)]}(\Upsilon(t)) \leq \frac{q_2}{q_1} R_0 \Upsilon(0) e^{-\gamma(t-\overline{D})}$, if $X(t) \in C$ for all $t \geq \overline{D}$.

Finally, let us denote $\sqrt{\rho_0^\star}$ such that $B(0, \sqrt{\rho_0^\star}) \subset C$ and observe that $|X(t)|^2 \leq \Upsilon(t)$ for all $t \geq 0$. Consequently, a sufficient condition for $X(t)$ belonging to C is $\Upsilon(t) \leq \rho_0^\star$ and the result follows with $\rho^\star = \frac{q_1}{q_2 R_0} \rho_0^\star$ and $R = \frac{q_2}{q_1} R_0 e^{\gamma \overline{D}}$. Theorem 1 is then proved.

5 Conclusion

In this chapter, we extended our previous work [18] to the case of nonlinear systems, using a controller based on a constant time horizon prediction to robustly compensate for stochastic input delays. A first direction of improvement of this work will be to relax the technical assumptions made in this chapter on the dynamics, in particular on the global lipschitzness of the vector field.

Besides, the sufficient condition we obtain for closed-loop stabilization bears on the range of the possible delay values, which should be small enough, and on the prediction horizon which should be close enough to this range. Hence, this condition does not distinguish between the delay probability distributions. It is thus likely to be somehow conservative. Relaxing this condition to involve the delay distribution into the stability analysis is an important direction of future work.

Similarly, to increase the closed-loop performances, other prediction horizons could be considered in the future, such as the current expected value of the delay.

Finally, our analysis methodology grounds on the probabilistic delay averaging approach [16], modeling the input delay as a Markov process with a finite number of states. Another important direction of future works is to extend this analysis methodology to the case where the delay takes values in a continuum.

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