Distributed and backstepping boundary controls to achieve IDA-PBC design

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Outline

1. Introduction
2. Boundary IDA-PBC
3. Tokamak example
4. Conclusion
**IDA-PBC control for infinite PH systems and issues**

[A. Macchelli, A. van der Schaft and C. Melchiorri] Port hamiltonian formulation of infinite dimensional systems.

i. modeling. Proc. 50th IEEE Conference on Decisions and Control (CDC), 2004


European Journal of Control, 2003
**Infinite dimensional PH systems**

**Definition**

General form of port-Hamiltonian system:

\[
\begin{align*}
\dot{x} &= [\mathcal{J}(x, t) - \mathcal{R}(x, t)] \frac{\partial \mathcal{H}}{\partial x} + g(x) u_1(x, t) \\
\begin{pmatrix} u_2 \\ y \end{pmatrix} &= Bx
\end{align*}
\]

- $x$ the energy state variables,
- $\mathcal{H}$ the Hamiltonian function, the total energy smooth function
- $u_1, u_2, y$ the inputs - outputs (port) variables, and $g(x)$ control mapping
- $\mathcal{J} = -\mathcal{J}^*$ skew-symmetric differential operator, $\mathcal{R} \geq 0$ self-adjoint dissipation operator, and $B$ boundary differential operator
**IDA-PBC for PHS**

**IDA-PBC theorem**

If we can find a feedback $u_{1,IDA}$ to drive the canonical system to the desired system

$$
\dot{x} = \left[ J(x) - R(x) \right] \frac{\partial H}{\partial x} (x) + g(x) u_1(t)
$$

then the closed-loop system is stable (locally) at $x_d = \text{arg min} \ (H_d)$.

- new interconnection $J_d(x) = J(x) + J_a(x)$, $J_d = -J_d^T$
- new damping $R_d(x) = R(x) + R_a(x)$, $R_d = R_d^T \geq 0$
- total energy $H_d(x) = H(x) + H_a(x)$

**Issues: Matching equation is (technically) difficult to solve!**

$$
\left[ J(x, t) - R(x, t) \right] \frac{\partial H}{\partial x} + g(x) u_1(t) = \left[ J_d(x, t) - R_d(x, t) \right] \frac{\partial H_d}{\partial x}
$$

**Solution**

- Matching equation **relaxation**
- **Boundary** (backstepping) control
Backstepping boundary IDA-PBC control for infinite PH systems

Matching equation relaxation

**Average IDA-PBC for PHS**

Average control law over the domain $\mathcal{Z} = [0; 1]$: 

$$ u_1(t) \int_0^1 g(x, t) \, dz = \int_0^1 [J_d(x, t) - R_d(x, t)] \frac{\partial H_d}{\partial x} \, dz $$ 

$$ - \int_0^1 [J(x, t) - R(x, t)] \frac{\partial H}{\partial x} \, dz $$

The closed loop system obtained with the average finite rank distributed control:

$$ \dot{x} = [J - R] \frac{\partial H}{\partial x} + gu_1 = [J_d - R_d] \frac{\partial H_d}{\partial x} + F(x) $$

where $F(x)$ is the matching error.
Proposition

Consider the Volterra (or backstepping) state space transformation with the kernel \( k(z, y) \):

\[
\mathbf{w} = \mathbf{x} - \int_0^z k(z, y) \mathbf{x}(y, t) \, dy
\]

if there exists a corresponding kernel \( k(z, y) \) satisfying:

\[
\begin{align*}
F(x, z) &- \int_0^z k(z, y) [\mathcal{J}_d - \mathcal{R}_d] Q_d (y) \mathbf{x}(y) \, dy \\
- \int_0^z k(z, y) F(x, y) \, dy + [\mathcal{J}_d - \mathcal{R}_d] Q_d \int_0^z k(z, y) \mathbf{x}(y, t) \, dy &= 0
\end{align*}
\]

then the closed-loop system is asymptotically stable in the sense of Lyapunov:

\[
\begin{align*}
\begin{pmatrix}
\dot{x} \\
\dot{u}'_2 \\
\dot{y}'_2 \\
\mathbb{H}_d
\end{pmatrix}
&= \begin{pmatrix}
\mathcal{J}_d - \mathcal{R}_d \\
0 \\
0 \\
\frac{1}{2} \int_\mathcal{Z} x^T Q_d x
\end{pmatrix} \\
\mathbb{B}_d \mathbf{x}
\end{align*}
\]

\[
\begin{align*}
\dot{\mathbf{w}}
&= \begin{pmatrix}
\ddot{u}_2 \\
\ddot{y}_2 \\
\mathbb{H}_{d\mathbf{w}}
\end{pmatrix} \\
&= \begin{pmatrix}
\mathcal{J}_d - \mathcal{R}_d \\
0 \\
\frac{1}{2} \int_\mathcal{Z} \mathbf{w}^T Q_d \mathbf{w}
\end{pmatrix} \\
\mathbb{B}_d \mathbf{w}
\end{align*}
\]

with the boundary control \( u_2 = \mathbb{B}_d \mathbf{x} = \mathbb{B}_d (\mathbf{w} + \int_0^z k(z, y) \mathbf{x}(y, t) \, dy) \)
Solving new matching equation

**Parameter conditions**

Matching condition:

\[
\frac{\partial}{\partial y} F(x, y) + \frac{\partial}{\partial y} k(z, y) iQ_d(y) x(y) + iQ_d(z) \frac{\partial}{\partial z} k(z, y) x(y) + k(z, y) (-F(x, y) + i\frac{\partial}{\partial z} Q_d(z) x(y)) + R_d(y) Q_d(y) x(y) - R_d(z) Q_d(z) x(y)) = 0
\]  

(1)

Boundary condition holds: \[ F(x(0)) + k(z, 0) iQ_d(0) x(0) = 0 \]

**k (z, y) propositions**

- if \( \frac{\partial}{\partial y} F(x, y) = 0 \) then equation (1) has analytical solution by using the variable separation method

- if \( k(z, y) = k(y) \)

\[
\begin{cases}
\mathcal{A}(y) \frac{\partial}{\partial y} k(y) + \mathcal{D}(y) k(y) + \mathcal{C}(y) = 0 \\
i \frac{\partial}{\partial z} Q_d(z) - R_d(z) Q_d(z) = 0
\end{cases} \iff \begin{cases} k(y) = e^{-\alpha(y)} \left( \int \beta(y) e^{-\alpha(y)} dy + \kappa \right) \\
i \frac{\partial}{\partial z} Q_d(z) - R_d(z) Q_d(z) = 0
\end{cases}
\]

with \( \alpha(y) = \int \frac{D(y)}{A(y)} dy \), \( \beta(y) = -\frac{C(y)}{A(y)} \) and where \( \kappa \) is a constant of integration depending on the initial condition.
Control of the resistive diffusion of the plasma poloidal magnetic flux


Tokamak facility

Figure: ITER Tokamak model
**Figure:** Schematic view of a Tokamak reactor from 1950 by Sakharov, Tamm and Artsimovich

**Figure:** Tokamak magnetic field with safety factor $q = \frac{B_\phi}{B_\theta}$
Control objectives

**Objective**
Reach a pre-defined safety factor \( (q_0 \text{ and } I_p \equiv q_1^{-1}) \) by regulating:

- the distributed non inductive current-drive heating source \( J_{ext} \)
- the boundary loop voltage \( V_{loop} \) (the voltage at the boundary of the plasma)

**Issue**
A differential distributed system with:

- non linear time and state dependence parameters (resistivity coefficient, bootstrap current,...)
- non-linearity and technological constraint on \( J_{ext} \) (finite rank distributed control)

**Solution**
- Equilibrium generation: non-linear feedforward control
- Boundary (backstepping) IDA-PBC feedback control
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The PCH model for the resistive diffusion equation

Electromagnetic (EM) 1D model

The Maxwell-Faraday and Maxwell-Ampère equations in PH form

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \begin{array}{c} \frac{D}{B} \\ x \end{array} \right) &= \left[ \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \end{array} \right] \frac{\partial}{\partial z} - \left( \begin{array}{ccc} \frac{C_3(z)}{\eta} & 0 & 0 \\ 0 & \frac{C_2(z)}{\mu} & 0 \end{array} \right) \left( \begin{array}{c} \frac{1}{\epsilon C_3(z)} \\ 0 \\ \frac{1}{\mu} \end{array} \right) \left( \begin{array}{c} \frac{D}{B} \end{array} \right) + \left( \begin{array}{c} -\overline{J}_{\text{ext}} - \overline{J}_{\text{bs}} \\ 0 \end{array} \right) \\
\mathcal{B} \left( \frac{\partial}{\partial x} \mathcal{H} \right) &= \left( \begin{array}{cc} f_\partial \\ e_\partial \end{array} \right) \bigg|_{z=1} = \left( \begin{array}{c} V_{\text{loop}} \\ I_p \end{array} \right) \\
\mathcal{H} &= \frac{1}{2} \int_{\mathcal{Z}} x^T Q x, \quad \mathcal{Z} = [0, 1]
\end{align*}
\]

where \( C_2, C_3 \) are the toric coordinate coefficients and the total current

\[
J = J_\Omega + J_{\text{ext}} + J_{\text{bs}}
\]

with the ohmic current \( J_\Omega = \frac{C_3(z)}{\eta} \left( \frac{1}{\epsilon C_3(z)} \overline{D} \right) \) (resistivity \( \eta \)), external current \( J_{\text{ext}} = f_{\text{ext}}(P_{\text{ext}}, B) P_{\text{ext}} \) and bootstrap current \( J_{\text{bs}} \).
Test case

- RAPTOR simulator with the configuration of TCV (Tokamak of Configuration Variable) is used
- The controller starts at $t = 0.2s$, the **reference q profile** is set as:
  \[
  \begin{align*}
  0.2s \leq t \leq 0.8s, & \quad q_{ref} = (0.85, 1, 6.3) \\
  t > 0.8s, & \quad q_{ref} = (1.05, 1.1, 7.5)
  \end{align*}
  \]

**Control description**

Control tuning

- IDA-PBC parameters: $J_d = J$, and
  \[
  R_d = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} > 0
  \]
- Boundary damping $\tilde{u}_2 = -K_p\tilde{y}_2$, $K_p > 0$
Simulation results

Figure: Simulation results of boundary IDA-PBC control
- without integrator (left)
- with integrator (right)
Conclusions

The following results were obtained:

- IDA-PBC feedback control has been designed for the infinite dimensional PH systems
- Backstepping boundary control is employed to compensate the matching error
- The proposed control law is tested by RAPTOR code, tokamak simulator base on TCV facility (Lausanne, Switzerland) configuration

First nuclear reactions in the new ITER reactor around 2030 ...
THANK YOU FOR YOUR ATTENTION