Abstract
Automation has the potential to improve efficiency, precision, and safety of pressure and flow control during underbalanced drilling (UBD). In addition, advanced estimation theory can be used to extract more information from existing measurements to increase knowledge of the downhole conditions during operation.

An essential part of an advanced (model based) pressure control system is the hydraulic model. Even with high-bandwidth distributed downhole measurements, a calibrated hydraulic model is required to ensure robustness, e.g. to sensor loss, and obtain real time estimates of unmeasured quantities and reservoir characteristics.

In UBD operations, in contrast to Managed-Pressure-Drilling (MPD) operations, the flow in the annulus is inherently a multiphase gas liquid flow, which severely complicates the modelling. Much effort has been put into developing multiphase flow models, however, to date; most of these are very complex, requires extensive configuration and are not well suited for real-time applications. Consequently, a major gap with respect to automation of UBD is the lack of a fit-for-purpose model capable of reproducing the main dynamics of the multiphase flow in the annulus, while being sufficiently robust and suitable for real-time applications.

In this work, we present recent advances on the development of a simplified fit-for-purpose model of the distributed gas-liquid dynamics, suited for advanced control of UBD operations. Using an automated calibration procedure, the model is shown to retain accuracy in a realistic case study. It is also shown that its reduced complexity enables real time coupling with measurements to obtain estimates of unmeasured quantities such as gas distribution and perform reservoir characterization.

Introduction
As the end of ‘easy oil’ is reached, and the world’s reservoirs start depleting, increasingly challenging wells are considered for drilling. This results in progressively more demanding drilling operations, which is the source of the strong drive for automation currently seen in many aspects of drilling.

Following the demand of the drilling industry, advanced, high-fidelity simulators of the drilling process have been developed. Applications of these include training of drilling personnel and decision support. At the same time, automated control systems for controlling various aspects of the drilling process have also been developed and are gradually being accepted by the industry. But, in contrast to what is offered by the existing high-fidelity simulators, simple reliable models are desired when designing automated control solutions. In the case of Managed Pressure Drilling (MPD), model-based control and estimation techniques have been developed based on a simple hydraulic model of the pressure and flow dynamics of the drilling system for several applications: downhole pressure estimation (Stamnes, Aamo, and Kaasa 2011b; Stamnes, Aamo, and Kaasa 2011a); downhole pressure control (John-Morten 2011); downhole pressure control with kick detection and handling (Stamnes, Aamo, and Kaasa 2011c), and; attenuation of downhole pressure oscillations due to heave (Landet, Pavlov, and Aamo 2013).

For conventional overbalanced drilling and MPD the hydraulic dynamics of the system are one-phase, but for Under-Balanced Drilling (UBD), where oil and gas are produced while drilling, the introduction of a gaseous phase makes modelling of the flow significantly more complicated. The simple models used to model one-phase flow are unsuitable for the gas-liquid two-phase dynamics. As of yet, there has been little focus on reducing the complexity of hydraulic UBD
models. In this paper we argue that simpler models of the UBD hydraulics are preferable to enable the design of robust and efficient automatic control algorithms for UBD.

An accurate model of a drilling operation requires careful calibration to the well in question and several papers have been published on the topic of calibration of hydraulic models of drilling (R. J. Lorentzen, Nævdal, and Lage 2003; Vefring et al. 2003; R. Lorentzen, Fjelde, Johnny, et al. 2001; Gravdal et al. 2005). Due to the complexity of these models, one has to resort to similarly complicated estimation schemes to perform such calibration. These schemes can in some cases be unreliable and give unexpected results, which make them unsuitable for online integration with a safety critical automated functions (see (Kaasa et al. 2011) for a discussion of the use of adaptive algorithms in safety critical applications).

To overcome this obstacle, this paper proposes a two-tiered architecture for combining the accuracy of a complex estimation technique with the predictability of a simpler one.

Existing results on state and parameter estimation using the drift-flux model
An early result on tuning of model parameters of a multiphase model of drilling hydraulics was presented in (R. Lorentzen, Fjelde, Johnny, et al. 2001). The paper presents a 3 state mechanistic drift flux model with constant slip parameters. The slip and friction parameters are tuned in an optimization procedure where the model predictions are fitted to the following measured data: pump pressure, bottomhole pressure and liquid and gas rate at the outlet. The procedure is tested against full scale experimental data, and it is concluded to produce reliable predictions.
A continuation of this work is presented in (R. Lorentzen, Fjelde, Jonny, et al. 2001) and (R. J. Lorentzen, Nævdal, and Lage 2003) where the parameters are tuned using an Ensemble Kalman Filter, see (Evensen 2003; Aanonsen et al. 2009). In these papers a slightly more complex model is used since the slip and friction parameters are state dependent. The numbers of states in these studies are in the order of a few hundreds, and the number of ensemble members is set to 100. This means that for every time step, 100 simulations of the model must be made. In (Vefring et al. 2003) it was also noted that due to the stochastic element in the ensemble Kalman filter, the model error has to be chosen somewhat lower than desired to ensure stable simulations.

The ensemble Kalman filter has also been used for reservoir characterization in coupling a simple reservoir model to the drift-flux model (Vefring et al. 2003; Vefring et al. 2002) and for updating permeability fields for near-well reservoirs (Geir et al. 2003).

In (Bloemen et al. 2006) the extended Kalman filter is used on a 3 state drift-flux model with slip and friction parameters independent of the states, but with two different flow regimes. The estimation algorithm is tested against experimental data and proves capable of estimating gas velocity and void fraction. It is noted that estimating both of the slip parameters simultaneously results in unrealistic values. Similar problems are noted in (R. J. Lorentzen, Nævdal, and Lage 2003) and are due to insufficient excitation of the dynamics.

The drift-flux model
The model consists of the mass conservation laws of the gas and the liquid separately, and a combined momentum equation. The mud, oil and water are lumped into one single liquid phase. For \( k = L, G, m \), denoting liquid, gas or mixture, we denote \( \alpha_k \) the volume fractions, \( \rho_k \) the densities, \( v_k \) the superficial velocities, and \( P \) the pressure. All of these variables are functions of time and space. We denote \( t \geq 0 \) the time variable, and \( s \in [0, L] \) the space variable, corresponding to a curvilinear abscissa with \( s = 0 \) at the bottom hole and \( s = L \) at the outlet choke position. The equations are as follows

\[
\begin{align*}
\frac{\partial \alpha_L \rho_L}{\partial t} + \frac{\partial \alpha_L \rho_L v_L}{\partial s} &= 0 \\
\frac{\partial \alpha_G \rho_G}{\partial t} + \frac{\partial \alpha_G \rho_G v_G}{\partial s} &= 0 \\
\frac{\partial \alpha_L \rho_L v_L}{\partial t} + \frac{\partial \alpha_G \rho_G v_G}{\partial s} + P + \alpha_G \rho_G v_G^2 + \alpha_L \rho_L v_L^2 &= -\rho_m g \sin \phi(s) - \frac{2f \rho_m v_m |v_m|}{D} 
\end{align*}
\]

In the momentum equation (3), the term \( \rho_m g \sin \phi(s) \) represents the gravitational source term, while \( \frac{2f \rho_m v_m |v_m|}{D} \) accounts for frictional losses. The parameters are detailed in Table 2. The mixtures are given as

\[
\rho_m = \alpha_G \rho_G + \alpha_L \rho_L, \quad v_m = \alpha_G v_G + \alpha_L v_L
\]

Along with these distributed equations, algebraic relations are needed to close the system.

\[
\alpha_L + \alpha_G = 1 \quad V_G = C_G v_m + v_{io}
\]

\[
\rho_G = Z_G R_G T P \quad \rho_L = \text{const.}
\]

Where \( Z_G, R_G, T \) are the gas compression factor, specific gas constant and temperature respectively, and \( C_G, v_{io} \) are the slip parameters giving the slip between the velocity of the gas and liquid phase. For relatively homogeneous operating conditions, using constant slip parameters is satisfactory. If the model is required to be able to simulate transitions to one-phase flow or liquid backflow the slip parameters must be state dependent such that \( C_G, v_{io} \rightarrow (1,0) \) when the liquid vanishes and \( C_G \rightarrow 1 \) when the gas vanishes (S Evje 2011; Shi et al. 2005). Further, for the case of vanishing gas, the liquid can no longer be assumed incompressible, i.e. use a relation of the form \( \rho_L = \rho_{L,0} + \frac{P-P_0}{a_L} \). These modifications further complicate the model. For suggestions on how to implement the model see (Steinar Evje and Fjelde 2002; Fjelde and Rommetveit 2003).

Boundary Conditions
Boundary conditions on the left (downhole) boundary are given by the mass-rates of gas and liquid injected from the drilling rig and flowing in from the reservoir.

\[
A \alpha_L(0) \rho_L(0) v_L(0) = W_{L,res} + W_{L,inj},
\]

\[
A \alpha_G(0) \rho_G(0) v_G(0) = W_{G,res} + W_{G,inj},
\]
The injection mass-rates of gas and liquid, \( W_{G,\text{inj}}, W_{L,\text{inj}} \), are specified by the driller and can, within some constraints, be considered as manipulated variables. The inflow from the reservoir is dependent on the pressure on the left boundary, usually given by a Vogel-Type Inflow performance relationship (IPR) (Wiggins, Russell, and Jennings 1996), but within the operational range of a typical UBD operation an affine approximation should suffice, i.e.

\[
W_{L,\text{res}} = k_L \max(P(0) - P_{\text{res}}, 0) \tag{9}
\]

\[
W_{G,\text{res}} = k_G \max(P(0) - P_{\text{res}}, 0) \tag{10}
\]

Here, \( P_{\text{res}} \) is the reservoir pore pressure and \( k_G, k_L \) are the production index (PI) of the gas and liquid respectively.

The topside boundary condition is given by a choke equation relating topside pressure to mass flow rates (Murdock 1962)

\[
A\sigma_L(L)\rho_L(L)v_L(L) + A\sigma_G(L)\rho_G(L)v_G(L) = C_u(Z) \sqrt{\frac{P(L) - P_S}{x_L}} + \frac{x_G}{\sqrt{\rho_L}} + \frac{Y^2}{\sqrt{P_G}} \tag{11}
\]

where \( x_{LG} \) denotes the mass fraction of liquid and gas, \( C_u(Z) \) the choke opening given by the manipulated variable, \( Z \), and \( Y \) is a correction factor for gas flow. Changing the choke opening is the primary control actuation for the drilling system.

**Proposed architecture: two tier estimation**

Real-time estimation becomes increasingly more difficult when the number of parameters that are estimated simultaneously increases. A discrepancy between the predicted and measured behavior of the system can in many cases be explained by several different combinations of altering the estimated parameters. In these cases, any given change in the estimated parameters that accommodates the discrepancy will lead to a convergence between the model prediction and the measurements, but with the risk of the estimated parameters not being correctly identified. This problem is especially difficult to handle when dealing with complicated, high order systems such as the DFM, and it has been noted as such in several of the previously cited investigations. The result may be unrealistic estimates and unpredictable behavior, something which should be avoided in safety critical operations.

The approach taken in this paper handles this by dividing the set of unknown or uncertain parameters in two and estimating them in separate estimation loops. It is designed to take the following into account:

- Empirical correlations that are used in the model to ensure good quantitative performance, but whose values are not of interest in their own right can be estimated simultaneously with others. Examples of these are the slip parameters.
- Parameters that are changing very slowly and which are known to be more or less constant over an extended period of time can be updated infrequently.
- Rapidly changing parameters that have significant qualitative impact on the behavior of the well, or are important for decision making purposes, should be estimated in real-time in a robust manner. A typical candidate for this kind of parameter is the PI.

To achieve robust, real-time estimation of the key, rapidly changing parameters, the number of parameters that are estimated simultaneously should be kept to a minimum.

The proposed model based estimation architecture is as outlined in Figure 1. It consists of an online estimation loop which estimates the model states and changes in the Production Index in real time, and an offline calibration loop which can be used to update the parameters of the online loop, e.g. on a daily basis.

Since the offline calibration procedure is not “in the control structure loop” it should be able to utilize more complicated estimation procedures to estimate multiple parameters without compromising the predictability and transparency of safety critical functions. Further, since it is only required to be run scarcely, it can rely on steady-state data. If the offline estimator crashes or outputs bad data, it can simply be discarded, and the process continue with the old calibrated parameters in the online estimator.

**Offline calibration**

Since the drift-flux model presented here is of a reduced complexity compared to the high fidelity models, certain parameters need to be adjusted in order to quantitatively reproduce the behavior of a given system. This can be done by formulating the fitting of the model response to measured or simulated data as an optimization problem, similar to what is done in (R. Lorentzen, Fjelde, Johnny, et al. 2001).

**Calibrated Parameters**

The number of calibrated parameters may vary according to the considered case, depending on how well known the parameters are in the given scenario. With the current formulation the following variables are calibrated:

- The slip law parameters \( C_0, v_0 \). In high fidelity models these are found from sophisticated relations based on flow regime predictions not included in the simplified model.
- The friction factor, \( f \), again, is dependent on flow regime.
In the ideal gas law \( P = Z_g \rho_g R_g T \), the empirical coefficient \( Z_g \) has been added and should be tuned. In addition it may be beneficial to tune the choke opening \( C_o \) and the gas correction factor \( Y \) in (11).

The calibration algorithm is a simple optimization routine. Given a set of \( n \) measurements at steady state \( z_i, i = 1, \ldots, n \), and a vector of \( p \) parameters to calibrate \( \theta \in \mathbb{R}^p \), we consider the following minimization problem

\[
\theta^* = \arg\min_{\theta \in \Omega} \begin{bmatrix} \hat{z}_1(\theta) - z_1 \\ \vdots \\ \hat{z}_n(\theta) - z_n \end{bmatrix}^T M \begin{bmatrix} \hat{z}_1(\theta) - z_1 \\ \vdots \\ \hat{z}_n(\theta) - z_n \end{bmatrix},
\]

where \( \hat{z}_i(\theta) \) is the output of the model corresponding to the measurement \( z_i \) for the vector of parameters \( \theta \). \( M \) is a weighting matrix used to normalize the outputs and \( \Omega \) is the set of allowable values which \( \theta \) can take. The vector of parameters \( \theta^* \) which solves (12) is found using a gradient based optimization procedure in MATLAB.

This approach has proved to be a simple yet effective way of calibrating the simple DFM. A particularly attractive feature is that it works very well even when the number of available measurements is low. In the following, we use only measurements of WHP, BHCP and the produced gas mass-rate, \( W_{\text{g, res}} \), at two steady-state points that have been encountered previously in the operation. It is a fair assumption that this data is readily available as the amount of produced gas can be more accurately measured after separation. Steady state measurements are also more reliable than measurements from transients as an average over a set of data points can be used. Further, using steady-state points also allows for fast calibration, as transient simulation of the DFM is not required; typical runtime of the procedure is 60 seconds when running in MATLAB using 2.60GHz Processor with 8 GB RAM.

Calibrating the four parameters \( \theta = [C_o, v_w, Z_g, f]^T \) gives good results when comparing the simple DFM with OLGA simulations, as is illustrated in simulations given in the following sections. Having calibrated the model, it can be used to generate the operating envelope.

**Online estimation of production index**

When drilling ahead, most parameters describing the well are constant or changing very slowly. Hence real-time estimation, or frequent updates, of these parameters is not necessary. Instead they can be updated using the offline calibration when good steady state data become available. Other parameters of the system can rapidly change in a manner which have significant impact on the operating envelope and which may require some counter action to keep the system within operating constraints. These parameters should be monitored online such that when a change is detected the operating envelope can be updated accordingly.

Parameters which fall in the last category will vary from well to well, but a typical candidate is the Production Index (given by \( k_G \) in the drift-flux model). The production index can change significantly over a few meters of drilling when encountering a formation fracture, and large variations of its value have major impact on WHP, BHCP as well as production rates, all of which may be subject to constraints.

There are several techniques which allows for online state and parameter estimation through injection of measurements. In this paper the Extended Kalman Filter is used (see (Bloemen et al. 2006) for implementation details of the Extended Kalman Filter on a drift-flux model). In the implementation the state vector is augmented with the estimated parameter \( k_G \). Continuing with our minimalist approach to instrumentation, we assume only BHCP and WHP to be measured in the following simulations.

**Case study**

The proposed architecture and procedure outlined in the previous sectioned is now illustrated with a case study. We consider the underbalanced drilling of a 2530 meter long vertical well drilled into 1.1 SG, dry gas reservoir (i.e. \( W_{\text{g, res}} = k_L = 0 \)). The data is generated synthetically by the dynamic multiphase flow simulator OLGA.

**Operating envelope**

In the first two hours of the simulation, the well is allowed to settle to a steady-state at the two choke openings of 10% and 7%. The WHP and BHCP at these two points are then used to carry out the offline calibration procedure described above, with the PI assumed to be initially known. Having calibrated the model, the model is used to compute the operating envelope for this well under the current operating conditions.

(Graham and Culen 2004) have proposed an alternative operating envelope which has several benefits to the conventional one. In particular, by not assuming the WHP to be constant, it allows one to include the backpressure choke to find the operating points. This is a very attractive feature in control-oriented modeling as opening and closing the choke is the main form of actuation to manipulate the well dynamics.

The operating envelope computed with the calibrated model is showed in Figure 5. The blue curve is found by specifying a BHCP, using the Production Index to find produced gas, and then solving the equations of the DFM at steady-state to find the corresponding WHP. This procedure is repeated for the whole range of BHCPs. Since the amount of gas and liquid mass flowing into the well for a given BHCP is known, we can use the choke equation of the backpressure choke to calculate the
required WHP for a given choke opening, \( Z \). Hence the steady state corresponding to a given choke opening is given by the intersection between the ‘WHP’ and ‘Required WHP’ curves. This figure clearly shows how the system can be moved along the WHP curve by closing and opening the choke, and gives information to the operator on how the well will respond to further changes in choke opening. This information can also be used to calculate production rates as discussed in (Graham and Culen 2004).

**Online estimation**

The calibrated model is used by the online estimation algorithm to estimate the distributed unmeasured states and the PI, using only the measured WHP and BHCP. Estimated vs actual values of the gas mass rate and gas and liquid velocities at the wellhead is shown in Figure 4. These estimates can be used to calculate whether there is sufficient velocity for hole-cleaning, and will also track the dynamics of transient events in real-time which can be used by an automated control algorithm or for decision support by the driller.

After 3.5 hours there is a sharp increase in the Production Index from 0.072 kg/s/bar to 0.12 kg/s/bar. The online estimation algorithm detects the discrepancy between estimated and measured WHP and BHCP and adapts the PI accordingly, ending up with 6.4% error at hour 6 (shown in Figure 2). Although there is a significant discrepancy between the estimated and actual PI after such a rapid change, it is clear that the estimation algorithm is able to detect that a large change has occurred and the estimated PI after 6 hours is good enough to generate an updated operating envelope, shown in Figure 6. This data is available in real-time and can be immediately put to use to manipulate the process to accommodate the change in operating conditions. Since the increase in PI has caused an increase in WHP, we may decide that it is too high and should be decreased. Normally this would be done by opening the choke, but in this case we want to limit our gas production to 4 kg/s. The updated operating envelop predicts that both can be achieved by closing the choke further. This is confirmed by the simulations when the choke is closed to 6% at 6 hours, and further to 5.5% after 8 hours. On both occasions the well settles to a new steady-state at with a lower WHP after a short transient.

**Conclusion**

We have presented a novel approach for control oriented modeling and estimation of multiphase flow during Underbalanced Operations. The approach is based on a simplified, yet representative, fit-for-purpose model for gas-liquid two-phase flow. The key feature of the parameter identification method is to separate slowly varying and empirical parameters from the key, and potentially more rapidly changing, parameters such as the Production Index. This enables the design of a scheme with two time constants: most of the parameters are updated scarcely, while distributed states and critical parameters, that need to be closely monitored, are estimated in real-time. The offline calibration loop can also be used to re-evaluate the operating envelope in real-time making it a tool for continuous decision support under changing operating conditions. The promising results advocate for the development of such simple drift-flux models that enable the design of algorithms from control theory.

**References**


Figure 1: Process schematic of the proposed architecture.
Figure 2: Estimating PI using only pressure measurements.

Figure 3: Measured WHP and BHCP used to estimate PI.
Figure 4: Unmeasured OLGA data and estimated gas mass rate and gas and liquid velocities at the wellhead.

Figure 5: Operating envelope showing operating points for the choke openings: $Z=10\%$ and $Z=8\%$. 
Figure 6: Updated operating envelope after the increase in the production index.

<table>
<thead>
<tr>
<th>Time</th>
<th>0-1h</th>
<th>1-2h</th>
<th>2-6h</th>
<th>6-8h</th>
<th>8-10h</th>
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<tr>
<td>Choke Opening</td>
<td>10%</td>
<td>8%</td>
<td>7%</td>
<td>6%</td>
<td>5.5%</td>
</tr>
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Table 1: Showing the choke openings used in the simulation.