CONTROL-ORIENTED DRIFT-FLUX MODELING OF SINGLE AND TWO-PHASE FLOW FOR DRILLING

Ulf Jakob F. Aarsnes
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Trondheim 7491
Norway
Email: ulf.jakob.aarsnes@itk.ntnu.no

Florent Di Meglio
Centre Automatique et Systèmes
MINES ParisTech
Paris 75006
France

Steinar Evje
Department of Petroleum Engineering
University of Stavanger
Stavanger 4036
Norway

Ole Morten Aamo
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Trondheim 7491
Norway

ABSTRACT

We present a simplified drift-flux model for gas-liquid flow in pipes. The model is able to handle single and two-phase flow thanks to a particular choice of empirical slip law. A presented implicit numerical scheme can be used to rapidly solve the equations with good accuracy. Besides, it remains simple enough to be amenable to mathematical and control-oriented analysis. In particular, we present an analysis of the steady-states of the model that yields important considerations for drilling practitioners. This includes the identification of 4 distinct operating regimes of the system, and a discussion on the occurrence of slugging in underbalanced drilling.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>G, L</td>
<td>Indices denoting gas and liquid</td>
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<tr>
<td>α</td>
<td>Volume fraction</td>
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<tr>
<td>ρ</td>
<td>Density</td>
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<td>A</td>
<td>Area of flow</td>
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<td>c_L</td>
<td>Velocity of sound, liquid</td>
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<td>c_G(T)</td>
<td>Velocity of sound, gas</td>
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<tr>
<td>K</td>
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<td>S</td>
<td>Slip parameter</td>
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<td>C_v</td>
<td>Choke constant</td>
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<td>D</td>
<td>Hydraulic diameter</td>
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<td>g</td>
<td>Gravity constant</td>
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<td>f</td>
<td>Friction factor</td>
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<tr>
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<td>Liquid inflow parameter</td>
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<tr>
<td>m</td>
<td>Liquid mass variable</td>
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<td>n</td>
<td>Gas mass variable</td>
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<tr>
<td>P_res</td>
<td>Reservoir pressure</td>
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<tr>
<td>P_s</td>
<td>Separator pressure</td>
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<tr>
<td>p_0</td>
<td>Reference liquid pressure</td>
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<tr>
<td>T(s)</td>
<td>Temperature</td>
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* Address all correspondence to this author.
1 INTRODUCTION

The process of drilling for hydrocarbons consists in creating a borehole, sometimes extending several thousand meters into the ground, until it reaches an oil or gas reservoir. There is an increasing drive for automation in many aspects of drilling, to increase efficiency and reduce the inherent risk of the operation. As the world reaches the end of ‘easy oil’ and the world’s reservoirs start depleting, increasingly challenging wells are considered for drilling.

Following the demand of the drilling industry, advanced, high-fidelity simulators of the drilling process have been developed. Applications of these include training of drilling personnel and real-time decision support. At the same time, automated control systems for controlling various aspects of the drilling process have been developed and are gradually being accepted by the industry. In particular, model-based estimation and control strategies exist for Managed Pressure Drilling (MPD) [1, 2]. These are based on a simple hydraulic model of the pressure and flow dynamics of the drilling system. Indeed, in conventional over-balanced drilling and MPD, the borehole pressure is maintained above the reservoir pressure by circulating liquid (usually mud or water) into the well. Conversely, in Under-Balanced Drilling (UBD), oil and gas are produced while drilling. The introduction of a gaseous phase makes modelling the fluid dynamics significantly more complicated. The main contribution of this paper is to present a simple and reliable model for two-phase flow, amenable to the development of model-based control and estimation techniques for UBD.

In [1] it is stated that the goal of that paper is to ‘attract researchers from the control community to problems within drilling automation’. We submit to that goal but will focus on Under-Balanced Drilling and the unique challenges posed by the particulars of this operation, such as the coupling with the reservoir and the complex dynamics of a distributed multiphase flow system.

The layout of the paper is as follows: In section 2 we explain the basics of the underbalanced drilling process. In section 3 we discuss how this process can be modelled using the Drift Flux model which can be solved with an implicit numerical scheme as suggested in Section 4. Section 5 suggests a simple procedure to match this model to more detailed models or experimental data. In Section 6, we use the model to derive a novel analysis of the steady-states and dynamics encountered in underbalanced drilling operations. Finally, in Section 7, we propose some interesting control and estimation problems from UBD, one of which is a novel idea derived from the preceding analysis.

2 UNDERBALANCED DRILLING

Consider the drilling system schematically depicted in Fig. 1. It consists of a circulation system: The drilling fluid is pumped into the top of the drill string, and circulated out at the bottom at the drilling bit. It then flows up through the annular section around the drill string, transporting the drilled formation particles, called cuttings, and any produced fluids before it is injected into the drill string again.

One of the main concerns of the driller is to control the pressure in the lower sections of the well. Too high pressure results in low ROP (rate of penetration), loss of drilling fluid (which in some cases can be excessively expensive) and degrade the future production rate of the well. Low pressure in the well causes influx of gas and/or other reservoir fluids, and excessively low pressure may result in wellbore collapse and a stuck drill string. The biggest danger in drilling is that of a blowout: this happens when the gas influx goes unnoticed and starts to displace the much heavier drilling fluid. Hence this causes a further reduction in downhole pressure and yet more gas influx, i.e. a positive feedback loop which can spiral out of control. For these reasons it is important to keep the downhole pressure within a specified pressure window at all times, and to closely monitor the dynamics in the well.

In UBD operations the downhole pressure is deliberately
kept below the reservoir pore pressure\(^1\), causing continuous inflow of produced fluids from the reservoir. The reservoir inflow is related to the downhole pressure by the Production Index (PI) and pore pressure. It is highly desirable to estimate the Production Index and pore pressure of the reservoir since these parameters have a large influence on the dynamics of the system and play an important role in determining the systems operational constraints.

Before starting an UBD operation, initial estimates of the reservoir properties are combined with various requirements posed by equipment constraints and operational considerations to develop the so-called operating envelope. The operating envelope defines the allowable range of the controlled and manipulated variables in the process. Staying within the operating envelope is paramount to minimizing risk and optimizing drilling performance. To achieve this, a good understanding of the multiphase flow and pressure dynamics of the coupled well-reservoir system is required. This has led to the development of several advanced, high-fidelity simulators of both over- and under-balanced drilling. The application of these, however, have largely been limited to training for and planning of operations.

### 3 MODELLING

#### Choosing Model Fidelity

Underbalanced drilling operations is an inherently multiphase process. Drilling mud, and in some cases injection gas, is pumped into the well through the drill string. In addition, both oil, gas and water may be produced from the reservoir, and cuttings from the drilled formation constitutes a phase of solid particles. In the literature, the most used model of multiphase flow in drilling is the Drift Flux Model (DFM) which requires one distributed state for each phase to model the mass balance while the momentum of the mixture is lumped into one equation (a seminal report on the DFM is [3], relevant references for the use of DFM in drilling is [4, 5]). Including an energy equation, this results in a total of \(n+2\) distributed states, where \(n\) is the number of phases. The resulting complexity makes these models ill suited for control and estimator design. Instead it has been proposed that a good compromise with sufficient accuracy is achieved by using a static momentum balance which reduces the number of distributed states down to 3. Further simplification can be achieved by using a static momentum balance which reduces the number of dynamic distributed states down to 2. The downside of this reduction is that the pressure waves propagate instantaneously, reducing the model accuracy during fast transients, [8, 9].

Low order lumped (i.e. ODE) models have been suggested for one-phase drilling [1, 10, 11], but these are unsuitable for two-phase flow. Low order lumped models have seen successful use in control of severe slugging in two-phase production risers [12–15], and has also been considered for application in UBD [16], but are not able to reproduce the operating envelope which is essential for UBD operations.

The rationale behind reducing model complexity is to enable estimation and control techniques. Besides, the simplification should give insight into the dynamics of the model. Any reduction in complexity from that of the high fidelity models, however, will require calibration or compensation by feedback from measurements of some sort. In the following we will focus on a mechanistic drift flux model which gives a reasonable compromise between complexity and accuracy, see Table 1. We stress that this model on its own have limited quantitative predictive power, but produces excellent predictions after being calibrated to actual data or a high fidelity model.

#### 3.1 The drift flux model:

The model is developed by expressing the mass conservation law for the gas and the liquid separately, while combining the momentum equation to obtain a third order hyperbolic PDE. In developing the model, we use the following mass variables

\[
m = \alpha_L \rho_L, \quad n = \alpha_G \rho_G
\]

where for \(k = L, G\) denoting liquid or gas, \(\rho_k\) is the phase density, and \(\alpha_k\) the volume fraction satisfying

\[
\alpha_L + \alpha_G = 1.
\]

Further \(v_k\) denotes the velocities, and \(P\) the pressure. All of these variables are functions of time and space. We denote \(t \geq 0\) the time variable, and \(s \in [0, L]\) the space variable, corresponding to a curvilinear abscissa with \(s = 0\) corresponding to the bottom hole and \(s = L\) to the outlet choke position (see Fig. 1). The equations are as follows,

\[
\frac{\partial m}{\partial t} + \frac{\partial mv_L}{\partial s} = 0, \quad \frac{\partial n}{\partial t} + \frac{\partial mv_G}{\partial s} = 0,
\]

\[
\frac{\partial mv_L}{\partial t} + \frac{\partial (mv_L^2 + mv_G^2)}{\partial s} + \frac{\partial P}{\partial s} = -(m+n)g \sin \phi (s) - \frac{2f(m+n)v_m^2v_m}{D}.
\]

In the momentum equation (5), the term \((m+n)g \sin \phi\) represents the gravitational source term, while \(-\frac{2f(m+n)v_m^2v_m}{D}\) accounts for

\(^1\)I.e. the pressure in the fluids in the reservoir surrounding the borehole.
frictional losses. The mixture’s velocity is given as:

\[ v_m = \alpha_G v_G + \alpha_L v_L. \]  

(6)

Along with these distributed equations, algebraic relations are needed to close the system.

**Closure Relations:** Both the liquid and gas phase are assumed compressible. This is required for the model to handle the transition from two-phase to single-phase flow. The densities are thus given as functions of the pressure as follows:

\[ \rho_G = \frac{P}{c_G(T)}, \quad \rho_L = \rho_{L,0} + \frac{P}{c_L^2}, \]  

(7)

where \( c_k \) is the velocity of sound and \( \rho_{L,0} \) is the reference density of the liquid phase at vacuum. Notice that the velocity of sound in the gas phase \( c_G \) depends on the temperature as suggested by the ideal gas law. The temperature profile is assumed to be known and fed into the model.

Combining (7) with (2) we obtain the following relations for finding volume fractions from the mass variables:

\[ \alpha_G = \frac{1}{2} - \frac{\rho_{L,0} c_G^2 n + m + \sqrt{\Delta}}{2 \rho_{L,0}}, \]  

(8)

\[ \Delta = \left( \rho_{L,0} - \frac{\rho_{L,0} c_L^2 n - m}{c_L^2} \right)^2 + 4 \frac{c_G^2}{c_L^2} n \rho_{L,0} \]  

(9)

Then the pressure can be found where we use a modified expression to ensure pressure is defined when the gas vanishes

\[ P = \left\{ \begin{array}{ll}
\left( \frac{m}{\alpha_G c_G} - \rho_{L,0} \right) c_L^2, & \text{if } \alpha_G \leq \alpha_G^* \\
\frac{n}{\alpha_G c_G} c_G, & \text{otherwise.} \end{array} \right. \]  

(10)

Because the momentum equation (5) was written for the gas-liquid mixture, a so-called slip law is needed to empirically relate the velocities of gas and liquid. Traditionally, the Zuber-Findlay [17] slip law is used

\[ v_G = C_0 v_m + v_{\infty} \]  

(11)

where \( C_0 \) and \( v_{\infty} \) are constant parameters for a given flow regime. However, for smooth transition between single and two-phase flow, a relation with state-dependent parameters is needed [18, 19]. More precisely, we use the following slip law

\[ v_G = (K - (K - 1) \alpha_G) v_m + \alpha_L S \]  

(12)

where \( K \geq 1 \) and \( S \geq 0 \) are constant parameters.

### 3.2 Boundary Conditions

Boundary conditions on the downhole boundary are given by the mass-rates of gas and liquid injected from the drilling rig and flowing in from the reservoir. Denoting the cross-sectional flow area by \( A \) the boundary fluxes are given as:

\[ m v_{L,j} = \frac{1}{A}\left( W_{L,\text{res}}(t) + W_{L,\text{inj}}(t) \right), \]  

(13)

\[ m v_{G,j} = \frac{1}{A}\left( W_{G,\text{res}}(t) + W_{G,\text{inj}}(t) \right). \]  

(14)

The injection mass-rates of gas and liquid, \( W_{G,\text{inj}}, W_{L,\text{inj}} \), are specified by the driller and can, within some constraints, be considered as manipulated variables. The inflow from the reservoir is dependent on the pressure at the downhole boundary, usually given by a Vogel-Type Inflow performance relationship (IPR) [20], but within the operational range of a typical UBD operation an affine approximation should suffice, i.e.

\[ W_{L,\text{res}} = k_L \max(P_{\text{res}} - P(0), 0) \]  

(15)

\[ W_{G,\text{res}} = k_G \max(P_{\text{res}} - P(0), 0) \]  

(16)
Here \( P_{res} \) is the reservoir pore pressure and \( k_G, k_L \) are the production index (PI) of the gas and liquid respectively.

The topside boundary condition is given by a choke equation relating topside pressure to mass flow rates

\[
\frac{mv_L}{\sqrt{P_L}} + \frac{mv_G}{\sqrt{P_G}} \bigg|_{s=L} = C_r \left( Z(t) \right) \frac{\sqrt{A}}{\max(P(s=L,t) - P_s,0)},
\]

(17)

where \( C_r \) is the choke opening given by the manipulated variable \( Z, Y \in [0, 1] \) is a gas expansion factor for the gas flow and \( P_s \) is the separator pressure, i.e. the pressure downstream the choke. Changing the choke opening is the primary control actuation for the drilling system.

4 NUMERICAL SCHEME

Model (3)–(17) rewrites as the following nonlinear 3-state hyperbolic system of conservation laws [9]

\[
\frac{\partial q_1}{\partial t} + \frac{\partial f_1(q_1,q_2,q_3)}{\partial s} = 0
\]

(18)

\[
\frac{\partial q_2}{\partial t} + \frac{\partial f_2(q_1,q_2,q_3)}{\partial s} = 0
\]

(19)

\[
\frac{\partial q_3}{\partial t} + \frac{\partial f_3(q_1,q_2,q_3)}{\partial s} = F_W(q_1,q_2,q_3) + F_G(q_1,q_2)
\]

(20)

where \( q = (q_1,q_2,q_3) = (n,m,mv_G + mv_L) \) is the set of conservative variables. Traditionally, explicit numerical schemes are favored for such systems, because they preserve shocks and limit numerical diffusion [6, 21]. However, to ensure their stability, the time and space steps \( \Delta t \) and \( \Delta s \) are required to satisfy, at all times, Courant–Friedrichs–Lewy (CFL) types of conditions, of the form

\[
\left| \lambda_{\text{max}}(\mathbf{u}) \frac{\Delta t}{\Delta s} \right| \leq 1
\]

(21)

where \( \lambda_{\text{max}}(\mathbf{u}) \) is the largest (in absolute value) characteristic velocity of the problem, i.e. the largest eigenvalue of the matrix \( \frac{\partial f}{\partial q}(\mathbf{u}) \). In the case of two-phase flow, the largest eigenvalue, which corresponds to the propagation of pressure waves in the gas, is of the order of 300 m.s\(^{-1}\). In the case of single-phase liquid flow, the order of magnitude jumps to around 1000 m.s\(^{-1}\).

For, e.g., a 3000 meter-long well with 100 space steps, this imposes a time step of the order of \( \Delta t < 0.1 \)s.

For this reason, we choose an unconditionally stable implicit scheme to numerically solve the equations, therefore not subject to CFL conditions. More precisely, consider a time-space grid \( t \in \{0,\Delta t, \ldots\}, s \in \{0,\Delta s, \ldots, (P-1)\Delta s\} \). Denoting \( q(p\Delta s,n\Delta t) = q^n(p) \), we consider the following approximate equations

\[
q^n_{1,2} - q^{n-1}_{1,2} + \frac{f_{1,2}(q^n(p+1)) - f_{1,2}(q^n(p-1))}{2\Delta s} = 0
\]

(22)

\[
q^n_3 - q^{n-1}_3 + \frac{f_3(q^n(p+1)) - f_3(q^n(p-1))}{2\Delta s} = F_W(q^n(p)) + F_G(q^n(p))
\]

(23)

These equations are valid for \( p = 1, \ldots, P-2 \), yielding \( 3 \times (P-2) \) implicit nonlinear equations to be solved at each time step. The boundary conditions (13)–(17) yield 3 more equations, that can be written in the following implicit form

\[
h_{\text{bottom,1}}(q^n(0)) = h_{\text{bottom,2}}(q^n(0)) = h_{\text{top}}(q^n(P)) = 0
\]

(24)

The last 3 equations are given by using a de-centered second-order discretization of the spatial derivative at the boundaries

\[
\frac{\partial f_{1,2}(q^n)}{\partial s}(P-1)\Delta s = \frac{3f_{1,2}(q^n(P-1)) - 4f_{1,2}(q^n(P-2)) + f_{1,2}(q^n(P-3))}{2\Delta s}
\]

(25)

\[
\frac{\partial f_3(q^n)(0)}{\partial s} \approx \frac{-3f_3(q^n(0)) + 4f_3(q^n(1)) - f_3(q^n(2))}{2\Delta s}
\]

(26)

This yields a set of \( 3P \) equations with \( 3P \) unknowns to be solved at each time step. To increase computational speed, the Jacobian of these equations is computed analytically, and a simple Newton algorithm is used to solve them. More precisely, the algorithm takes the following steps

1. At \( t = 0 \), pick a suitable initial condition, e.g. an equilibrium profile.
2. At \( t = n\Delta t \), considering that \( q^{n-1}(p) \) is known for all \( p = 0, \ldots, P-1 \) from the previous iteration, do the following to compute \( q^n(p) \)
   (a) Use \( q^{n-1}(\cdot) \) as the initial guess of \( q^n(\cdot) \).
5 MODEL CALIBRATION

Since the model described in Section 3.1 is of a significant reduced complexity compared to the high fidelity models, certain parameters need to be adjusted in order to quantitatively reproduce the behavior of a given system. This can be done by formulating the fitting of the model response to measured or simulated data as an optimization problem, similar to what is done in [22].

5.1 Calibrated parameters

The number of calibrated parameters may vary according to the considered case, depending on how well known the parameters are in the given scenario. Here we use the the slip law parameters \( K, S \), and the friction factor \( f \) which are empirical parameters. In high fidelity models these are found from sophisticated relations based on flow regime predictions which introduces significant extra complexity to the model. In addition it may be beneficial to tune the choke coefficient \( C_v \) and the gas expansion factor \( Y \) in (17), to ensure good performance of the model as small changes in these parameters can have a large effect on the model dynamics.

The calibration algorithm is a simple optimization routine. Given a set of \( n \) measurements at steady state \( z_i, i = 1, \ldots, n \), and a vector of \( p \) parameters to calibrate \( \theta \in \mathbb{R}^p \), we consider the following minimization problem

\[
\min_{\theta \in \Omega} \begin{bmatrix} z_1(\theta) - z_1 \\ \vdots \\ z_n(\theta) - z_n \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} z_1(\theta) - z_1 \\ \vdots \\ z_n(\theta) - z_n \end{bmatrix}
\]  

where \( z_i(\theta) \) is the output of the model corresponding to the measurement \( z_i \), for the vector of parameters \( \theta \), \( M \) is a weighting matrix used to normalize the outputs, and \( \Omega \) is the set of allowable values which \( \theta \) can take.

The vector of parameters, \( \theta \), which satisfies (29) is found using a gradient based optimization procedure in MATLAB. The result of using this procedure on a data set generated with OLGA can be seen in Fig. 2. In this example only BHCP and WHP from each steady state were used to tune the four variables \( \theta = [K, S, f, C_v, Y]^T \). The resulting calibration of the simple model has a good performance in the operating range covered by this dataset, but may deteriorate when the system moves to a distinctly different operating regime.

6 APPLICATION OF MODEL TO UBD

In this section, to be more clear, \( P(0) \) will be referred to as Bottom-Hole Circulating Pressure (BHCP) and \( P(L) \) as Well-Head Pressure (WHP). The current approach to controlling pressure in UBD is manual control of the outlet choke. Often, operators monitor the WHP and try to stabilize it around a constant
value. Their a priori knowledge of potential steady-state operating points is given by graphs similar to Fig. 3, taken from [23]. These graphs determine, for a constant WHP, the correspondence between BHCP and gas flow rate, and determine a set of operating constraints referred to as the operating envelope [24].

We now perform a steady-state analysis of the model. The key difference of this approach compared with state-of-the-art steady-state analyses software such as the one used in [23, 24], is to include the choke model, thus “classifying” operating points in terms of choke opening rather than WHP. This allows us to identify a broader variety of operating points, as well as clearly defining control challenges.

6.1 Classifying operating points

To understand the need for a characterization of steady-states, consider the simulation depicted in Fig. 4, corresponding to a UBD operation in a dry gas well (i.e. \( W_L,_{res} = 0 \)). The choke opening is decremented after 3, 6 and 9 hours, leaving enough time between each decrement for the system to reach equilibrium. Between \( t = 0 \) and \( t = 6h \), the system exhibits an intuitive behavior: both BHCP and WHP increase as the choke opening is decreased. However, during the next step, between \( t = 6h \) and \( t = 9h \), the WHP increases before settling at a lower value than the previous equilibrium. The counter-intuitive notion that lowering the choke opening can lower the equilibrium WHP complicates in itself the manual control of the operation. This difficulty is increased by the inverse response of the WHP, which seems to indicate the presence of unstable zeros in the transfer function between this output and the choke opening. When the choke opening is decreased further after \( t = 9h \), the system drifts towards overbalanced conditions. We now use the model described in Section 3.1 to get insight into these mechanisms.

2The reason for this is that WHP determines the flow rates through the choke, thus the amount of gas and liquid that the separator has to handle at once. This is one of the main constraints in UBD.

6.2 Steady-state analysis

At steady states of the system (3)-(12), the mass-rates are constant throughout the well, while the pressure gradient is given by the three terms: acceleration due to the expanding gas, friction and hydrostatic pressure. Given a value of the BHCP, integrating these equations yields the corresponding equilibrium value of the WHP, as depicted by the solid line in Fig 5. However, this curve does not take into account the choke model on the topside boundary \( s = L \). Indeed, to each value of the BHCP corresponds unique values of the inflow rates of gas and liquid, through Equations (13)-(17). Only the WHPs which yield a total out flow rate (given by (17)) equal to the sum of the in flow rates can be steady-states. This is represented by the dotted lines in Fig. 5, which, for various values of the choke opening, are the WHP required for the in flow and out flow rates to match. The intersections of the solid and each dotted line is the steady-state equilibrium for the choke opening corresponding to the dotted line.

6.2.1 Steady-state taxonomy

Depending on the operating point, the system exhibits significantly different dynamics. Specifically, we identify the four distinct operating regimes of the UBD system, ordered by increasing equilibrium BHCP, and illustrated on Fig. 6.

Intuitive regime This regime corresponds to the first 6 hours of the transient simulation shown in Fig. 4. It is stable and minimum phase.

Non-intuitive regime A small increase in BHCP yields a lower influx of gas into the system, which increases the hydrostatic pressure and eventually decreases the WHP. This regime is stable for a constant choke opening but with a inverse re-
FIGURE 5. Alternative operating envelope showing operating points for different choke openings.

FIGURE 6. Using the choke as actuation, the system can have 4 distinct operating regimes.

Unstable regime
For larger values of the equilibrium BHCP, there is no stable equilibrium. The system will then either enter a limit cycle of severe slugging, or, drift towards the steady-state in overbalanced conditions.

Stable, overbalanced regime
The system only contains the liquid phase and the difference; BHCP-WHP, is constant.

6.3 Control Envelope
The behaviour of the system w.r.t. to changing choke opening can perhaps more clearly be seen in Fig. 7. In this so-called control envelope the steady state values of BHCP and WHP (i.e. the green and red dots in Fig. 5) is plotted over a range of choke openings given on the x-axis.

Consider a well initially in the overbalanced regime. Closing the choke will cause the system to move along the red line until a choke opening of 0.38. This corresponds to a choke opening where the red dotted line in Fig. 5 is below the WHP minima occurring at the transition to underbalanced conditions. The system will then move to the stable steady states given by the blue curve in Fig. 7. Reducing choke opening when the system is in this state will make the system move along the blue curve. The end of the blue curve, moving towards left, in Fig 7, is the limit to the unstable regime. Closing the choke past this point will either cause the system to go to the overbalanced regime (as in the simulation shown in Fig. 4) or enter a severe slugging limit cycle as shown in Fig. 8.

6.4 Slugging
By a slight modification to the parameters used to obtain the simulation shown in Fig 4 (increasing $P_{res}$ and decreasing $Y$), the system becomes capable of severe slugging. Specifically, instead of moving to the overbalance steady-state when the choke is closed sufficiently, it enters a slugging limit-cycle (see Fig.8 and 9). We note that the limit-cycle takes the system into the overbalanced regime for a brief period, which requires the model to be capable to handle the transition to liquid only flow.

The possible occurrence and control of severe slugging in production risers have been studied extensively (see e.g.\cite{25--27}), but slugging in underbalanced drilling operations have not yet gotten significant attention. Due to the different boundary conditions the causes for slugging in production risers, such as density waves and casing heading, cannot necessarily explain slugging during UBD. Hence, the causes for severe slugging in UBD is not yet well understood. To move forwards toward increased automation of UBD this must be amended, and as such, the investigation of the causes for severe slugging in UBD was major motivation for the implementation of the current model.

7 FURTHER CONTROL AND ESTIMATION PROBLEMS IN UBD

This section contains suggestions for how applied control theory can make an impact in UBD.
FIGURE 7. Control envelope showing steady state points plotted against choke opening. The system shows a hysteresis-like behaviour in that it will converge to different steady-states depending on whether the system is over- or under-balanced.

7.1 Estimation of PI and pore pressure

Estimation of the Production Index (PI) and pore pressure amounts to identifying the amount of produced fluids and relating it to the measured BHCP as by (13)-(14). In typical UBD operation available real-time measurements of significance are BHCP and WHP. Additionally, measurements of the amount of produced gas is available after it has been separated from the drilling fluid with a delay of 2 - 12 hours. When drilling, the PI and pore pressure will remain more or less constant until a fracture in the reservoir is encountered, at which time \( k_G \), \( k_L \) will increase. Rapid estimation of the PI using the immediately available measurements of BHCP and WHP has considerable benefits as it enables the drillers to optimize the process on-line as well as pinpointing the exact position of the fracture based on the position of the drilling bit when the fracture was encountered [28].

7.2 Connection scenario

For every 30 meters drilled, the drill string has to be extended in a procedure called a drill string connection. During this operation the main pump is shut down, stopping the circulation of drilling fluid. This causes a separation between the phases as
the gas travels to the topmost part of the well when gravity pulls the liquid down to the bottom. The gas expands as it travels upwards increasing the pressure in the well. During this transient process, it is desirable to keep the BHCP at a specified set-point towards increasing the pressure in the well. During this transient the liquid down to the bottom. The gas expands as it travels up-the gas travels to the topmost part of the well when gravity pulls

7.3 Extending the Operating Envelope the Unstable Regime

Often during UBD operations, produced gas is flared after separation from the drilling fluid. In these cases it can be desirable to minimize the amount of gas produced due to economic or environmental considerations of the gas flaring. As discussed in Section 6.2 and shown in Fig. 7, the operating points corresponding to the least amount of produced gas (i.e. closest to the reservoir pressure) are in the unstable regime. Hence, these potential operating points are currently not part of the operating envelope and drilling currently does not occur in this regime. Using feedback control, however, these steady-states can be stabilized thereby extending the operating envelope, potentially enabling more cost effective and environmentally friendly operations.

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References
